Self-Diagramming Lace

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Abstract

For the past year or so, I have been exploring frieze symmetries in knitted lace through artworks, talks, and my most recent Bridges paper. As part of this endeavor, I have produced frieze symmetries in my knitting and diagrammed the symmetries in my expository slides. Here, I describe my latest work, which incorporates the symmetry markings from the diagrams into the knitting itself with colored beads.

Symmetries in Knitted Lace

As described in my previous Bridges paper on mathematical fiber arts [3], a *complete symmetry sampler* is an artwork composed of a collection of designs, most often incorporated into a single object, with exactly one design for each type of symmetry in a particular class (e.g., wallpaper groups or frieze groups) that is attainable in the given craft. In addition to pieces by various artists in temari, counted cross stitch, bead crochet, two-color knitting, and blackwork embroidery, this earlier paper included one of my series of knitted lace pendants displaying all seven frieze groups. Figure 1 shows my fourth and, to date, final entry in the original series, *Linear Lace in Burgundy*. The lace panels are in the style of traditional Estonian lace [1], which includes a distinctive stitch called a *nupp* (which rhymes with "soup") that forms a decorative cluster of yarn.

Apart from making nupps, the knitter forms the lace design by incorporating two special types of stitches into the fabric: *increases* and *decreases*, so named because an increase adds a stitch that does not correspond to a stitch in the previous row and a decrease combines multiple stitches into one, effectively subtracting stitches from the previous row. The most efficient way to convey the arrangement of stitches for a particular lace design is through a knitting chart[1] like those in Figure 2. The charts here correspond to the top center and top right designs in the pendant in Figure 1, and do not include the thicker, colored lines and points, whose significance will be explained below. The increases, in this case (and most commonly in lace knitting) *yarn overs*, are denoted by open circles, reflecting the fact that each produces a small opening in the knitted fabric. The decreases are denoted by the slanted and forking line segments, which tell the knitter how many stitches to combine and which stitch to place in front of the other(s). A nice feature of these charts is that they roughly approximate the appearance of the actual lace fabric.

Because lace charts convey an approximation of the knitted fabric itself, they lend themselves to geometric diagrams that explicate the symmetries of the lace. In Figure 2, which is adapted from a slide in one of my talks, the different symmetries of each frieze pattern are marked with colored lines and dots. The red lines show reflection axes (the significance of solid and dotted lines will soon be clear), the yellow line shows a glide reflection axis, the dark blue dots show centers of rotation, and the light blue arrows indicate the shortest translations.

[1] These charts, which I actually used in knitting *Linear Lace in Burgundy*, are simpler than conventional Estonian lace charts in that they depict a nupp with a light grey dot in a single stitch square. In fact, nupps are formed over multiple rows and are usually diagrammed to reflect this, which makes the usual cluster of symbols less reflective of the shape and symmetry of the actual nupp.

Figure 1: *Linear Lace in Burgundy, from the JMM 2018 art exhibition.*

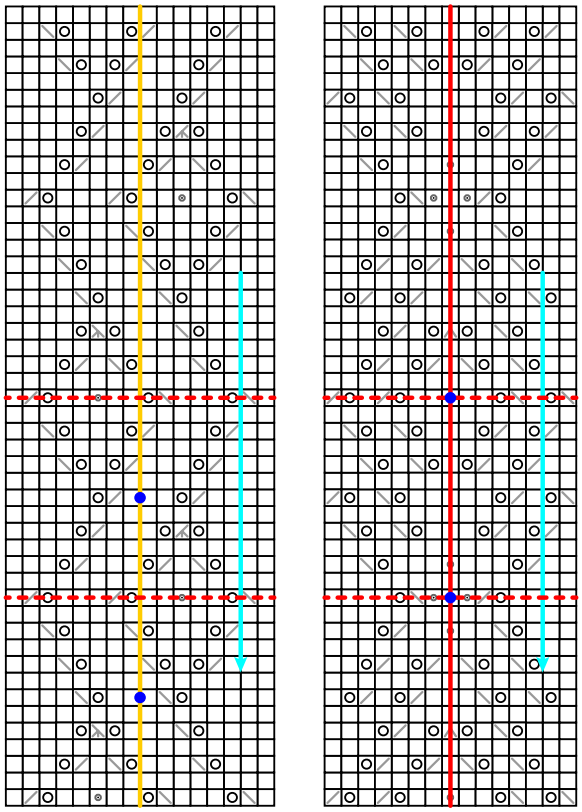

Figure 2: *Knitting charts for the top center and top right patterns in Linear Lace in Burgundy with the symmetries marked. The red lines are reflection axes, the yellow line is a glide reflection axis, the dark blue dots are centers of rotation, and the light blue arrows show the translation symmetries*

Alert readers may have already noticed that while the reflection through the vertical solid red line preserves the chart on the right in Figure 2, the reflections through the horizontal dotted red lines do not preserve either chart. More precisely, the yarn overs and nupps[2] retain their positions when reflected across a marked horizontal axis, but the decreases do not. The reason why these reflections seem to preserve the lace itself is that both plain knit stitches (the squares without symbols in the chart) and decreases create solid fabric. Unless you inspect the lace very closely (so, in a higher resolution than in Figure 1), all you register is where there is yarn, where there is an opening, and where the nupps protrude from the fabric. Note that the same holds for the rotations of the charts; although the rotations change the positions of the decreases, they preserve the positions of the yarn overs and nupps.

[2] Again, this is because of the simplified nupp markings in these charts.

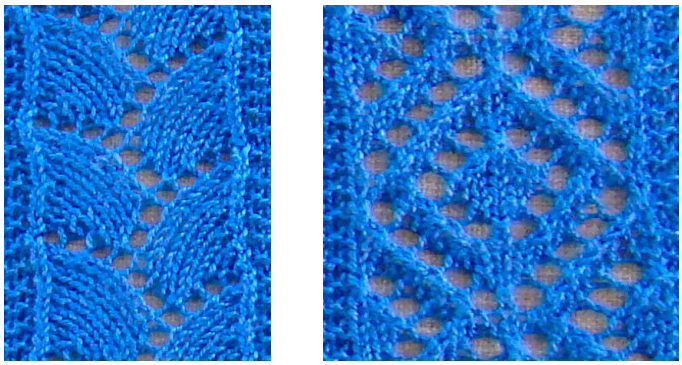

Figure 3: *Details of the shawl* Linear Lace *[2]. The design on the left has only symmetries that preserve the shapes and arrangements of the stitches, but the design on the right has symmetries that do not.*

To give a better view of what this looks like in the fabric itself, Figure 3 shows two details of a prototype from my shawl pattern, *Linear Lace* [2], in which the stitch detail is clearer. Like *Linear Lace in Burgundy*, the shawl has its own set of seven lace designs (although without nupps) that correspond to the seven frieze groups. In the design on the left, which has only translations and glide reflections along a vertical axis, all of the symmetries preserve both the shape of the individual stitches and their arrangement in the fabric. In the design on the right, the situation is complicated by the asymmetry of knit stitches—each individual stitch is shaped like a small V—and by the asymmetric placement of the decreases, which form the slanting lines of stitches just below the yarn overs. Here, the reflection through the vertical axis still preserves the finer stitch details (as do the translations that this close up is too small to show), but the horizontal reflection and the rotation about the center turn all the stitch V's upside down and change the positions of the slanted decreases.

Marked Symmetries in Knitted Lace

Having tweaked my lace frieze pendant concept three times to get to *Linear Lace in Burgundy*, I thought that this artistic vein was finally tapped out. But at the beginning of this year, the thought occurred to me: what if I used beads (a common embellishment in lace knitting) to superimpose the symmetry diagrams onto the lace itself?

My first crack at lace that contains its own symmetry diagrams was *Coded Symmetries*, shown in Figure 4. As a comparison to Figure 1 makes clear, this pendant was adapted directly from *Linear Lace in Burgundy*. In fact, the only substantive change to the underlying lace was the removal of the nupps to avoid distracting from the beads. The symmetry axes and centers are marked exactly as in the Figure 2 charts, with reflection axes in red, glide reflection axes in orange, and centers of rotation in blue. The teal beads on the edges of the designs mark the translations; in all of the designs except the left design in the second row and the design in the bottom fringe, these markers are duplicated so that each of the seven frieze groups preserves both the lace and the beads.

While I was quite pleased with *Coded Symmetries*, it suffers a bit from its design process: the lace came first, and the beads were an afterthought. This meant that some of the bead placements were made inconvenient by the lace, and some of the symmetry markings seemed tacked on after the fact. I redesigned the lace around the beads to get *The Symmetries Diagram Themselves* (Figure 5). Here, the reflection axes are white, the glide reflection axes are yellow, the centers of rotation are bronze (and slightly larger than the rest of the beads), and the translation markings are red.

Figure 4: *Coded Symmetries, 2018.*

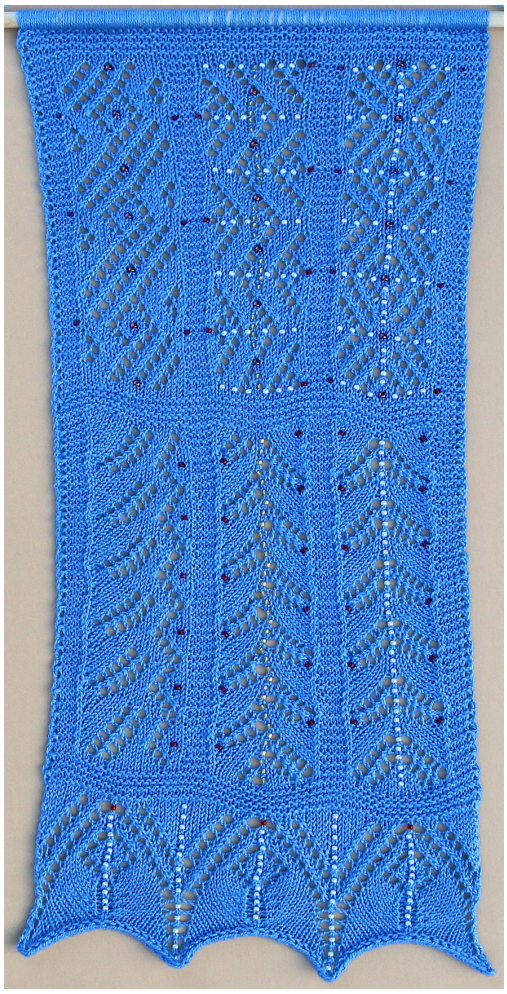

Figure 5: *The Symmetries Diagram Themselves, 2018.*

It was a delightful surprise to find that an idea I thought had run its course has life left in it. Going forward, I hope to explore how to clearly mark the distinction between symmetries that do and do not preserve the stitches, and to try using beads to highlight fundamental regions. I like to think that if people who had not seen symmetry samplers before were to encounter *Coded Symmetries*, *The Symmetries Diagram Themselves*, and future self-diagramming lace projects, they would have a reasonable chance of deciphering what the beaded markings mean on their own. And isn't it the goal of any artist to have her art speak for itself?

References

[1] N. Bush. *Knitted Lace of Estonia*. Interweave Press LLC, 2008.

[2] S. Goldstine. "Linear Lace" knitting pattern. Ravelry, 2016. https://www.ravelry.com/patterns/library/linear-lace-2.

[3] S. Goldstine. "A Survey of Symmetry Samplers." *Bridges Conference Proceedings,* Waterloo, Canada, July 27—31, 2017, pp. 103—110. http://archive.bridgesmathart.org/2017/bridges2017-103.pdf.