Compound Parallelohedra Building Blocks with Creature-Like Morphologies

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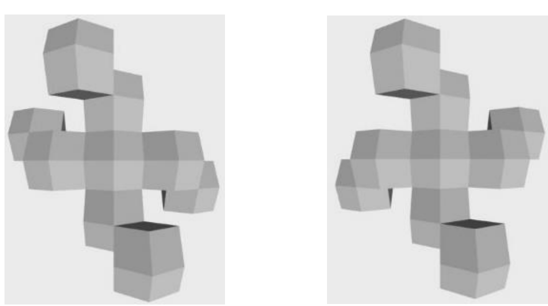
Abstract

We present three types of building blocks using rhombic dodecahedra and truncated octahedra, which are geometric with some symmetry yet have creature-like morphologies. Basic connections and constructions with a set of the blocks are explored and confirmed using 3D models and prototypes realized with wood and paper.

Introduction

Tiling and packing have been studied in fields such as mathematics, crystallography, material science, design, and architecture. They have been also applied to industrial products, art and design pieces, and educational materials in the related fields. One of the acclaimed usage of tiling in art was done by M.C. Escher for his engraving art pieces, which have broad influence to art lovers, artists, scientists, and mathematicians among others. Besides 2-dimensional pieces, some creators and researchers have extended his idea to 3-dimensional pieces. For example, the authors in [4] designed and fabricated a pair of space-filling 3-dimensional blocks, "elephants" and "frilled lizards", by using a set of icosahedra as primitives. The authors in [3] created a 3-dimensional space-filling block that looks like a tree as a teaching material for crystallographic symmetry. There is another line of development of new building blocks for creating various constructions, such as LEGO bricks, building blocks by the toy company Naef, SL blocks [2], and the triskelion blocks presented at Bridges 2017 [1].

In this paper, we make full use of the sliding property of space-filling parallelohedra and present three types of building blocks which we call Rhombic Dragons and Wild Octahedra in Figures 1 and 2 that use rhombic dodecahedra and truncated octahedra as primitive parts. All of the blocks have simplicity with some symmetry, have extendability of connecting blocks in various directions, and also have creature-like appearances with four crooked extremities, which evoke us a body and the limbs of some animals. These properties make the constructive play more feasible and image-provoking. In the sequel, we first show the 3D models of the blocks and their basic connections and constructions. Then, we show the prototypes of the blocks made of wood and paper and we also demonstrate the actual connections and constructions. Here we note that wooden blocks are rigid while blocks made of paper are soft and can be slightly twisted.



(a) (b)

Figure 1: *Rhombic Dragons: (a) block A, (b) block B.*

Figure 2: *Wild Octahedra.*

Two Types of Building Blocks Using Rhombic Dodecahedra

A rhombic dodecahedron is a parallelohedron that consists of 12 rhombi with diagonals of ratio $\sqrt{2}$: 1 as shown in Figure 3. It has squares and hexagons as orthogonally projected images. Here we present two building blocks both using nine rhombic dodecahedra. The central five rhombic dodecahedra among the nine are shown in Figure 4 on the *xy*-plane. The centers of mass of these five dodecahedra are located at the *xy*-plane with $z = 0$ with the coordinates O (the origin), $(\pm 1, 0, 0)$, and $(0, \pm 1, 0)$. The remaining four rhombic dodecahedra are attached at the four ends as shown in Figure 1(a) and (b), where the coordinates of their centers of mass are $(\pm 1/2, \mp 3/2, \sqrt{2}/2)$ and $(\pm 3/2, \mp 1/2, -\sqrt{2}/2)$ for (a) and $(\pm 1/2, \mp 3/2, \sqrt{2}/2)$ and $(\pm 3/2, \pm 1/2, -\sqrt{2}/2)$ for (b) with double signs in the same order. Since the appearances of the blocks are just like (Komodo) Dragons, we call them *Rhombic Dragons A* and *B* (DA and DB in abbreviated forms). Geometrically, Rhombic Dragons have the following symmetries: DA has 2-fold rotational symmetry along the three orthogonal axes $y = \pm x$, $z = 0$ and the *z*-axis. DB has 2-fold rotational symmetry only along the *z*-axis and is changed to the mirror image when rotated for 180 degrees along axes $y = \pm x$, $z = 0$. We note that these blocks are not space-filling but can be connected and attached in various ways using the symmetry and parallelism they have in their structure.

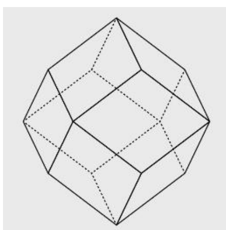


Figure 3: *Rhombic dodecahedron.*

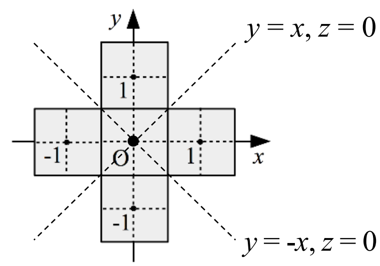
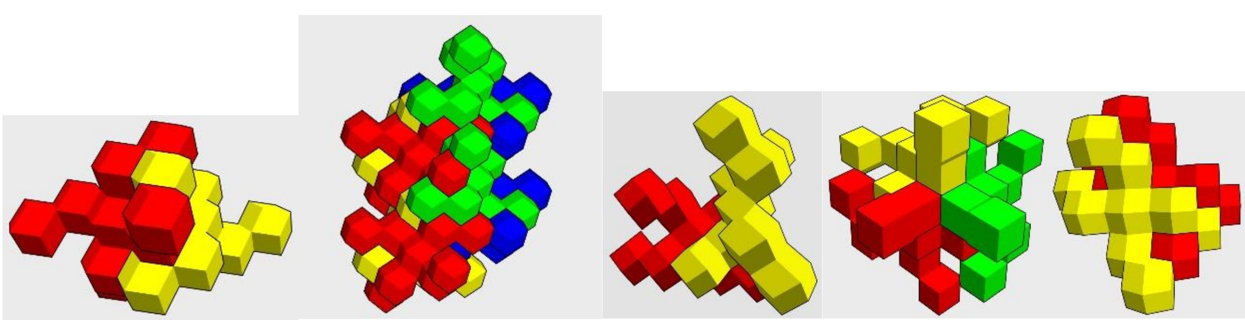


Figure 4: *Five rhombic dodecahedra with centers on xy-plane.*

To describe how the building blocks are connected, we first fix a DA (DA1) with the coordinates used in Figure 4. Locations of the other blocks are described by the following notation that uses transformations from DA's that are formerly defined. Now *tx*2, for example, means a translation by 2 units (of rhombic dodecahedra) in the *x*-direction; *tz*-0.5 means a downward translation (in the negative *z*-direction) by 0.5 unit; and $r(y = x, z = 0)$-90 means a rotation of 90 degrees clockwise around the line $y = x$, $z = 0$. We denote $r(y = z = 0)90$, that is, a rotation of 90 degrees counterclockwise around the *x*-axis, by *rx*90 in short.

Now, the location of the second block in Figure 5(a) is described by DA2 = DA1($r(y = x, z = 0)$-90, *ty*1, *tz*-1) and the connection is described by DA1 + DA2. In Figure 5(b), DA2 is the same with the one in Figure 5(a). Then, DA3 is determined from DA2 in the same way as DA2 is determined from DA1 by applying the transformation of the same functionality. This construction continues recursively and forms a spiral having 4-fold symmetry, where 8 DA's are used in two cycles in (b). In Figure 5(c), DA2 = DA1($r(y = -x, z = 0)90$, *rz*90, *tx*1, *ty*-0.5, *tz*1). In Figure 5(d), DA2 = DA1($r(y = x, z = 0)$-90, *rz*90, *tx*-1.5, *ty*-0.5, *tz*-0.5). DA3 is then determined from DA2 in the same way. This construction has 3-fold rotational symmetry.



(a) (b) (c) (d) (e)

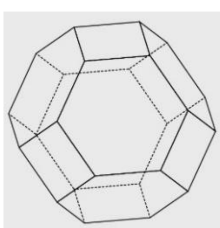
Figure 5: *Basic connections and constructions of Rhombic Dragons A.*

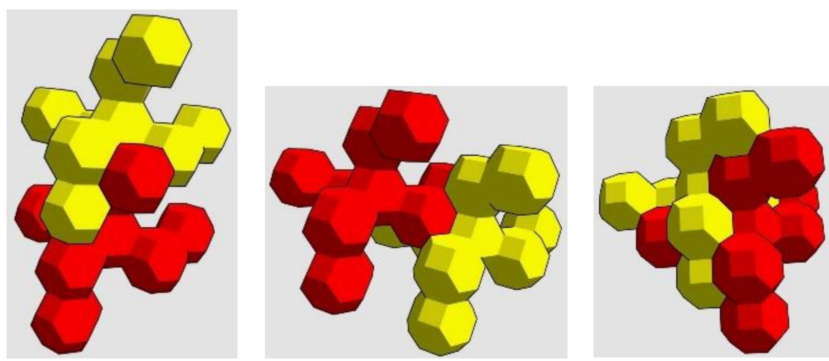
In Figure 5(e), DA2 = DA1(*tx*-0.5, *ty*-0.5, *tz*0.5). DA1 and DA2 contact with each other the most. We note that the first four constructions (Figure 5(a)-(d)) are realized with rigid building blocks while the last one (Figure 5(e)) can only be achieved with blocks made from flexible material such as paper or rubber. We also note that when the blocks are fabricated with reasonable accuracy, connections of (a), (b), and (e) are tight that even if they are picked up by one part, the other parts do not easily fall away. On the other hand, the blocks in (c) and (d) are only closely attached at their facets and are easily fallen apart.

DB's also have various connections and constructions. Here we describe two of them. Again, we fix DB1 to the coordinates in Figure 3. In the first connection, we set DB2 = DB1(*rx*180, *tx*1, *ty*2). Then, DB1 + DB2 is realized using rigid building blocks and is very tight. In the second connection, we set DB2 = DB1(*tx*-0.5, *ty*-0.5, *tz*0.5). Then, similarly to the connection in Figure 5(e), DB1 and DB2 contact with each other the most and can only be connected using blocks made from flexible materials.

A Building Block Using Truncated Octahedra

Figure 6 shows a truncated octahedron, which is also a parallelohedron. We first attach four truncated octahedra to the central truncated octahedron at the four non-neighboring regular hexagons. Then four more truncated octahedra are attached at the four ends of the block as shown in Figure 2, resulting in nine truncated octahedral as primitive parts. Though the appearance of the block seems not like an existing creature, it gives dynamic and lively impression so we call it *Wild Octahedra*. It has 2-fold rotational symmetry along the axis passing the centers of two antipodal squares of the central truncated octahedron. The three types of basic connections using Wild Octahedra are shown in Figure 7. Similarly to Rhombic Dragons, the first two connections of Wild Octahedra in Figure 7(a) and (b) are realized with rigid blocks and the connection in Figure 7(c) can only be realized with blocks made from flexible materials.





(a) (b) (c)

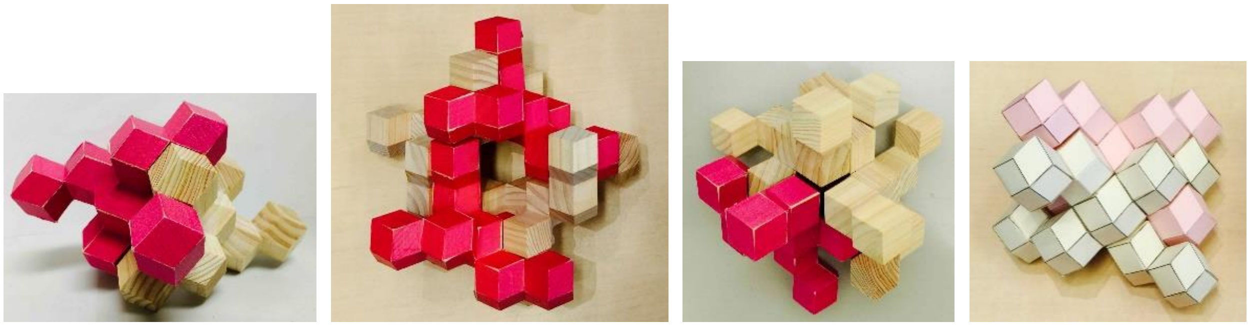
Figure 6: *Truncated octahedron.*

Figure 7: *Three types of basic connections of Wild Octahedra.*

Prototyping of Blocks Made of Wood and Paper

We first demonstrate the actual connections and constructions of Rhombic Dragons A that correspond to those in Figure 5. Connections in Figure 8(a) to (c) are realized with blocks made of wood and (d) with blocks made of paper. (a) shows the most fundamental connection. (b) shows the connections of four blocks in a spiral form having a cylindrical hole in the center. The connections from Figure 8(a) to (c) can be also realized using materials such as ABS and nylon resin. The connection in Figure 8(d) is realized with paper this time but soft rubber is also an appropriate alternative.

For Wild Octahedra, four types of connections including the three in Figure 7 are demonstrated using blocks of wood and paper. Similarly to the case of Rhombic Dragons, the tight connection in Figure 9(d) is possible only with soft paper or rubber.



(a) (b) (c) (d)

Figure 8: *Connections of Rhombic Dragons A: (a) to (c) made of wood, (d) made of paper.*

(a) (b) (c) (d)

Figure 9: *Connections of Wild Octahedra: (a) to (c) made of wood, (d) made of paper.*

Summary and Conclusions

We presented three types of building blocks using rhombic dodecahedra and truncated octahedra. We have fabricated the blocks using wood and paper and confirmed that a variety of connections and constructions are possible using these blocks. Here we note that possible connections can change according to the material of the blocks. We plan to print the blocks using nylon resin and rubber to further investigate the usability and playability of the blocks.

Acknowledgements

This work was supported by JSPS Grant-in-Aid for Scientific Research(C) 16K00507. The author would also like to thank Hiroki Tone for his support in 3D modeling.

References

[1] A. Matsuura and H. Shirane. “Triskelion Block Families.” *Bridges Conference Proceedings*, Waterloo, Canada, July 27–31, 2017, pp. 371–374.

[2] S. G. Shih. “On the Hierarchical Construction of SL Blocks – A Generative System that Builds Self-Interlocking Structures.” S. Adriaenssens, F. Gramazio, M. Kohler, A. Menges, and M. Pauly Eds. *Advances in Architectural Geometry*, 2016, pp. 124–137.

[3] Y. Teshima, Y. Watanabe, Y. Ikegami, K. Yamazawa, S. Nishio, and T. Matsumoto. “Development of Teaching Materials to Learn Crystallographic Symmetry.” *Acta Cryst.*, 2014, A70, C1280.

[4] Y. Watanabe, Y. Ikegami, K. Yamazawa, and Y. Murakami. “World of Scientific Puzzle Art Using Layer Manufacturing.” *Forma*, vol. 21, no. 1, 2006, pp. 37–48.