Folding Space-Filling Bisymmetric Hendecahedron for a Large-Scale Art Installation

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Abstract

This article discusses the bisymmetric hendecahedron, a space-filling polyhedron that was used to create a large-scale art installation at the site of the famous North Christian Church by Eero Saarinen in Columbus, Indiana. The article focuses on the geometric construction of the bisymmetric hendecahedron as well as its transformation into a large-scale art installation. The focus is on the artistic design, the material construction, and the assembly techniques.

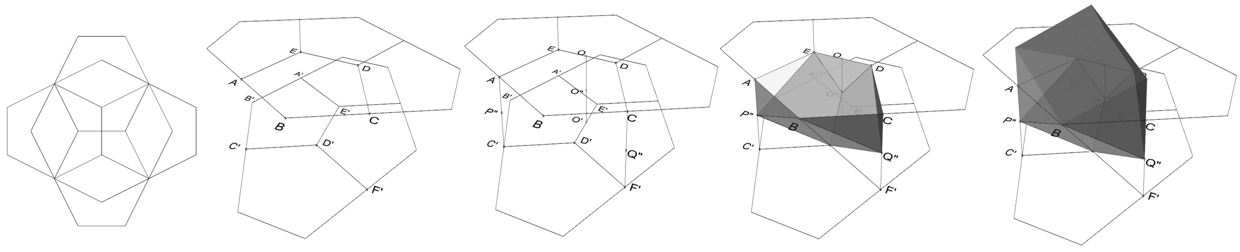
Introduction

In geometry space-filling tiling of polyhedrons form honeycombs that can be found in naturally occurring structures such as living cells, crystals, beehives, etc. Nature's strategies in filling space in this manner have inspired developments in material science, chemistry, biology and in architecture [8]. Buckminster Fuller's octet truss [6] was a structure resulting from the regular tiling of tetrahedrons and octahedrons in space. The structure in an octet truss is related to the densest possible arrangement of equal-sized non-overlapping spheres in space, which Fuller called a vector equilibrium configuration, or the shape of spaces [3]. The largest architecture to apply octet truss was the Festival Plaza built for Osaka Expo in 1970 [5]. While many space-filling polyhedrons have been found since ancient times, the search for interesting and distantly rich tiling of space still baffles mathematical minds, as such findings have implications for new material and new architecture and design developments and could lead to advances in communication systems and computer security[1, 7]. Perhaps one such interesting space-filling polyhedron is the bisymmetric hendecahedron that one of us found in 1996 [4]. This article discusses the bisymmetric hendecahedron and its transformation into a large art installation that was completed in 2017. The focus is on the artistic design, the material construction, and the assembly techniques.

Space-Filling Bisymmetric Hendecahedron

A hendecahedron is a polyhedron with eleven faces (the Greek word for *eleven* is *héndeka*), seven of them are quadrilaterals and four of them are triangles. It has two planes of symmetry; hence it is called a bisymmetric hendecahedron. The space-filling bisymmetric hendecahedron can be constructed as shown in Figure 1:

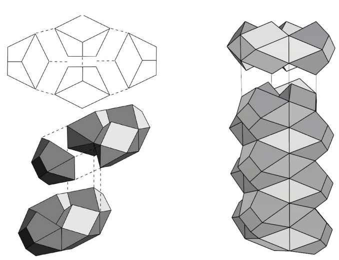
1. Construct two perpendicular hexagonal units, each are tessellated with four pentagons. Move one unit at an arbitrary distance above another unit (Figure 2a).
2. Let point A, B, C, D, and E be the vertices of one of the pentagons on the top, and let A′, B′, C′, D′, and E′ be the vertices of one of the pentagons on the bottom that is rotated 90° clockwise (Figure 2b).
3. Let point O be the mid-point of line DE and let O′ to be the mid-point of the line D′E′. Let points O″, P″, and Q″ be the mid-points of lines OO′, AC′, and CF′ respectively (Figure 2c).
4. Join the vertices between D′ and O″, D′ and P″, D′ and Q″, B and P″, B and Q″, O″ and E, and O″ and D to produce a half unit of the bisymmetric hendecahedron (Figure 2d).
5. Mirror the half unit to produce a full unit of the bisymmetric hendecahedron (Figure 2e).



(a) (b) (c) (d) (e)

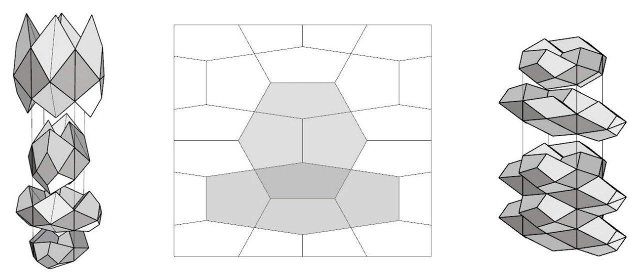
Figure 1: *Construction of a bisymmetric hendecahedron: (a) Two perpendicular hexagonal unit, (b) vertices of the pentagons, (c) midpoints, (d) half unit, (e) full unit*

Figures 2a and 2b shows how four identical hendecahedrons can form a hexagonal boat-like shape that can be stacked to fill space. The hexagonal boat thus can be understood as a translation unit that can be packed in a lattice without any rotation or reflection. It is important to note that the bisymmetric hendecahedron can have unlimited variations while maintaining its space-filling quality. In Figure 1, line AC′, line CF′, and line OO′ have an equal length and the length can be set arbitrarily; the height of a bisymmetric hendecahedron can be set arbitrarily and can still be space-filling (Figure 3a). Furthermore, the Cairo pentagonal tessellation can also be distorted, resulting in the space tessellation of two distinct bisymmetric hendecahedrons of different proportions (Figure 3b and 3c). Such variations allow for the artistic elaboration of a space-filling structure. While the bisymmetric hendecahedron can be easily reproduced as a small-scale paper model and as a 3D printed structure, transforming the small scale model to a full-scale outdoor architecture installation is not trivial.



(a) (b)

Figure 2: *Periodic bisymmetric hendecahedron (a) four identical units connected to form a hexagonal boat-like shape, (b) stacked to fill the space.*

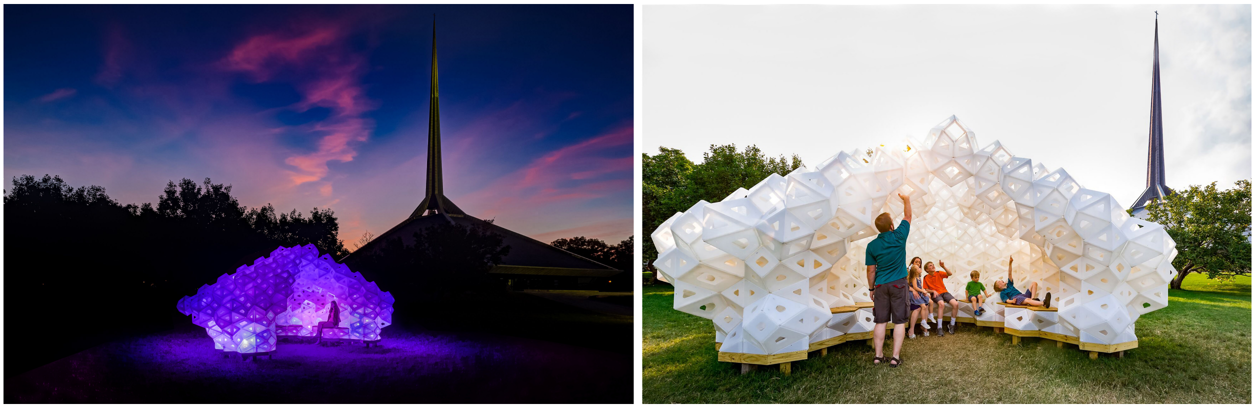


(a) (b) (c)

Figure 3: *Aperiodic bisymmetric hendecahedron: (a) stacked using units of different heights, (b) distorted Cairo pentagonal tessellation, (c) stacked using two units of different proportions.*

Synergia Installation in Columbus, Indiana

Synergia is part of a bi-annual design exhibition in Columbus, Indiana (Figure 4). The site is on the grounds of the North Christian Church, designed by Eero Saarinen between 1959 and 1961. One of the main purposes of the Synergia installation was to pay tribute to Eero Saarinen, a well-known modernist architect who was very keen on understanding the relationship between mathematics and architecture. In the North Christian Church, Saarinen used many geometric shapes – triangle, square, cross, hexagon, and octagon – in both the plans and the sections of the building [2]. For instance, the main level of the church features an elongated hexagon that can be seen on both the inside of the sanctuary floor and outside in the berm-framed moats, the stepped concrete, and the sloped roof. The hexagonal form found in the space-filling bisymmetric hendecahedron therefore was seen as central to the Synergia installation. In Synergia, hundreds of identical units of bisymmetric hendecahedrons were connected and stacked in a total of ten layers to create the final structure that not only echoed Saarinen's hexagonal building (Figure 5), but also provided a quiet and light-filled space for people to enter and to sit and relax.



(a) (b)

Figure 4: *Synergia at night; the building on the right is the North Christian Church by Eero Saarinen. (a) night view, (b) day view. Photo Courtesy of Tony Vasquez.*

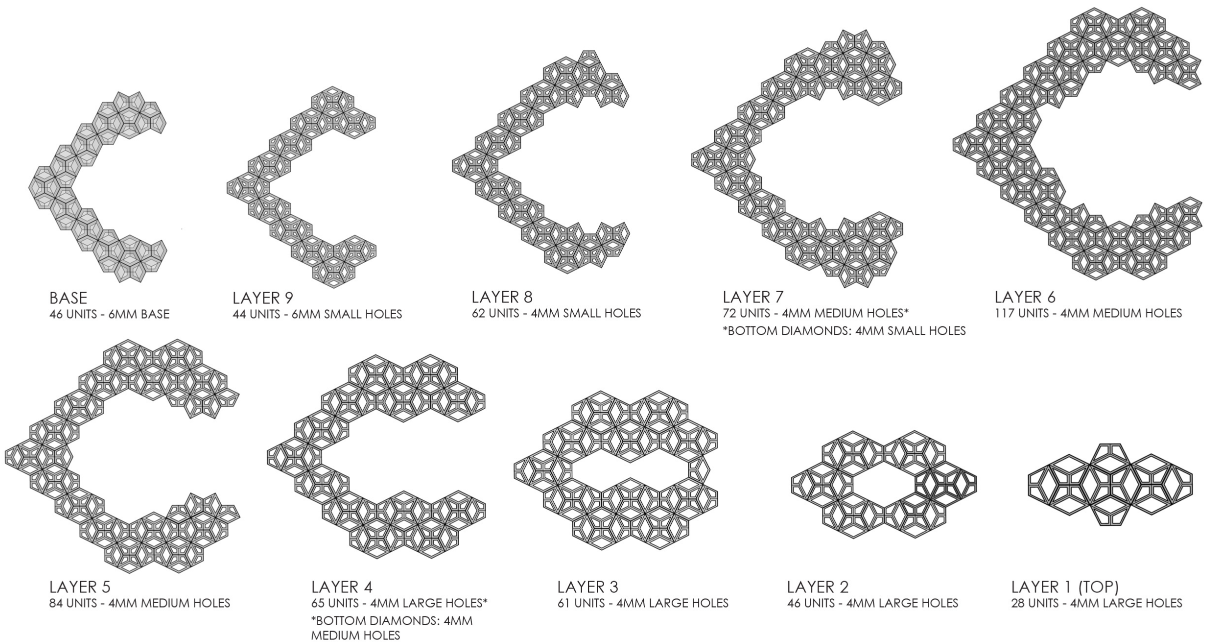
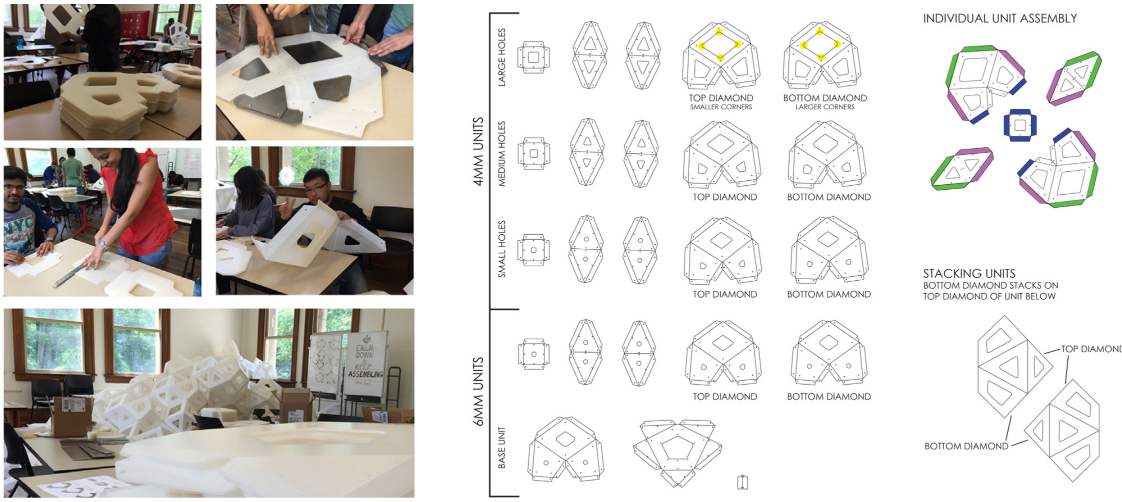


Figure 5: *The layer assembly diagram used in Synergia installation.*

Synergia was intended as a faculty led student project. Therefore, all the fabrication and assembly needed to be accomplished in design studios at Indiana University Bloomington. Because of the limitation in regard to accessing industrial scale productions in an educational environment, Synegia was designed to use the techniques of folding and bending so that only simple tools and hardware were required at the point of assembly. The main material chosen is a type of thin and translucent corrugated plastic board that is not only easy to cut and fold, but also produces desired lighting effects both during the day and in the night. To create structurally rigid forms using the plastic boards, each hendecahedron was folded from five unique panels with various geometric shapes cut out in the panels in order to create porosity for the structure and to allow for easy assembly. In general, the layers on the top of the structure have larger openings, while the layers on the bottom of the structure have smaller openings. In order to create structurally sound installation and the bench tops for people to sit, both 4mm and 6 mm thick corrugated boards were used. Because of the variation in material thickness, digital models were carefully constructed so that various cut out patterns could be generated in order to create seamless space-filling connections. Overall, twenty-two unique cutout patterns were required. These patterns were first sent to a laser cutter to cut and then hand scored and folded in the design studios (Figure 6a). Roughly 20,000 sets of hex-shaped plastic nuts and screws were used in the overall assembly that took several weeks to complete (Figure 6b).



(a) (b)

Figure 6: *The layer assembly diagram used in the Synergia installation: (a) unit assembly in the design studio; (b) unit assembly diagram showing twenty-two unique cutout patterns*

Summary and Conclusions

Space-filling geometries allow structurally sound forms to be generated from thin sheet materials, in ways similar to how soap bubbles and biological cells congregate in order to build forms in nature. While this articles discusses a type of periodic space-filling geometry, non-periodic geometry would also need to be explored in order to fully integrate nature's strategies in space filing designs in order to create artistic and sustainable solutions.

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