Mathematics in Drafting Japanese Crest Designs

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Abstract

Japanese crests, called *mon*, are a simple circular designs most commonly used as a form of Japanese heraldry. Many Japanese families have a design associated with their clan. Many of these designs can be created with a compass and ruler. In this paper, I discuss the connections between Japanese culture, geometry, and symmetry that appear in these *mon* designs, and also my exploration in drafting these designs.

Mon

Mon are Japanese crests or emblems, often associated with clans or establishments, that are black and white designs usually set in a circular arrangement, as shown in Figure 1. *Kamon*, are crests specifically associated to clans, and are analagous to the European coats of arms. They are used to embellish a variety of objects, ranging from battle flags and armor to everyday objects such as clothing. The simplicity of the designs and the cultural restriction to a circle are what makes the designs beautiful and interesting mathematically.

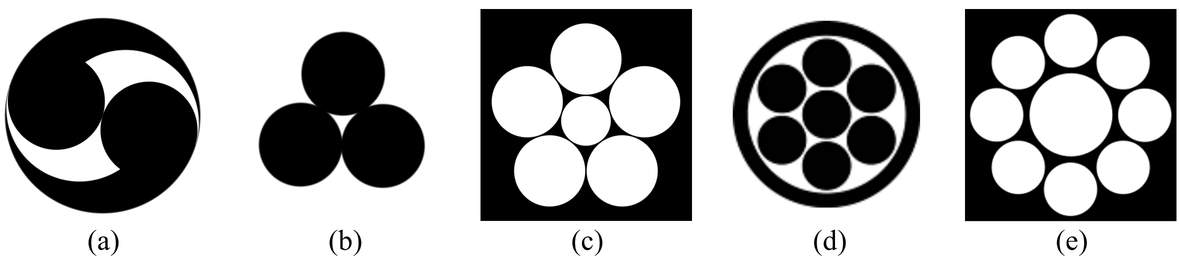


Figure 1: *Examples of mon designs with different symmetries: (a) Two comma-shapes (Futatsu-Domoe), (b) General Stars (Mitsuboshi), (c) Star Plum Blossom (Hoshi Umebachi) [7], (d) Round Ring and Seven Luminaries of Heaven (Maru wa sichiyou) [7], (e) Nine Stars (Kuyou) [7]*

There exist many *sangaku*, or mathematical problems posted on wooden tablets at temples in Japan from the Edo period which feature geometry problems that are beautifully illustrated in designs that resemble those of Japanese crests [2]. One such problem featured in [3] is a geometry problem that resembles the star plum blossom design in Figure 1 (c), states "…a circle of radius r is surrounded by a loop of five equal circles of radius R. Find r in terms of R". As there exist many *mon* designs that are essentially tangential circles, such as the ones in Figure 1, this naturally leads to the question of how to draft them. As *mon* designs are much older than the *sangaku* problems, it is possible that the *sangaku* problems are reflecting the art and design of *mon*.

On textiles, *mon* designs are transferred to the fabric by a process that involves bleaching out the design from the fabric with the help of stencils that are formed in the rough shape of the *mon* design. The *mon* artist fills in the details after bleaching, but the basis of the design is formed on the fabric from the stencils, which are made of paper. Thus, the designs are closely related to the method of stencil cutting [4]. I am interested in looking at how designs can be obtained from drafting stencil designs with compass and straightedge, and in the future investigating paper folding and cutting, which are related to a Japanese craft called *monkiri*. Most of my focus has been in researching the types of symmetries appearing in the designs, and how that relates to compass-straightedge constructions. There are some sources about the history of *mon* in English, but I have not found any English source about the mathematics of creating *mon*. In this paper, I will discuss my findings in the types of symmetries that occur in *mon* and my work in trying to construct drafts of *mon*.

Symmetries of *Mon*

Bilateral symmetry is the most popular symmetry appearing in *mon*. The bilateral symmetry is likely popular because of the relationship of *mon* to cutting stencils, in that the bilateral design requires only one fold of the paper to cut the design. Three- and five-fold symmetries are next in popularity, possibly because Japanese flora used in motifs, such as the cherry blossom or wood-sorrel, are more likely to have symmetries of odd orders. Another possible reason is that in Japanese culture, there is a preference for odd numbers, which is exhibited socially, such as purchasing items or giving gifts of multiple items in odd numbers, but also is exhibited aesthetically in art and design.

Shown in Figure 2 is my tally of all of the *mon* designs in [1], with Dn denoting the n-th dihedral group and Cn denoting the n-th cyclic group. Symmetries which were very rare, or that I could not find an example of, are seven-, nine-, and 11-fold symmetries. The designs with symmetry more than 10-fold were either 12- or 16-fold. It is probable that there are not many designs with symmetry higher than 10-fold as these designs were often printed on clothing with a diameter of one to two inches, so a design with a high symmetry number will be indistinct visually. Given these observations, one might wonder if the occurrence, or lack of, of certain n-fold symmetries in Japanese crest designs are related to which regular n-gons are constructible with compass and straightedge.

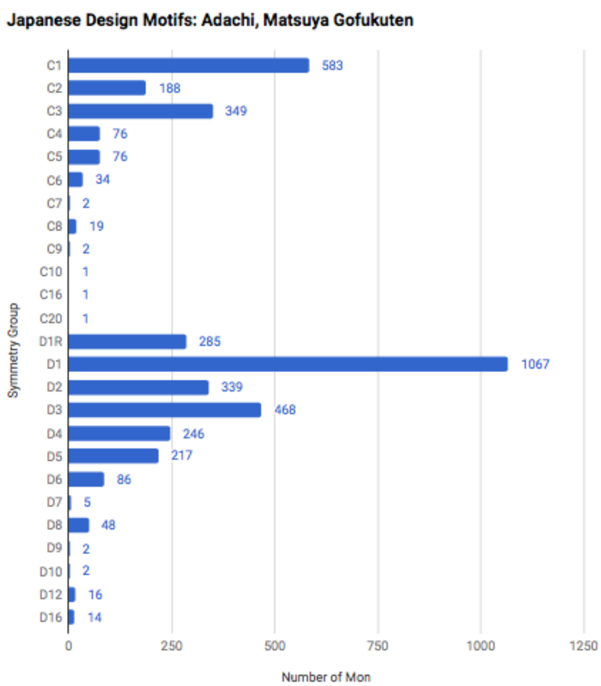


Figure 2: *Bar graph of my tally of mon classified into symmetry groups (dihedral and cyclic) from mon from the text Japanese Design Motifs by Adachi, Matsuya Gofukuten.*

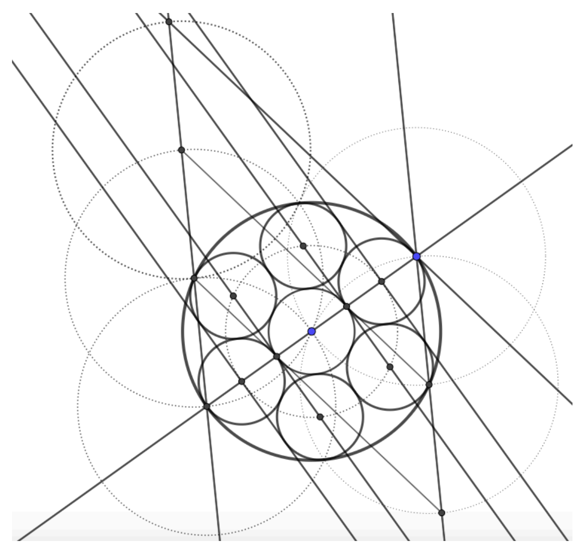


Figure 3: *Compass and straightedge construction for the Seven Luminaries crest created in GeoGebra.*

Drafting *Mon*

Many *mon* designs can be drafted using only a compass and straightedge, although there are no rules such as in the permissible constructions of Euclidean geometry. The compass is free to be placed anywhere, and is not necessarily "collapsible".

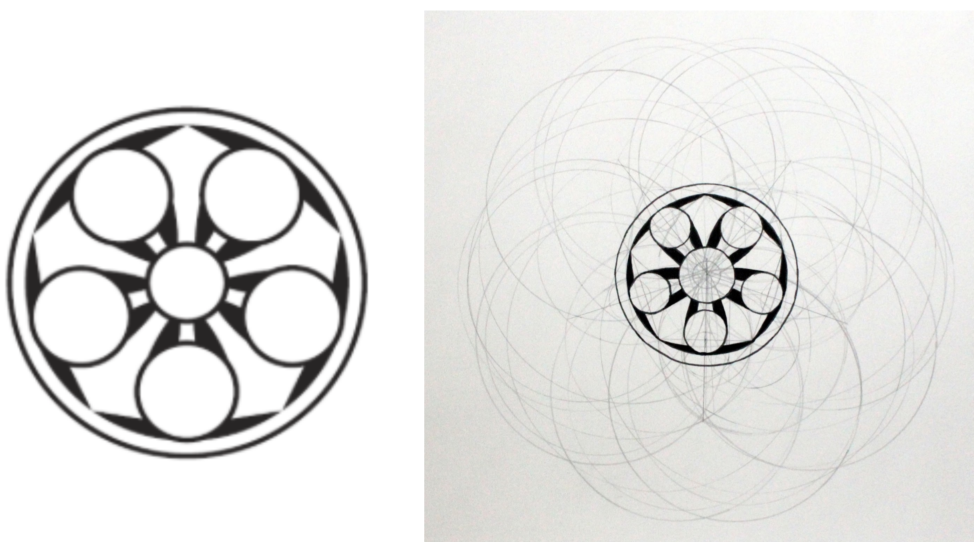
Inscribing Polygons in Circles

If the *mon* design needs to be of a certain diameter, such as in making a stencil to use on a piece of clothing, we should start drafting with creating an outer circle that will serve as the boundary of the design. As many of the designs feature rotational or reflection symmetry and are associated with a dihedral group or cyclic group, we need to next inscribe the circle with a regular polygon. The regular polygon should be an n-gon where n is the n-fold symmetry in the design. As inscribing a polygon in a circle is the first step of the design, it seems that this may restrict the types of designs that can be made in this way.

Kusuda Mon

The first *kamon* I encountered was my boyfriend's family's *mon,* Figure 4 (a): a stylized plum blossom, with five circles for the petals, set inside a circle with outline, with kite-shaped stylized swords set between the circular petals. Seeing the Kusuda *mon*'s inherently geometric design led me to start researching *mon*.

Shoryu Hatoba is a *mon* designer that specializes in the dying art of painting *mon*. His website [4] contains beautiful works he calls "Mon-Mandala" which feature the *mon* design with the all of the precise drafting arcs from the compass, with parts painted to create the positive-negative space of a *mon* design. My own attempt at drafting the Kusuda *mon* in the same style as Hatoba is shown in Figure 4 (b), where I use a compass and straightedge by hand. The basis of this design was in starting with an outer circle to establish the size of the design, and inscribing a pentagon. The pentagon served as a basis for the placement of the petals, and the size of the circular petals approximated by my guess as to how large they might be from the original draft.



(a) (b)

Figure 4: *Kusuda mon: (a) Kusuda mon drafted by Cindi Kusuda, (b) Drafting a variation of the Kusuda mon by hand using a compass and straightedge.*

The Tangential Circles Problem

There exist thousands of distinct *mon* designs [5]. While many designs are geometric, there are a set of designs that feature tangential circles that resemble circle-packing problems in *sangaku*. In some designs, the circles are not actually tangential, but in order to draft the design, one needs ideas of how to create symmetrical, tangential circles that are also tangential to an outer circle.

One of the easiest examples to draft is the *Round Ring and Seven Luminaries of Heaven* design shown in Figure 1 (d), with compass and straightedge construction shown in Figure 3. Once the outer ring is drafted, to pack seven circles of the same radii into the larger circle, one would trisect the diameter of the circle. The length of one-third of the diameter forms the diameter of each of the seven circles. Once the inner-circle is drawn, the placement of the other six circles is quite easy, and they are all tangential to each other and the outer ring. This one is easy to draft, along with the *General Stars mon* shown in Figure 1 (b), because it is easy and quick to inscribe a regular triangle into a circle, so it is quite easy to inscribe a regular hexagon.

On the other hand, it is quite difficult to draft the *Hoshi Umebachi* version of the *mon* shown in Figure 1, (c) when starting with an outer ring and having tangential circle petals, because we have to not only inscribe a pentagon, but find the radius of the flower petal circles so that they touch tangentially. This design is one that I have not yet fully mastered because in all of my attempts, I am not so sure if the petals are truly tangential; they seem to overlap at more than one point.

Summary and Conclusions

I mentioned discovering that certain symmetries are more popular than other symmetries in *mon,* and speculated that the reasons for this are either cultural or mathematical. Mathematically, if most *mon* are designed and drafted only from a compass and straightedge with a base of a regular n-gon, then indeed n-fold symmetries for which it is impossible to construct a regular n-gon should be rare. On top of that, even with *mon* with symmetry corresponding to a constructible n-gon, when that *mon* has tangential circles, my exploration of reconstructing it met further difficulties.

In this paper, I have focused my investigation on the connections between mathematics and *mon* to compass and straightedge constructions, but there is much more to investigate. Given that *mon* design and stencil cutting are integral parts of the art, and the existence of the paper cutting activity *monkiri*, I am curious as to how adding paper folding increases the design possibilities.

References

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