Juan Sánchez Cotán: Seeing Mathematically

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Abstract

Juan Sánchez Cotán (1560–1627), a Spanish painter and Carthusian monk, created austere but startlingly vivid still-life pictures. Cotán's works are arresting both for their hyperfine observational detail and for subtle mathematical references that can be seen and studied by mathematically versed observers—including students.

Juan Sánchez Cotán

Mathematics and art can have deeply symbiotic marriages, ones that benefit both partners. As evidence that such marriages can thrive, I adduce Juan Sánchez Cotán, a Spanish painter born in 1560. Cotán had a successful art career around Toledo until his early 40s, when he shuttered his workshop and took holy orders in a Carthusian monastery. Thereafter, Cotán painted mainly religious scenes.

Figure 1: *Juan Sánchez Cotán, "Still Life with Game Fowl, Vegetables and Fruits", 1602 (Museo del Prado Madrid)*

Cotán is now famous mainly for his still lifes, particularly of the refined contemporary style called *bodegones*. (For more on Cotán, still life painting, and musings on mathematical content, see [1].) I find the one above (Figure 1) quite beautiful. It's also arrestingly strange. Notice the eye-popping colors and the flash-photo contrast: bright birds and produce against a pitch-black background. How did Cotán achieve these effects, or even imagine them, 300 years before the light bulb? Notice, too, Cotán's eye, and his care, for geometry and structure. The apples are aligned in classic sphere-packing style. (As we well know, Johannes Kepler—a German contemporary of Cotán—conjectured in 1611 that the "standard"

sphere packing is indeed the tightest possible. This obvious-seeming result was first proved almost 400 years later, by Thomas Hales.) The lemons form a triangle, the vegetables suggest geometric forms, the six little birds are lined up in parallel, and the bunch of celery suggests a family of mathematical curves emanating from a single point. These observations would all be dully pedantic, like the contrived story problems in too many mathematics textbooks, or the facile "discovery" of golden rectangles everywhere in art, were it not for Cotán's gorgeous textures, saturated colors, and high botanical detail. We can almost smell the lemons and apples, perhaps overlaid with a hint of decaying meat.

Before proceeding we should acknowledge a key fact: Cotán was not completely, or even mainly, a mathematician. He was an artist and a Carthusian monk. The geometric simplicity and even austerity of Cotán's *bodegones* is surely informed by religious as well as artistic, let alone mathematical, motives. But mathematics is certainly here to be found, even if more in service to art than the other way around. That mathematics, as I argue, can inform and deepen how we and our students study, interpret, and appreciate the work.

Figure 2: *Juan Sánchez Cotán, "Quince, Cabbage, and Cucumber", 1602, (San Diego Museum of Art).*

The painting in Figure 2 is aesthetically more austere than that in Figure 1, but to my eye even better, both artistically and mathematically. Notice the contrast between the curvaceous natural forms and the sharp geometry of the shadows and the rectangular window opening. Where is the light source, and how can it be so bright? Why are two fruits hanging from strings? (To this question there is a likely but prosaic answer: In Cotán's day meats and even produce were often suspended to avoid touching each other and thus to retard spoilage.)

Here the mathematical elements are even clearer: the spherical quince and cabbage, the cut-spherical melon with part of its wedge, and the cone-shaped cucumber, approaching us from back to front, and arranged in a curve that looks tantalizingly mathematical. How is the window opening really shaped? What kind of curve do the fruits and vegetables describe: Parabolic? Exponential? Something else? That's a question we (and our students) might ask, and profitably investigate, at least for the pedagogical sake of comparing families of curves, fitting data, and the like.

Before getting into details we should acknowledge another fair question: How "intentional'' is the mathematics to be seen in Cotán's *bodegones*? Is the cluster of apples in Figure 1 intentionally sphere-packed, or simply a natural way of hanging fruit? Such matters, although probably unknowable with certainty, are addressed at least briefly in the critical literature. The art historian Norman Bryson [1], for instance, finds quite explicit mathematics in "Quince, Cabbage, and Cucumber," even citing topology in his discussion of the various geometric forms. The curve described by the foods' centers of gravity is described first as "precisely logarithmic" and later as "hyperbolic". This analysis seems to me both questionable mathematically (a curve cannot be both transcendental and algebraic) and, more important, overreaching: that Cotán really meant to carry all this mathematical freight seems to me unlikely.

Fitting Curves

Whatever mathematics Cotán may have intended, it's rewarding for young students, and amusing for anyone, to try fitting curves from various families to Cotán's "vegetable data points". With *Mathematica* or similar software the mechanics of doing so is easy. Figure 3 shows three possibilities—quadratic, cubic, and exponential—each found by coordinatizing each vegetable's (rough) center of gravity, and applying simple least-squares methods (thanks, *Mathematica*) to find best-fitting curves of each type. The mathematical details are all simple and standard, and not really the point here. But we and students might enjoy, for example, the modest challenge of deciding which curve is which. Students might also notice, profitably, that for small numbers of data points curves of various families can be chosen to "fit" reasonably well.

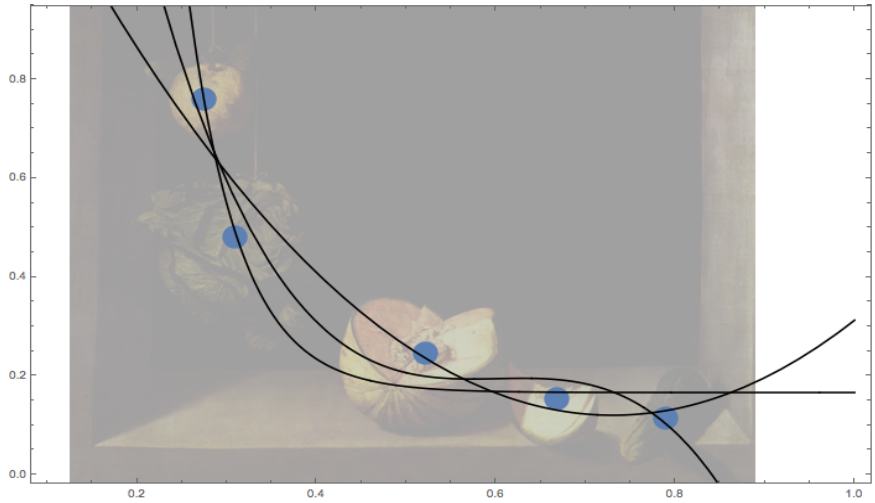

Figure 3: *Fitting three candidate curves (quadratic, cubic, and exponential) to Cotán's produce. Which is which? Which is "best"?*

We can't really know, of course, what the artist had in mind—if indeed *anything* strictly mathematical. But Cotán surely had a mathematically sophisticated eye, as his treatment of shadows and perspective in the window opening attest.

One more possibly "fitting" curve family—hyperbolas—deserves mention, for two reasons: let the reader (or student) gauge how convincing either may be. First, hyperbolas are conic sections, and the cucumber brings cones to mind. Second, it's an easy (but interesting!) exercise to show that the tip of a sundial's shadow describes a hyperbolic curve over any given day. Might Cotán have noticed such curves from the window of his monastic cell?

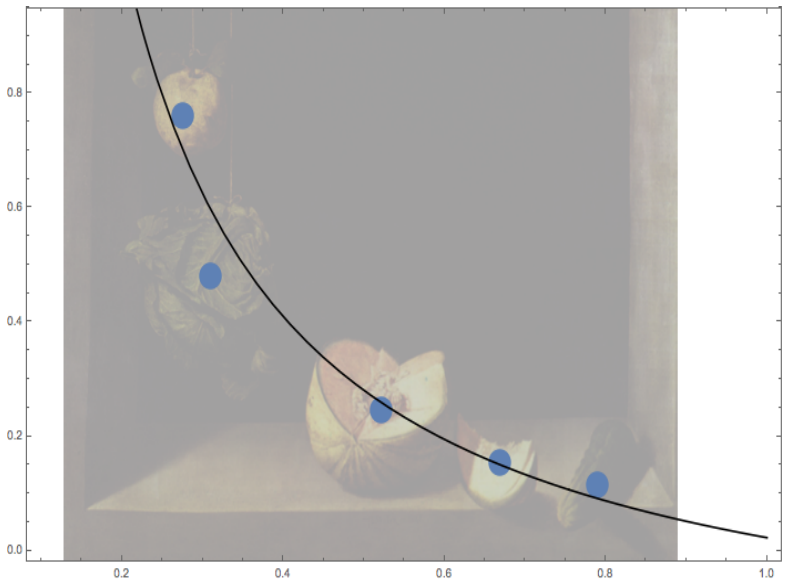

Figure 4: *Fitting a hyperbola to Cotán's produce.*

In any event, Figure 4 shows a hyperbolic curve (in this case a simple linear combination of $1/$□ and 1) also calculated by Mathematica to reasonably fit the vegetable data. To this author's eye the hyperbolic curve does a little better than its quadratic, cubic, and exponential competitors.

Summary and Conclusions: Mathematical and Pedagogical Lessons from Art

How much of this mathematical stuff is really *in* these pictures? How much mathematics did Cotán truly have in mind? A reasonable guess (all that's possible from this distance in time) is that Cotán was mathematically sophisticated in an informal way, but that he kept his mathematics—quite literally—in perspective. Cotán's *bodegones* are works of art, not mathematics lessons; we see this in the exquisite detail of each cabbage leaf, the overripe and dented quince, and the faithfully-rendered textures of melon, quince, and cucumber. Perhaps the artist is saying, figuratively, that both the universal forms of geometry and the particulars of real quinces and cabbages and melons are worth caring about, and paying close attention to.

There are pedagogical lessons here, too. Cotán's art does not aim mainly to teach us mathematics, but it links to mathematics in at least two important ways. First, bringing some mathematics to one's view of almost anything can help us and our students appreciate both the art and the mathematics. Second, Cotán's "monastic" style, which rejects needless decoration in favor of simple forms, has itself something mathematical about it. In mathematics as in the *bodegones*, austerity itself can be, paradoxically, a form of beauty. Cotán's message for us and our students may be to look closely, deeply, and intensely at simple but beautiful things.

Reference

[1] N. Bryson, *Looking at the Overlooked: Four Essays on Still Life Painting*. Harvard University Press, 1990, pp. 60–70