Mapping from *e* to Metaphor

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Abstract

Metaphors based on math concepts can be useful to poets because mathematics inhabits a liminal space between abstract ideas and real-world applications. To be effective, however, such metaphors have to be meaningful to both mathematicians and non-experts. The paper outlines the development of one such metaphor in a poem based on the relationship of e to the catenary curve.

Mathematics as a Liminal Space

The relationship of math to the real world has been a conundrum for philosophers for centuries, but it is also an inspiration for poets. The patterns of mathematics inhabit a liminal space—they were initially derived from the natural world and yet seem to exist in a separate, self-contained system standing apart from that world. This makes them a source of potential metaphor: mapping back and forth between the world of personal experience and the world of mathematical patterns opens the door to novel connections. This paper is a case study in developing such a metaphor in a long poem, "Catena."

There are challenges for a writer trying to work in this particular space. A successful metaphor enables us to see a relationship and feel it is genuinely meaningful. This is fairly straightforward for mappings that involve familiar concepts—for instance, a relationship between 'sunset' and 'death'. We all have some familiarity with both ends of the comparison. Developing a metaphor connecting 'death' and a concept that isn't generally familiar, such as an asymptotic curve, is more difficult. Yet such a comparison might be rich and satisfying if it is made sufficiently accessible.

The other side of the challenge is that a math metaphor really should respect the math. The process of making it accessible to a non-mathematician shouldn't cause someone with expertise to roll her eyes at the inaccuracies or distortions. The result, ideally, is meaningful to both non-mathematicians and experts. Can this be done? Mathematicians have been known to bristle at attempts to express their concepts through metaphors, feeling this will inevitably over-simplify and that only the mathematical formulations can make for accuracy. "Metaphors deny distinctions between things: problems often arise from taking structural metaphors too literally [7]."

However, *any* attempt to communicate a math-inspired concept to a broader audience (whether for the purpose of poetry or pedagogy) will have to depend on real-world analogies and examples. Moreover, a good metaphor is not a small, neat bull's-eye on a target. It's more like a mathematical projection, which physicist Frank Wilzcek describes as "… a mapping from one shape to another, by which information about the first shape is presented in a new form. Often (but not always) some of the information is lost [8]." This does not invalidate the mapping.

Form vs. Subject: Where to Start?

Creation of any poem involves negotiating the continuous tension between form and content—sometimes one of these partners takes the lead and sometimes another, but both are always in play. In the case of my particular poem, the specific impetus for the form came from a paper by Sarah Glaz, "Poems Structured by Integer Sequences [5]." In it, she noted that a number of poets have been inspired to use the decimal expansion of *pi* as a structuring device. However, to her knowledge, other famous irrational numbers have not been used to do so—notably, the Euler number, *e*.

Meanwhile, I had been looking for a way to deal with a particular subject: the recent deaths of two family members, both of whom had to live with congenital illnesses that irrevocably shaped their lives. My niece had died at the age of 40 from a form of muscular dystrophy that progressively limited her physical abilities, while a brother-in-law died in his late 60s after living with schizophrenia since his early teens. I wanted to express something of the hardship this had meant for them and for their families, the arbitrariness of the luck that shaped their experience, the continuity and discontinuities that resulted from genetic malfunction.

I began to wonder how I might put these two ideas together, which meant thinking about what *e* means in terms of mathematical pattern. It is perhaps most familiar as the base of natural logarithms, the number that turns up in continuous exponential growth of all kinds in our world, from calculating compound interest to population growth. All equations of the form $y = e^{ax}$ (where *a* is any fixed positive number) will graph a curve of exponential growth, rising steeply rising to the right of the y axis. Equations of the form $y = e^{-ax}$ draw the mirror curves of exponential decay.

Although these exponential curves are infinite, the number *e* itself is a kind of endpoint—the limit to which the sum of 1/n! converges, 2.718281828… You might think of it as a relatively small number that nevertheless encompasses *all* numbers.

e is irrational; its decimal expansion is a random spatter of digits that never settles into a regular pattern. And (like *pi*) it is transcendental—there is no polynomial equation with integer coefficients that has *e* as its root. Words like ‘irrational’ and ‘transcendental’ are instances of the way in which language maps back and forth between mathematics and ordinary speech. In English, “irrational” was originally a term created specifically for mathematical use, constructed from Latin and French roots that pertained to calculating but also to reason, logic, our human capacity to perceive relationships correctly. However “irrational” was soon imported back into ordinary speech to describe absurdity, the *il*logical.

The concept of exponential growth and decay was interesting but didn’t seem immediately useful as a target for the subject I wanted to address. However, Alex Bellos’ chapter “All about *e”* in his book *The Grapes of Math* [4] gave me another possibility: for every fixed number *a*, the formula

$$y = \frac{e^{ax} + e^{-ax}}{2a}$$

This equation generates the catenary curve, a relationship in which exponential growth and exponential decay are inextricably linked. Catenary curves are everywhere in our natural and man-made worlds. They are drawn by any material suspended by its own weight: a line of spider silk, a chain, a phone cable, a swinging bridge. The curves can appear different because of the variable *a,* which reflects how far apart the ends of the string are. But every variation is governed by that constant, *e*.

Developing the Metaphor

The shape of the catenary curve and its generating *e* gave me a number of points that might be linked on a mapping between math and personal experience:

- The sequence of digits in the decimal expansion of *e* can’t be predicted. We don’t know what the n^{th} digit will be until we get there, as we do not know which genes from the menu inherited from ancestors will be ordered into the chromosomes of an individual.
- Though the curves of exponential growth and decay are infinite, in the real world, catenary curves are defined by two fixed points—their beginning and their end—as is any individual human life.
- The catenary is not a consistent shape; it is a ‘family’ of curves. The individual expression of it depends crucially on that variable *a.*
- The word ‘catenary’ itself, which comes from the Latin, *catena,* meaning chain. This is another word that has moved back and forth over the centuries between mathematical usage and ordinary speech. Among other things, it refers to a kind of Biblical commentary (like the *Catena Aurea* of Thomas Aquinas) that constructs a chain of commentary about a verse of scripture [1].

- The relationship between discrete integers and continuous geometry that is inherent in *e* can be seen as analogous to the relationship between an individual and the bonds connecting him/her to others.
- An object hanging in a catenary curve is, all along its length, continuously balancing two forces: tension that would pull it apart, and connection.
- The concept of *e* as a number that converges towards a limit echoes for me the relationship between the aspirations of individuals and the inescapable boundaries of our physical bodies and minds.

This mapping influenced various aspects of the poem:

- *Structure:* I chose a chain of stanzas with the number of lines in each one determined by the series of digit in *e*'s decimal expansion. (The poem is constructed using the first 30 digits of e's decimal expansion.)
- *Formal constraints:* Though the series of digits doesn't have any predictable pattern, they're not random; they are determined by their place in the chain, by what has gone before. I wanted to echo this using a pattern of end rhyme. The first or second line in each stanza had to rhyme with a line near the end of the previous one.
- *Imagery:* Patterns of imagery throughout the poem refer to catenary curves that people would find familiar, notably the rope bridge. Another significant pattern of imagery concerns DNA/chromosomes/genes, which tie together the issues of congenital illness with mathematical concepts like integer vs. continuous line.

Conclusion

Finding the right metaphor can help a poet render her individual experience universal by providing a framework for the strictly personal. Mathematical metaphors have the particular gift of coming from the intersection of the abstract and the real world, which provides some distance and structure for deeply felt human emotion, which might otherwise become simply a blurt on the page.

Working through this particular metaphorical map that could help me express the human impact of congenital illness was a special source of joy. This emotion is not restricted to poets and artists. As Werner Heisenberg said in a conversation with Einstein: "You must have felt this too—the almost frightening simplicity and wholeness of the relationships that nature has spread out for us [3]." Though I am hardly doing earth-shattering physics, I recognize the feeling that almost magically you've seen a pattern and can work out its consequences in some detail. Perceiving a metaphorical relationship may not involve working out a pattern within such strict constraints as solving a mathematical challenge. Nevertheless, creativity in mathematics does involve noticing previously unsuspected relationships, and it is this experience I imagine as being similar.

To conclude, I offer the first few stanzas of the resulting poem:

Catena [6]

2.71828 1828 4590 4523 5360 2874 7135 2…

2 Crap shoot. Snake eyes. The double-twine
of chromosome, super-coiled, coils on coils

. Point. Nit-pick of gene.

7 And then the downward plunge, the ravine
that splits a life away from fair beginnings.

Catenary curve—line of a rope bridge slung
above a chasm. Sickening sway
at its lowest point, suspended tension.
The chain of accident, curvilinear
that shapes a life.

1 Each life so wholly singular

8 inimitable outcome of equations –
universal patterns driving the particular.
Your brothers bend above you
in farewell. The summer twilight fades
to the murmured kaddish. Your graven face
lays pale as feldspar. Three brothers, chains
of DNA – so sibling-similar
and yet unique. And the long chain
binding you to breath now broken.

2 Why do bad things happen
to bio-diverse people? The random chosen?

8 Catenary—the dark arc mapping
the path of grief, its exponential plunge
from *no! This is not happening*
to the comprehended: *It is.*
The slowing increments of loss
when it can't get any worse, or
any better. The sad slog up, to stand
on something that approaches solid ground.

1 The curve by which decay and growth are bound ….

References

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[6] A. Major. "Catena." *Welcome to the Anthropocene.* University of Alberta Press, 2018, pp. 61-65.
[7] D. Pimm. "Metaphor and Analogy in Mathematics." *For the Learning of Mathematics*, vol. 1, no. 3, 1981, pp. 47-50.
[8] F. Wilczek. *A Beautiful Question: Finding Nature's Deep Design.* Penguin Books, 2015, p. 378.