When Mathematics Meets Art: How Might Art Contribute to the Understanding of Mathematical Concepts?

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Abstract

This paper presents a mixed methods study whose objective was to learn about a possible contribution of art to the understanding of mathematical concepts learned in the online asynchronous course "When Mathematics Meets Art." The math concepts were tessellations, zero and infinity, golden section, spatial vision, dimension and self-similarity. The research sample consisted of 130 pre-service elementary school math teachers who submitted 20 different assignments created to reflect their math knowledge. The assignments were graded based on a pre-designed rubric. The preliminary results suggested that there is partial overlap between math and art assessments which makes the math-art connection innovative, intriguing, fun and inspiring.

When Mathematics Meets Art

The course "When Mathematics Meets Art" was designed to implement principles of an interdisciplinary approach to learning mathematics via exploring artworks. This learning approach is more suitable to the 21st century [4], enriches students' inner world, and can set them an example for a way of including art in teaching mathematics [3]. The connection to art helps learners to express themselves; to develop critical and creative ways of thinking while seeking solutions to questions, problems and natural phenomena; to look for multiple solutions and not to follow already-known approaches; to develop the ability to bridge different areas of knowledge by transferring knowledge from one field to another; to pay attention to the finest details that can ultimately make a big difference in the outcome; to find expression in forms more complex than words and numbers; to think with and through given material; to experience through senses that cannot be reached in any way other than the arts; to adapt to different learning styles ([1], [3], [6]).

The course was offered for the first time in the fall of 2017 at Gordon Academic Educational College in Israel for pre-service elementary school teachers majoring in mathematics and science or in mathematics and special education. It was an online asynchronous semester-long course, i.e. 14 weeks. The objectives of the course were: expanding students' mathematical knowledge; executing a small-scale math inquiry tasks; creating a community of learners and; applying mathematical concepts to art, daily life and natural occurrences. The course consisted of six math topics presented in the following order: tessellations, zero and infinity (in calculating area and perimeter), golden section, spatial vision (focusing on impossible figures), dimension and self-similarity. The topics were selected using three criteria in relation to the elementary school curriculum: to deepen students' mathematical knowledge of topics included in the curriculum (zero and infinity, spatial vision); concepts mentioned in the curriculum but not studied in depth (dimension and self-similarity) and; concepts that are not taught in the curriculum but are tightly coupled with both mathematics and art (tessellations and the golden section). The learning course materials were developed especially for this project. After developing the course, my main role included facilitating student forums and resolving technical problems.

The course was designed on the Moodle learning management system and consisted of six units, one for each mathematical concept as indicated above. The student requirements for each course unit were: (1) to participate in a forum discussion or answer a survey – these tasks were designed to check their prior knowledge (15% of final grade; students were graded for participation and not for knowledge); (2) to study the mathematical concept theoretically through a specially prepared video, an article or a PowerPoint presentation – they then applied their acquired knowledge by solving exercises or carrying out an inquiry

task – they checked their knowledge through a short online test (test score is 20% of final grade); (3) to contribute an artwork (it was not required to be original) to an online collaborative gallery (9% of final grade); (4) to pass a final exam (56% of the final grade).

Student Math and Art Assignments

Mathematical assignments were designed based on prior knowledge of the students and the mathematical depth that could be achieved in the course. Due to limited space considerations, I have chosen to present an example of a task that focuses on zero and infinity. Students are familiar with the mathematical properties of zero and know the concept of infinity as well as the concepts area and perimeter. The purpose of this inquiry task was to show them the fascinating encounter between zero and infinity in one object – Koch's Island (Figure 1).

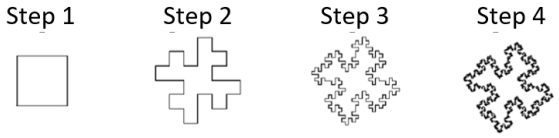

Figure 1: *The Koch island fractal.*
Retrieved from Brilliant. https://brilliant.org/discussions/thread/koch-island-fractals/.

The students were asked to answer the following questions for a given Koch Island:

1. If the side of the square in Step 0 is 1 cm long, what is the perimeter of the Koch Island in Step 1? If the side of the square in Step 0 is 1 cm long, what is the area of the figure that appears in Step 1? Step 2? Step 3? Step *n*?
2. Try to find examples of other planar figures where area and perimeter relations have the same nature as in the Koch Island.

The instruction for each art contribution to the online gallery was: "Upload an original artwork (or the artwork of an artist) that expresses your interpretation of the math concept studied. Add one short sentence explaining your choice." Although the art assignment was the smallest part of the course grade, many students invested a great deal of effort in it. Following are five representative original artworks (Figure 2):

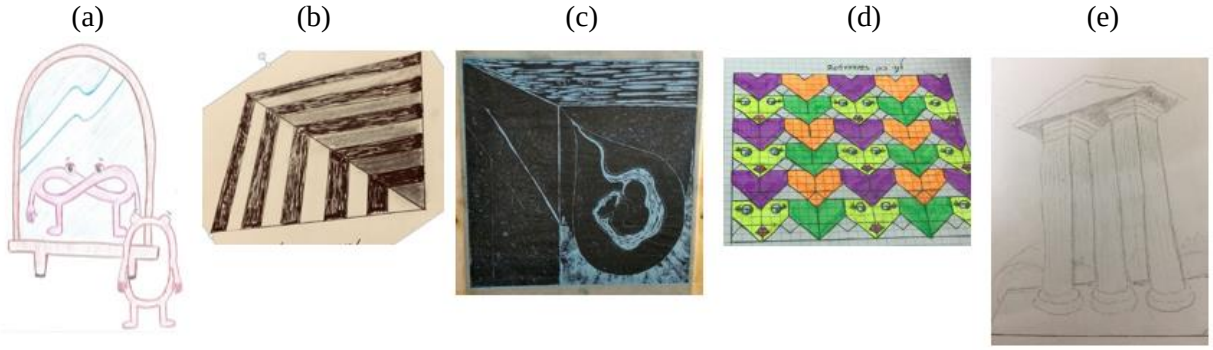

Figure 2: *Students' original art: (a) Student 1: Zero and infinity, (b) Student 2: Dimension, (c) Student 3: Golden ratio, (d) Student 4: Escher-style tessellations, (e) Student 5: Impossible gate.*

How Might Art Contribute to the Understanding of Mathematical Concepts?

To answer the question in the heading above, explanatory design research was found to be the most suitable approach from among the mixed method research strategies [2]. In the first phase of the study, quantitative data was collected and analyzed. In the second phase, the quantitative data was interpreted by the qualitative data. In this manner the advantages of the two research paradigms were exploited; using a quantitative method, one can examine the relationship between variables, while the qualitative method supplies the participants' interpretation of the quantitative findings.

The research questions: (a) Does artwork contribute to student understanding of mathematical concepts? (b) What is the essence of the contribution of artwork to student understanding of mathematical concepts?

The Research Tools

Online tests: At the end of each unit, the students took an online test, designed to examine the mathematical knowledge acquired. The students were allowed two attempts to answer the test. They could see their grade only after the second submission. The final test grade was the higher of the two. The test was checked automatically based on pre-determined criteria.

Collaborative galleries: The unit gallery consisted of the student art contributions. The students could choose to contribute an artwork (original or not original) with a short explanation of how the mathematical concept under study in the unit is expressed in it. Students could have commented on the contributions of their colleagues and indicate 'like.' The Padlet (https://padlet.com) website was selected for the collaborative art galleries.

Semi-structured interviews with students: An interview guide was designed for this purpose. On the course website the students were asked to volunteer for an interview. There were eight interviews in total, which took place at the college and lasted 30 – 60 minutes.

Course evaluations: At the end of each unit, students could voluntarily fill in an online survey that referred to the math tasks, the virtual gallery and the contribution of artwork to learning the mathematical concept presented in that unit. A total of 101 responses were collected for the first four units (42 for tessellations, 23 for zero and infinity, 24 for golden section and 12 for spatial vision).

Data Analysis

The quantitative data was statistically analyzed and an attempt was made to find a link between the math content (online tests) and the art. The qualitative data (interviews and online survey) were analyzed by the Strauss-Corbin method [6]: at the first stage, each of the interviews and surveys responses was analyzed in order to identify central and secondary themes. At the second stage, an attempt was made to identify links among the different categories and to define major categories and sub-categories. In the final stage of the analysis, an attempt was made to link the quantitative and qualitative findings.

The Results

In this mixed methods study, the quantitative data was the student scores on the online math tests and artworks; the qualitative data was comprised from semi-structured interviews and student course evaluations. Table 1 summarizes student grades of four course units (for which data is available at this time). The math test grades indicated that the chronological order of the units can be correlated with the complexity of the mathematical concepts. The artwork grades showed a different picture: on one hand the number of students who did not submit any artwork increases with time – from 13 in the first unit to 29 in the last. On the other hand, the largest number of original artwork was submitted in the first unit, but most of these (97/130) were of poor quality. As the course progressed, the quality of the artwork improved but the number of original artworks declined. It is interesting to note that the average grade for the artworks was quite stable; it ranged from 70 to 75, with the exception of the first unit.

Table 1: *Descriptive statistics of grades in online tests and artwork (N=130).*

	TT	TA	ZIT	ZIA	GST	GSA	SVT	SVA
Average grade	9.15	6.11	9.07	7.63	8.12	7.22	7.08	7.50
Std. Deviation	3.331	2.537	1.753	2.998	2.006	3.177	2.279	3.474

Abbreviations: Tessellation math test (TT); Tessellation artwork (TA); Zero and infinity math test (ZIT); Zero and infinity artwork (ZIA); Golden section math test (GST); Golden section artwork (GSA); Spatial vision math test (SVT); Spatial vision artwork (SVT).

The next step for defining the possible contribution of art to student understanding of mathematical concepts was to calculate Pearson correlations for the online math tests and the artworks. Table 2 summarizes the data analysis.

Table 2: *Correlations for online tests and artworks (N=130).*

		TT	TA	ZIT	ZIA	GST	GSA	SVT	SVA
TT	Pearson Cor.	1	-0.79	-.349**	.032	-.255**	-.007	-.238**	-.001
	Sig. (1-tailed)		.185	.000	.360	.002	.467	.003	.494
TA	Pearson Cor.		1	.283**	.230**	.229**	.482**	.176*	.343**
	Sig. (1-tailed)			.001	.004	.004	.000	.023	.000
ZIT	Pearson Cor.			1	.236**	.668**	.231**	.507**	.294**
	Sig. (1-tailed)				.003	.000	.004	.000	.000
ZIA	Pearson Cor.				1	.211**	.190*	.138	.161*
	Sig. (1-tailed)					.008	.015	.059	.033
GST	Pearson Cor.					1	.336**	.584**	.294**
	Sig. (1-tailed)						.000	.000	.000
GSA	Pearson Cor.						1	.183*	.217**
	Sig. (1-tailed)							.019	.006
SVT	Pearson Cor.							1	.438**
	Sig. (1-tailed)								.000
SVA	Pearson Cor.								1
	Sig. (1-tailed)								

* Correlation is significant at the 0.05 level (1-tailed).
** Correlation is significant at the 0.01 level (1-tailed).

The data in Table 1 point out that in three out of four units (ZIA, GRT and SVT) there are significant positive correlations between artwork and math knowledge. Moreover, in these same three units, there is a significant positive correlation between the online math tests and the artwork. That is, it is reasonable to postulate that the artwork helped students understand the mathematical concepts. To verify this conclusion, a single index of all math tests (that is, scores for all four math tests) was calculated, as well as for artwork; both indices showed reliability of Cronbach alpha: α=0.60 and α=0.63, which are reasonably good results. A positive correlation of r = .25, p <.01 was found between the two indices. This result suggests that there is a positive and significant partial overlap between the mathematics and art worlds; in what follows it will be referred to as 'math-art land.'

The grades of the online math test for tessellations stand out in Table 2 – the Pearson's correlation is negatively correlated with all other math tests and it is zero, or near zero, with different artworks. Four possible explanations for this are: (1) Beginner's adjustment – this unit was the first of the online course and students were not yet accustomed to this kind of learning. (2) Student learning habits – at this point in the academic year most students had not yet organized a community of learners (hypotheses 1 and 2 will be discussed later). (3) Lack of familiarity with the topic – this topic is not included in any school or college curriculum. (4) Beginning of the semester – students may join or drop the course during the first two weeks

of the semester. Each of these explanations should be explored in depth after all data is available. What is also noteworthy is that the tessellation artworks were positively correlated to all other variables.

The qualitative data analysis outcomes may be presented in two main categories: (a) math-art land is innovative, intriguing, fun and inspiring; (b) math and art have reciprocal contribution.

Math-Art Land is Innovative, Intriguing, Fun and Inspiring

The innovation of the course was expressed in several ways: the link between math and art and learning math in an online course. The students' responses point out that including art in learning mathematics triggered positive feelings about learning mathematics and a new way of looking at it. Here is how one of the students described her thoughts about math-art land:

> *This is amazing. The math-art connection was a new experience for me. I never thought about it in this way. It showed me a new way to look at math. The art gave me a comprehensive, spatial vision on the math topic… it was beyond anything I knew… it was an eye opener.* [Anonymous survey].

The words describing math-art land – *"fun, enjoy, interesting, fascinating"* appeared more than 60 times in the four relevant surveys. One of the students, Mia (all students names are alias), described her motivation to explore the math-art land by saying: *"It blew me away. I simply sat down and drew."* Another student, Laura expressed her curiosity about the art galleries in her interview:

> *I check the course site every day, sometimes even twice a day. I check out what's new and look to see whether or not the artwork the students post met your requirements. Some of them really do meet the requirements and some do not. Nevertheless, I find all the artwork interesting.*

Most of the students did not have any prior art education therefore it was only natural that mathematical complexity would be inversely proportional to the number of original artworks. Natalie describes her own experience:

> *At first, I tried to draw an original artwork for the golden ratio but it was too hard. I tried to draw the sunflower; it didn't work, so I uploaded something from the internet. It was simpler.*

Sasha talked about her motivation to contribute an original artwork to the gallery. Although she cannot draw well, she asked her brother to do it for her. As he was too busy to help her, she took the time to look for artwork, which did not already appear on the site:

> *First, my brother drew a picture for me and then he did not have any more time for it. So I looked at what others uploaded to the gallery and then found a few pictures I wanted to upload. I wanted something original so I searched for pictures with the golden ratio… Then I sat down and checked where the golden ratio was in the picture ... only then I uploaded the picture to the gallery.*

Some students liked the math-art land concept and would like to use this model in their future career:

> *I like the concept of math-art connection. It is good to teach kids some math and then to ask them to draw it. They like drawing. I will use this method when I will be a teacher.*

Reciprocal Contribution of Math and Art

The students pointed out the reciprocal contribution of math and art to their learning: the artwork helped them understand math concepts, and the math concepts triggered artwork. Here is how Iris experienced the contribution of artwork to her mathematical understanding: *"The artwork explained the measurements and it helped me implement what I learned."* Another student wrote in the survey:

> *I did not create an original artwork in the Escher style but it seems to me a great exercise to implement and to practice everything we are learning in this unit.*

Mary, whose hobby is drawing, described the contribution of math to her artwork:

> *It helped me formulate the idea and the way to draw it... I was thinking about the relationship between mathematics and the picture I am drawing... math inspired me to draw unconventional pictures.* [This student drew "The impossible gate" Figure 2e].

Most of the students found it difficult to express the contribution of the artwork verbally, but they sensed it. Here is one representative excerpt:

> *Of course the artwork helps to understand math. I don't know how exactly to explain it. I just feel it does help.*

Summary and Conclusions

The research objectives were to examine the possible contribution of art to students' understanding of mathematical concepts, and to explore math-art land in an online course. The preliminary research results confirm the hypothesis that there is a possible overlap between math and art, inspired by a mathematical concept. This overlap, math-art land, triggered positive feelings about learning mathematics that can help students deepen their mathematical ability to solve a given assignment or to perform an inquiry task; it can also help them develop mathematical intuition, since art enables expression that is beyond words and numbers [1]. Based on these preliminary research results, I believe that math-art land holds great potential for math education, particularly at the elementary school level, which should be further explored. One of the possible research directions can be true experimental design - randomly assigned students to two sections of a course, one section requiring students to engage in artwork activities, and a second section without such a requirement. This platform will allow to compare the contribution of the artwork to students' understanding of mathematics.

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