

## Some Memories of Koos Verhoeff (1927 – 2018)

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**Figure 1:** A 2017 oil painting on panel of Koos by his granddaughter Amy Verhoeff (note the smoke).

## Life

Koos Verhoeff was born on 27 February 1927 in The Hague, The Netherlands, in a neighborhood where all the streets were named after trees. His father worked as a financial auditor for the dairy industry and was an accomplished amateur photographer. Koos attended three schools, all in that tree neighborhood. Due to World War II, his school period was rather ‘chaotic.’ At an early age he already developed a keen interest in geometry, reading various textbooks of his older brother on geometric topics.

Right after the war, Koos went on to study mathematics with physics and astronomy at Leiden University, where he obtained his bachelor degree in two years. His studies were interrupted by military service, which brought him to Indonesia (then: Dutch East Indies). He was fortunate enough to end up as an assistant to Prof. Zaanen at the Technical University of Bandung. After this dark period (he never discussed it), he returned to study mathematics with philosophy at the University of Amsterdam, where he graduated in 1952.

Already before his graduation, Koos had started working at the Mathematical Center (now: Center for Mathematics and Computer Science) in Amsterdam as a researcher in pure mathematics (see e.g., [2]). There, he first encountered the computer and programming, and he met his wife Bertha Haanappel (Figure 2), who was employed as a computer. They married in 1955.



**Figure 2:** *Bertha Haanappel and Koos Verhoeff (28 Nov. 1952, engagement picture by Koos' father).*

In 1957, he got an appointment as lead researcher on the library automation project at Delft University of Technology. He invented an innovative system [3], where you could dial a book code, which turned on lights to guide the library assistant to the right book case and shelf for the requested book. The assistant then dropped it on a spiral chute connecting the four stories of the library building. A conveyor belt transported the book to the central dispatch station. His dissertation concerned decimal error-detecting codes [4].

After a brief period with Philips where he worked on factory automation, he was appointed full professor in Computer Science at the Erasmus University Rotterdam in 1971 as successor of Max Euwe. There he introduced computers into university education, and he drew attention to recreational computer applications. In his opening lecture at the Dutch Book Week of 1978, he announced the disappearance of the printed book as main means of conveying knowledge, introducing words like *reading tablet* and *hyper text* [1].

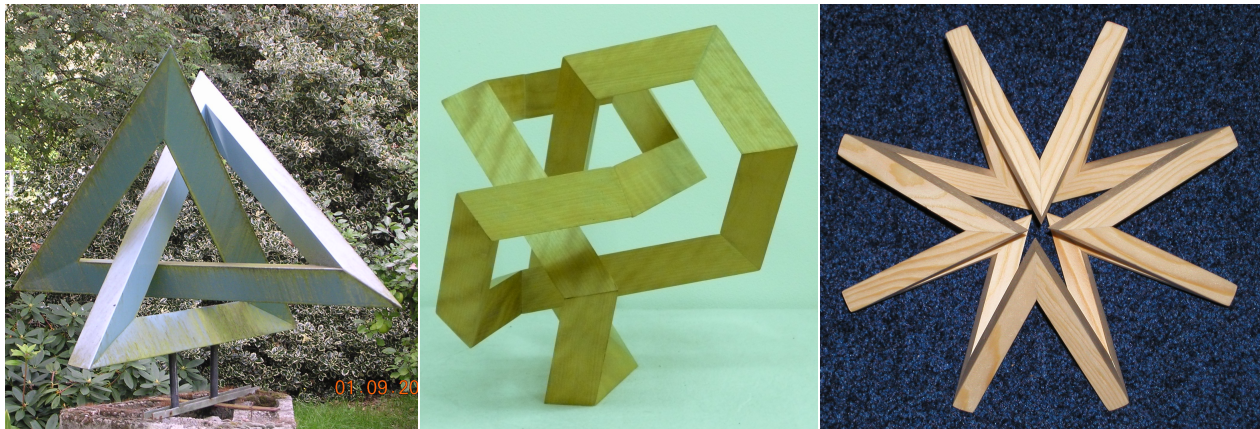
At the university in Rotterdam, Koos again got involved in library automation, this time using numerous microprocessors instead of relays and vacuum tubes to control robots that transported books in plastic boxes. He retired in 1988, and from then on devoted all his time to the design and construction of mathematical art.

## Work

Dutch artist Popke Bakker consulted Koos in the early 1980s on a problem concerning closed spatial paths involving miter joints. Koos initially calculated some designs for Popke, which Popke subsequently constructed. But Popke refused to construct the trefoil knot on the left in Figure 3, because it involved a triangular beam. From then on, Koos decided to do his own construction work as well.

The challenge with a closed spatial path constructed from beams connected by miter joints is to ensure that the longitudinal beam edges connect properly all the way round [6]. The path accumulates a so-called *torsion*: each joint spans a plane, and the directed angle between the planes spanned by two successive joints is called the torsion of the beam connecting those joints. The total torsion angle accumulated along the entire path needs to be a symmetry of the beam's cross section in order to obtain proper closure.

In hindsight, one can distinguish three periods in how Koos approached this challenge. At first, he *tinkered*, that is, he parameterized his designs and played with the locations of the joint points to ensure proper closing. In this approach, all beam segments can involve different lengths and cutting angles with awkward values. The trefoil knot on the left in Figure 3 is an example. The calculations involved are tedious and making another copy of an artwork is a major hassle because tools must be readjusted for every beam.



**Figure 3:** Work from three periods: tinkering (left), lattice walking (middle), constant torsion (right).

To simplify calculations and mass production, Koos resorted to *lattice walks*, where the path visits neighboring points in a lattice, such as the simple cubic (SC), face-centered cubic (FCC), or body-centered cubic (BCC) lattice. This gives you more control over the accumulated torsion angle, and it restricts the set of beam types considerably. The figure-eight knot in the middle of Figure 3 involves the BCC lattice. Koos used to compare this approach to navigation in a city where you have a road map to guide you.

Later, Koos realized that there is a more direct way to control the torsion angle without restricting oneself to a lattice. In this approach you only allow torsion angles that are a symmetry of the cross section [7]. That way, the total torsion is also a symmetry of the cross section, thereby ensuring proper closure. Koos compared this to navigation in the desert with a compass. The regular polygon of constant torsion on the right in Figure 3 is an example. Here, all beams have a square cross section, the same length, the same joint angles, and a torsion of  $\pm 90^\circ$  where the signs follow the pattern  $++--$  repeated 4 times.

Koos designed and constructed much more than just closed spatial paths. Some of his work is described in [5] and in his other Bridges papers [8, 9, 10, 11, 12, 13, 14, 15]. He considered himself not a creative artist but rather a discoverer, who explores mathematical spaces, and only selects interesting and intriguing structures that somehow had always existed. Besides showing the beauty of mathematics, his main goal was to make people wonder and think. Koos passed away in his sleep early on 19 March 2018 at the age of 91.



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