

# The Mathematical Circus Project

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## Abstract

Magic and mathematics appear hand in hand when it comes to promoting the interest and motivation for learning mathematics. Portugal, like other countries, has a high failure rate in school mathematics, which of course causes a negative attitude towards this subject in society. Mathematics is generally thought of as a difficult subject with little practical importance. Motivated by the need to fight this stigma, the *Mathematical Circus Project* emerged in 2012. Linking mathematics with the performing arts (theater and circus) is not as common as other connections such as with the visual arts. The *Mathematical Circus* team performs mathematical magic shows with a circus atmosphere in schools, science centers and other public spaces. In a traditional magic show, one often realizes that what causes the fantastic effects is the magician's ability to manipulate objects combined with the art of controlling the perception of the spectators. The tricks performed in the Mathematical Circus are exclusively based on mathematical principles. In this paper we describe the Mathematical Circus project: its context, goals, contents and achievements.

## The Portuguese *Mathematical Circus* Project

The issue of academic failure in mathematics has been debated for several years. Knowing its causes and finding ways to combat it is a priority at all levels of education. It is essential to reverse the existing and generalized negative feeling towards mathematics by motivating the students and thereby promoting their success in mathematics. One way of doing this is through recreational mathematics, which Singmaster [11] defines as the mathematics that is fun, popular and with pedagogical value.

In particular, the exploration of the relationship between magic and mathematics has enormous potential in the development of activities in schools or elsewhere, to effectively promote the interest for mathematics. In the classroom, the exploration and study of a magical effect, can become a mathematical problem and provide a source of reasoning and research. Outside the classroom, mathematical magic shows can promote a positive relationship with mathematics and stimulate the curiosity for understanding the tricks' explanation, and in turn, for learning mathematics. These resources are being used in Portugal as in other countries (as for instance by *Marcus' Marvellous Mathemagicians* in the UK [12], and by Fernando Blasco in Spain [5]) to engage students and promote enthusiasm for learning mathematics.

In March 2012, the Ludus Association [2] created the *Mathematical Circus* project coordinated by Jorge Nuno Silva, professor at the University of Lisbon. It is a project where complementary skills are merged to produce a high intervention capacity within a wide geographical range in the promotion of mathematics, an urgent task in Portugal.

When linking mathematics with arts, most of the connections appear in the visual arts and some in music. Interlacing mathematics with other forms of performing arts is less common though much can be done in this area. The performing arts landscape is evolving and diversifying in this ongoing 21<sup>st</sup> century and the circus is one of the areas where this evolution is more noticeable, with contemporary circus getting the most out of creativity and joining together different art forms, high skilled acrobatics and technology.

In this context, the *Mathematical Circus* project is an innovative project which uses some of the circus elements to create an original combination: magic, beauty, surprise, laughter and wonder, all associated with the circus, are the key ingredients used by the *Mathematical Circus*. All the activities in the repertoire of the *Circus* have a mathematical content and although mathematical magic is the most relevant component, the clown jokes and fooleries aren't less important and all based on mathematics.

### **Who We Are**

The *Mathematical Circus* has two teams, one in the north (hosted by the University of Aveiro) and one in the south (Lisbon). Both authors of this presentation are members of the north team which is coordinated by the first author. The north team is composed mostly of mathematics teachers from the University of Aveiro. Figure 1 contains some photos of the north team.



**Figure 1:** Members of the *Mathematical Circus* team from Aveiro and logo.

### **What We Do**

Magic shows are well known throughout the world. Using mathematics in magic tricks is not a novelty but making an entire show out of mathematical magic is not so common. Fernando Blasco is one of the few people to do so and Bridges Coimbra 2011 hosted one of his presentations [5] which played a part in the building up of the *Mathematical Circus* project. Not all mathematical magic tricks are appropriate for a wide audience performance and an important part of our project consists in developing/adapting tricks for large audiences. The *Mathematical Circus* performs shows mostly in schools, but also in science centers and universities (for visiting schools), typically for students up to eighteen years old. These shows consist of several magic tricks alternated with clown interventions, looking, as much as possible, like a live magic show with a circus atmosphere. As a norm, each show has a host, a magician, a mentalist, a contortionist, a clown and a DJ. The magician performs general magic tricks involving mathematics (tricks with cards, with numbers, etc.); the mentalist performs tricks that have to do with reading the public's mind; the contortionist performs magic tricks using topology (the contortion effects are done with ropes, paper folds and cutting, etc.); the clown fools around in most of the tricks and has his own tricks and diversions; the DJ plays selected music for each trick and also deals with projecting images and charts used in the tricks. The necessary sound equipment (wireless microphones, loudspeaker and mixer)

was obtained through a national project – *Ciência Viva PEC176-2013*. All the tricks involve the participation of the public (volunteers) thus creating a constant interaction between the team and the audience. Figure 2 illustrates the *Mathematical Circus* shows in schools.

In addition to the regular shows for students, the *Mathematical Circus* also performs other types of shows:

- Shows for the general public (including families) in science centers.
- Sessions exclusively for teachers. These sessions are intended to perform the tricks and explain in detail all the mathematical content so that teachers can then perform them with their students.
- Street performances and face-to-face interactions in science fairs and cultural festivals.
- Shows for the academia at international conferences such as: the International Conference ICME, International Congress on Mathematical Education, Seoul, South Korea, July 2012; Gathering for Gardner 2012 and 2014, USA; Hands-on Science, Aveiro, Portugal, July 2014; Conferência Internacional Espaço Matemático em Língua Portuguesa, Coimbra, Portugal, October 2015; Recreational Mathematics Colloquium IV and V, Lisbon, Portugal, January 2015 and 2017.



**Figure 2:** *The Mathematical Circus shows*

Overall the Lisbon team has performed an average of 50 shows per year for about 5000 spectators/year. The north team has performed about 20 shows per year for about 2500 spectators/year, mostly students. These differences are mostly due to regional differences (the Lisbon area has a much larger population than the region around Aveiro).

### ***Show Contents***

The *Mathematical Circus* uses a rich variety of tricks. Each show consists of about 10 contrasting tricks. Next we give an example of a possible program for a one hour show.

#### **Opening circus music and host welcome**

**Guessing a volunteer’s birthday** – A volunteer is shown several lists of numbers and of months. All he must do is tell if his day/month of birth is, or is not, in the given lists. Based on the answers and using binary code the magician “guesses” his date of birth.

(DJ: plays the happy birthday music to introduce the topic.)

**Clown guessing a volunteer's birthday** – The clown makes his first appearance and tries to guess another volunteer's birthday (month of birth) by playing a joke on him. He claims he can guess the volunteer's month of birth with only one list. His list has one only word – January – and the volunteer must do as before, say if his month of birth is, or is not, in the list. Most of the times the answer will be no and the clown finds himself in trouble which he solves by using logic – he firmly states that the volunteer was not born in January and claims for success as his statement is obviously true!

**Unraveling joined ropes** – Two volunteers are joined together by two ropes and then asked to unravel themselves. In all the shows we have done so far, not one volunteer has ever been able to find out that the “holes” around their wrists must be used to achieve the goal. This is an old topological effect described by Martin Gardner [8] using ropes and holes.

(DJ: plays humorous music as clown performs unexpected diversions.)

**Against probability** – After asking a volunteer to randomly shuffle (American shuffling) a prepared deck of cards, the magician is able to guess the colors, the suits and even the numbers of several sets of cards, until running out the deck. This trick is based on Gilbreath's principle [7] which concerns the preservation of some order properties in shuffled decks.

**Guessing a word** – The volunteer is asked to guess a word, previously hidden in the room, from a thick book (we use one of the Harry Potter books). He starts with a three digit number of his choice, reverses the digit order and subtracts the smallest number from the largest. He then reverses the order of the digits of the difference and sums this number to the difference obtaining 1089, irrespectively of the starting number. This number is the code to the word in the book – the participant is asked to look up at page 10, line 8, word 9 (trick based on Ball [3] and described in more detail below).

(DJ: plays music as volunteers do the calculations.)

**Unfair division of sweets** - the clown comes in with a bag full of sweets. Misusing arithmetic, he takes advantage of an unfair division of the sweets. This diversion is based on a scene by Abbot and Costello [1] where one of the actors misuses the division algorithm to conclude that 7 into 28 gives 13. He then misuses the multiplication algorithm to obtain  $7 \times 13 = 28$ , and finally by adding  $13 + 13 + 13 + 13 + 13 + 13$  he (wrongly) obtains 28. The same type of misleading procedures can be used to obtain  $25 \div 5 = 15$ , or  $24 \div 4 = 15$  or  $36 \div 6 = 15$ .

**Handcuffed undressing** – Staging a car accident where the volunteers put on reflecting vests (on the wrong side) a policeman (disguised clown) comes in and handcuffs them. Thereafter, they are challenged to turn the vests inside out, without removing the handcuffs. This trick is a stage adaptation of one of Gardner's tricks [8].

(DJ: plays realistic sirens and crash sounds during the staged car accident.)

**Faster than a calculator** – A volunteer is asked to write down the first 10 terms of a generalized Fibonacci sequence starting with two numbers of his choice. He is then asked to sum up the 10 numbers using a calculator while the magician uncovers his eyes and gives the result in a flash since the sum is equal to the 7<sup>th</sup> number multiplied by 11 (see Bastos [4] for a detailed explanation).

(DJ: plays music as volunteers do the calculations.)

**News** – The clown makes a news presentation made up of mathematical jokes like “This morning a student in New York ate his math homework. It seems his teacher said it was a piece of cake”.

(DJ: plays a news announcing music)

**Huge hole in an A4 paper** – Several volunteers are asked to cut a hole in an A4 piece of paper such that all of them can go through. The clown joins in the group and he is the first to achieve the goal! One possible solution may be found in Ball [3].

(DJ: plays background music while volunteers try to find a solution.)

**Crystal Ball** – Everybody in the room is asked to think of a two digit number, sum its digits and subtract the sum from the initial number. Then, by looking up the result in a projected table of numbers and symbols, the mentalist is able to guess which symbol is attached to everybody’s results. This trick is based on the flash applet The Flash Mind Reader [13] and is based on the divisibility rule for nine since the result is always a multiple of nine (starting with a two digit number,  $\overline{xy} = 10x + y$ , and subtracting the sum of the digits always gives a multiple of nine:  $(10x + y) - (x + y) = 9x$ ).

(DJ: plays “trance” music to create the atmosphere; lights are dimmed.)

A professional magician never reveals his tricks. It’s part of his business. In contrast, the *Mathematical Circus* team loves to explain all its tricks at the end of a show. The wonder of magic is never broken during the show, but once it is over it is important that the students have the opportunity to grasp the mathematics behind the tricks. Therefore, in all shows the public is informed that at the end of the show the team is available to explain all the tricks performed.

The mathematics behind the tricks covers different topics and has different levels of depth. Some tricks provide a wonderful opportunity to explore algebraic reasoning. As an example we provide one example of an explanation of a trick (Guessing a word), using algebraic expressions. We provide one possible way to show that when starting with a 3 digit number  $\overline{xyz}$  (the first digit must be different from the last one:  $x \neq z$ ), reversing its order, subtracting the smallest from the largest ( $x > z$ ), reversing the order of the sum digits and adding the sum with its reversed version always gives 1089 as the result. This is done through the usual subtraction and addition algorithms (in Portugal subtraction is usually performed using the equal additions algorithm as in the following example). This trick explanation is a good example to work algebra in middle school classes, providing students with a simple proof and showing them that variables do not appear only in equations and can be used in other contexts. We start with the subtraction: since  $x > z$  the units digit of the minuend has to be increased ( $z + 10$ ) and the tens digit of the subtrahend is increased accordingly ( $y + 1$ ) so that the difference is kept unchanged. Now the tens digit of the minuend ( $y$ ) is again smaller than that of the subtrahend ( $y + 1$ ) and the same procedure is applied to the tens and hundreds (underlined grey operations).

Hundreds	Tens	Units
$x$	$y + \underline{10}$	$z + 10$
$- \underline{z + 1}$	$\underline{y + 1}$	$x$
$x - z - 1$	$9$	$z + 10 - x$

Next we use the addition algorithm after reversing the order of the digits of the difference. For the units we have  $(z + 10 - x) + (x - z - 1) = 9$  and for the hundreds we have  $(x - z - 1) + (z + 10 - x) + 1 = 10$ . The final result is 1089.

$x - z - 1$	$9$	$z + 10 - x$
$+ \quad z + 10 - x$	$9$	$x - z - 1$
<b>10</b>	$\longleftarrow \textcircled{1}8$	<b>9</b>

### Concluding Remarks

Mathematical magic may be used to engage students with mathematics, showing them that mathematics can be fun, attractive and surprising. In Portugal there is a great need to change a common belief that mathematics is difficult and not really important. In an attempt to fight this belief and increase students’ motivation towards mathematics, the *Mathematical Circus* project was created. In this project mathematics is interlaced with the performing arts – theater and circus – in innovative shows made up of mathematical magic, fooleries and jokes (magic show with a circus atmosphere). The *Circus* teams have brought fun and surprise to more than 40 000 Portuguese students during the last 6 years.



The Mathematical Circus attracts not only students but also their teachers who find it interesting to use magic in their classes. Some pedagogical materials have already been developed stemming from this project (Bastos [4], Marques [9] and Rodrigues [10]).

By their nature, the *Mathematical Circus* shows consist of a one-time interaction between the public and the team. This makes it difficult to evaluate the impact of the project. However, evaluation can be done indirectly through the following aspects:

- During the shows there is a constant response/reaction of the public which clearly indicates that the students are completely delighted and absorbed with the tricks;
- Many schools invite the team back year after year.
- The *Mathematical Circus* does not promote intentional advertisement because it is not a commercial project and the human resources are quite limited. There is a home page [6] and a facebook page and most schools know about the project from other schools (word of mouth publicity). The increase of calls (and consequent number of shows) over the last years is a good indication of the popularity and success of the project.
- Several school teachers have engaged with the project and have been tutored in order to use the tricks in their classroom and explore the mathematical content with their students.

We believe the *Mathematical Circus* project has the potential to grow further, especially if members from the circus and theater communities would join the project. We hope to continue to develop this project and reach even more students and teachers throughout the country and abroad.

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