# Laminar Reciprocal Structures

Javier Barrallo[1], Francisco González-Quintial[2] and Antonio Sánchez-Parandiet[3]

UPV-EHU The University of the Basque Country, Donostia-San Sebastián, Spain
[1] javier.barrallo@ehu.eus, [2] 0.pgquintial@gmail.com, [3] antonio.sanchez@ehu.eus

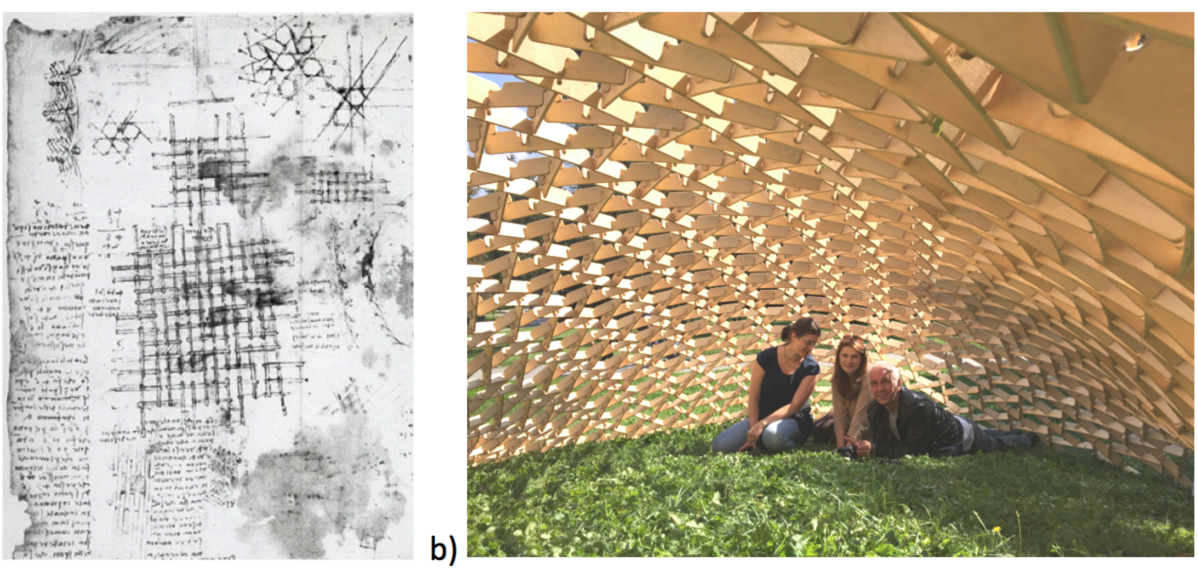
## Abstract

Linear reciprocal structures are constructed by using a wide variety of patterns. These designs are a good source of inspiration when working with laminar constructions (sheets). Using the same formal schemes, laminar constructions feature better bending of their elements along the shape of the model and the application of extra pressure on the connection joints. Many of the geometric constructions made at the University of the Basque Country and presented in this paper show structural, constructive and formal improvements in many reciprocal structures assembled using sheets instead of linear elements.

## Reciprocal Structures

Although reciprocal structures have probably been known since antiquity, the first detailed sketches are found in the folio 899v of Leonardo da Vinci's Codex Atlanticus [9]. In recent years the scientific community has begun to consider these structures as something more than a historical curiosity. Some scholars attribute to Villard de Honnecourt the first drawing of a reciprocal structure, found in the sketchbook he compiled between 1225 and 1250, but his sketch is imprecise and in any case, it should be considered more as a constructive detail than a real structure [3].

The principle of structural reciprocity is the use of elements that lean on each other mutually in a three-dimensional structure [6]. A minimum of three elements is needed, and some friction amongst them is required. Ideally, the unions of elements have no mechanical connections, just pressure and friction, but it is not unusual to find in these constructions notches, ropes, wires or rivets to secure the structure at the contact points [5].



a) b)

**Figure 1:** *Leonardo da Vinci sketches (a) and Sculptor Rinus Roelofs inside one of his domes (b)*

Reciprocal structures have been studied in depth for over twenty years by the Dutch sculptor Rinus Roelofs, who has constructed several impressive wood domes. This type of construction received the name of Leonardo Grids because they used the same constructive principles detailed by Leonardo da Vinci. Rinus Roelofs expanded extensively the sketches of Leonardo into a complex constructive system featuring many different patterns, changes of curvatures and increasing the number of layers [7].

## Passive and Active Bending

Reciprocal structures formed by the arrangement of rigid linear elements may become stable simply by the friction generated in the joints between bars. In these cases, the weight of the bars generates the necessary friction to balance the structure. However, if we work with bars flexible enough, the structure is allowed to change its curvature depending on the bending capability of the elements (Fig. 2a). The new structure may be stabilized under the action of a tensor (Fig. 2b) or create a closed structure linking the bars if the model bends until the ends meet (Fig. 2c).

If we substitute the bending passive rods by laminar elements, these are allowed to be folded in the direction of the axis of weakest inertia and the set turns to behave as a structure with bending active planar patches. Active bending increases the friction forces generated by the weight of the elements to the normal forces to the surface, tending to stabilize the ensemble.

As a consequence, the interaction amongst the laminar elements of the reciprocal structure implies a difference in the geometry of the resulting structure. The pieces are curved following a continuous surface. This difference can be seen in the organization of the elements depending on the position of the connecting flaps inwards or outwards, resulting in a passive or active bending structure. (Figs. 2d, 2e).

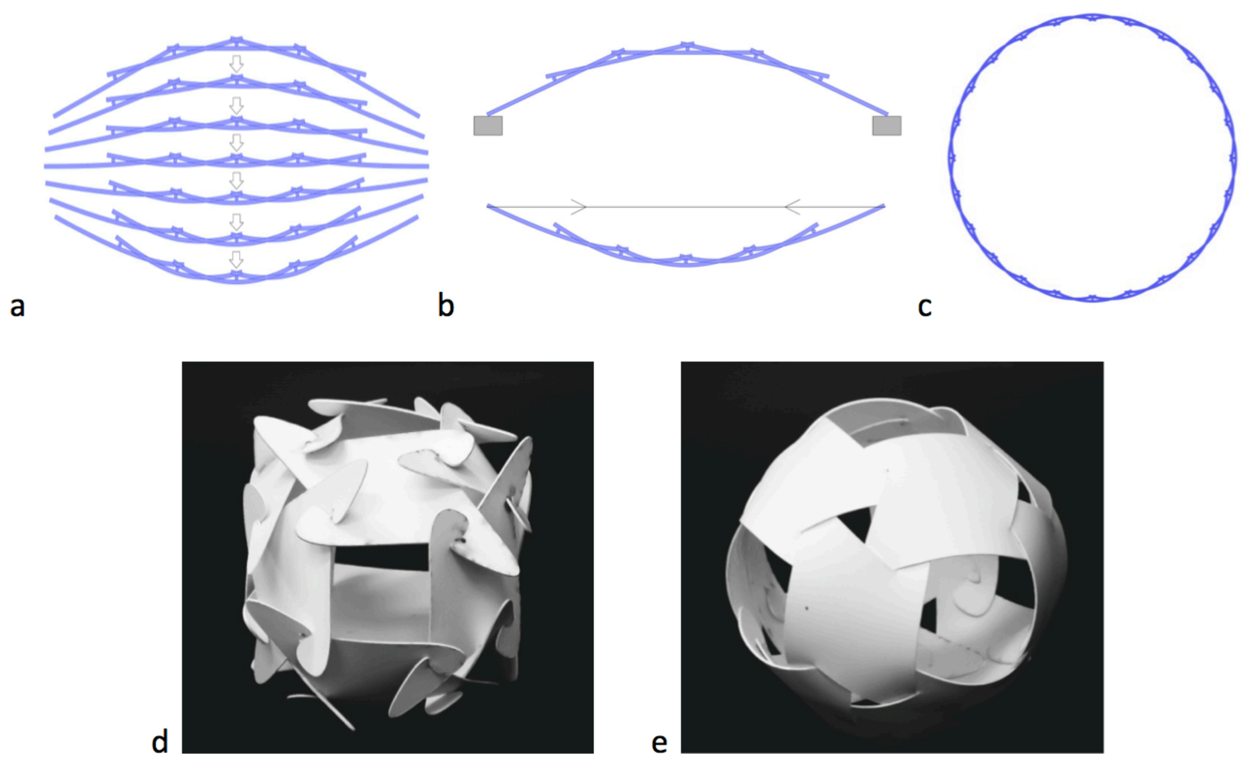



**Figure 2:** *a) Changes of curvature in a reciprocal structure; b) Structure under passive and active bending (with a tensor); c) closed structure (no tensor needed); d) Polyhedron with outwards flaps under passive bending; e) Same Polyhedron with same pieces but inward flaps under active bending*

## Laminar Constructions without Linear Elements

In spite of the increase of experiments involving reciprocal structures in recent years, the scarcity of references to constructions with laminar elements is striking. Some isolated cases seem to appear more by chance than by any systematic analysis with different geometries and settings. There also exist very interesting exceptions, such as the works of Olivier Baverel and Alberto Pugnale [2].

In most cases the planar elements are introduced in the reciprocal structures by expanding the thickness of linear beam elements. In this way, the length of these new elements allows the introduction of deep interlocked joints to fix the structure in a very similar way to that used in the London Serpentine Pavilion by Alvaro Siza, Eduardo Soto de Moura and Cecil Balmond [8].

By increasing the thickness of these pieces and the angle of the joints, very solid structures may be created. But far from being flat structures composed of planar elements, they are rather spatial structures, composed of sheets assembled in different planes following the scheme of an ordinary linear reciprocal structure.

One of the most interesting results of our work shows that the sheets covering the empty spaces of a linear reciprocal structure work perfectly as substitutes of the structural elements (rods). The sheets both support the construction and act as a planar closing surface without the presence of any beam element (Fig. 5). The example shown in Figure 3, which was assembled by associating pairs of rods with square and rectangular sheets shows simultaneously both linear and laminar reciprocal structures.

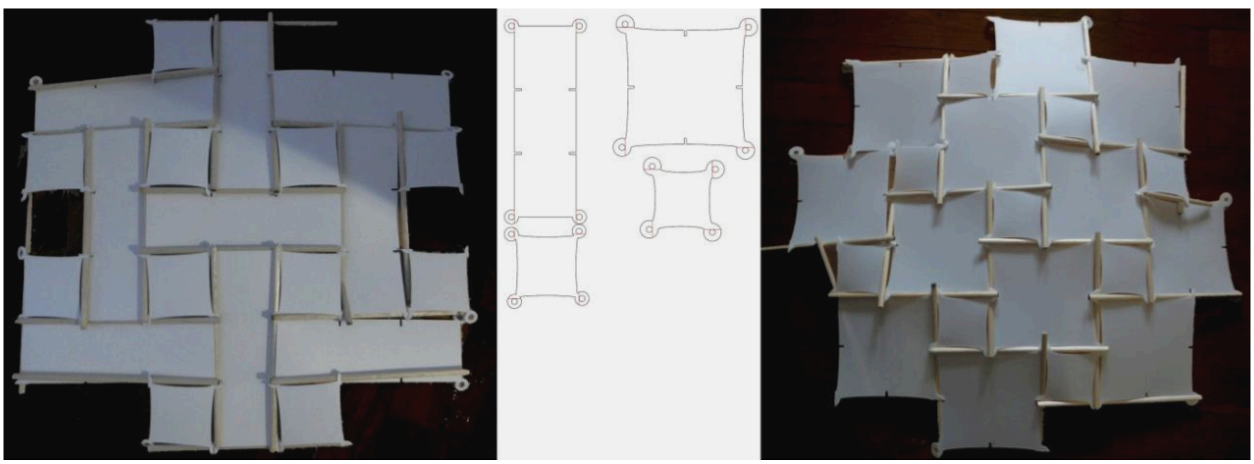


**Figure 3:** *Two basic structural frames with its laminar closing. The one on the left is based on planar elements 3x1 and 1x1, the one on the right has tiles 1x1 and 2x2.*

Another model, Fig. 4a, represents a reciprocal pattern with square pieces. The borders of the squares work like the rods, flexing due to the thrust over the edges of the adjacent pieces. The original shape is distorted because of the presence of four tabs around the square. Two of the tabs lap over two sheets, while the other two give support to another pair of sheets. The upper and lower view of this structure can be seen in pictures 4b and 4c. The arrows in Figure 4d show the direction of the forces that the four white sheets apply over the colored one, alternating up and down directions depending on the situation of the tabs. Note that the eight tabs are concealed in the upper view and are visible in the 4c view.

If the material of the sheets provides enough friction, the reciprocal structure will be stable; otherwise the use of tacks or glue is necessary. For many of the small scaled models developed in this paper we have used 0.8 mm polypropylene. Polypropylene is the lightest of commercial plastics and features a hinge effect that allows bending without breaking, unlike other similar plastics like PVC. Unfortunately it is very slippery (less friction), but it has a high resistance to many chemical products.

Polypropylene produces a straight apparent contour when bending (see the top image in Fig. 4e) meanwhile other materials like plastic foams produce more curved apparent contours (see the bottom image in Fig. 4e).

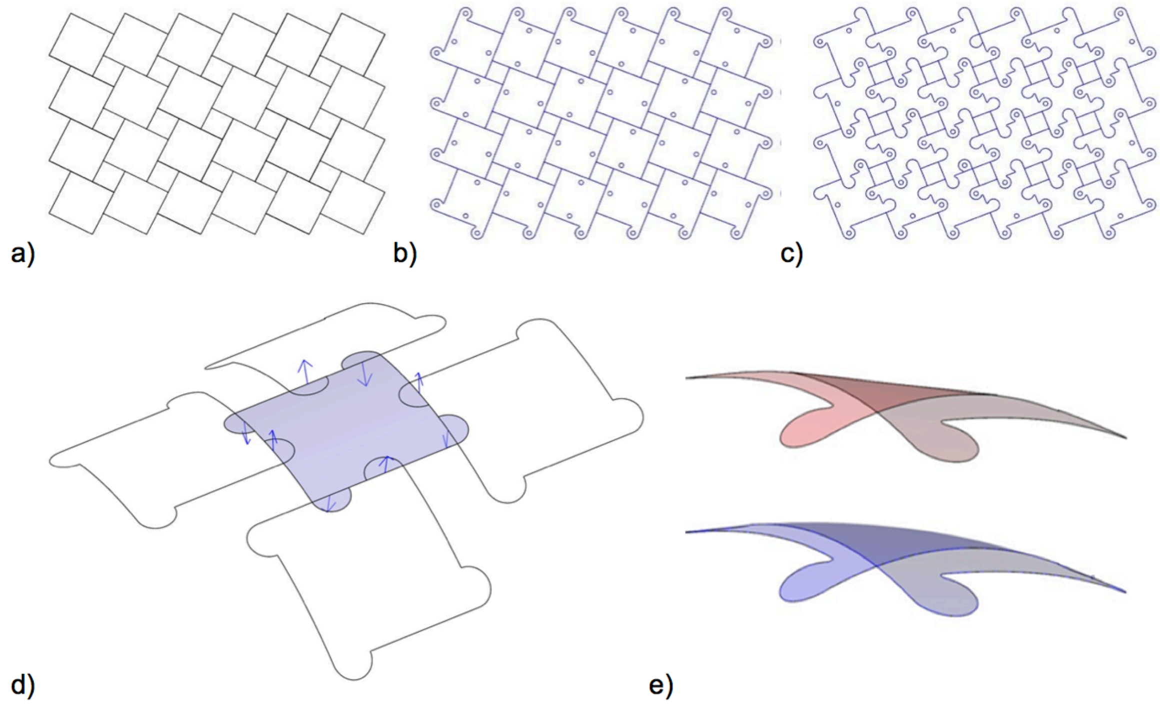


**Figure 4:** *a) Reciprocal frame structure pattern with square sheets and without rods; b) Upper view; c) Lower view; d) Direction of the forces from the white sheets over the blue one; e) Apparent contours of one sheet – red (above), straight; and blue (below), curved – due to the use of different materials.*

## Models Associated to Textile Reciprocal Patterns

Up to this point, our work has focused on the fact that the sheets can cover the reciprocal frame structure working as substitutes of the linear structural elements. Hence no kind of bar, rod or beam is required, as sheets function simultaneously as structure and enclosure. When the shape is filled up using sheets as strips following a nesting or weaving system we called them textile reciprocal patterns. The number of solutions to cover a surface using textile reciprocal patterns is really huge, depending on the kind of pattern we choose [1].

In order to join the sheets together it is necessary to add some tabs that work as joints between them. The bending of the sheets produces the necessary amount of force to fix the tabs. On every par of tabs, one laps over the sheet, while the other gives support to another, generating a reciprocal system, as shown in Fig. 4d. Materials with enough friction result in stable structures; otherwise the use of a mechanical joint system will be necessary.

## Comprehensive Analysis of Constructions Based on a Single Model of Sheet

In order to study the possibilities of the laminar constructions described in previous sections we want to show some works developed just with a single model of sheet (Figures 6 and 7). All the constructions are based on laminar reciprocal structures working with flexion. The selected piece is rhombus shaped based on two equilateral triangles with four tabs. Each tab occupies a quarter of the side of the diamond. The size of the tab is important because bigger tabs will produce tighter connections with more flexion while smaller tabs will leave more empty spaces in the model. If the tab occupies half of the side of the rhombus no empty spaces will be produced in the connections.

The possibilities of assembling depend on the design of the piece and its capabilities to be fitted with identical ones. In some cases, slight alterations in the geometry of the sheet or the tab decreased the difficulties during assembling, but the goal of our challenge was the use of a unique geometry.

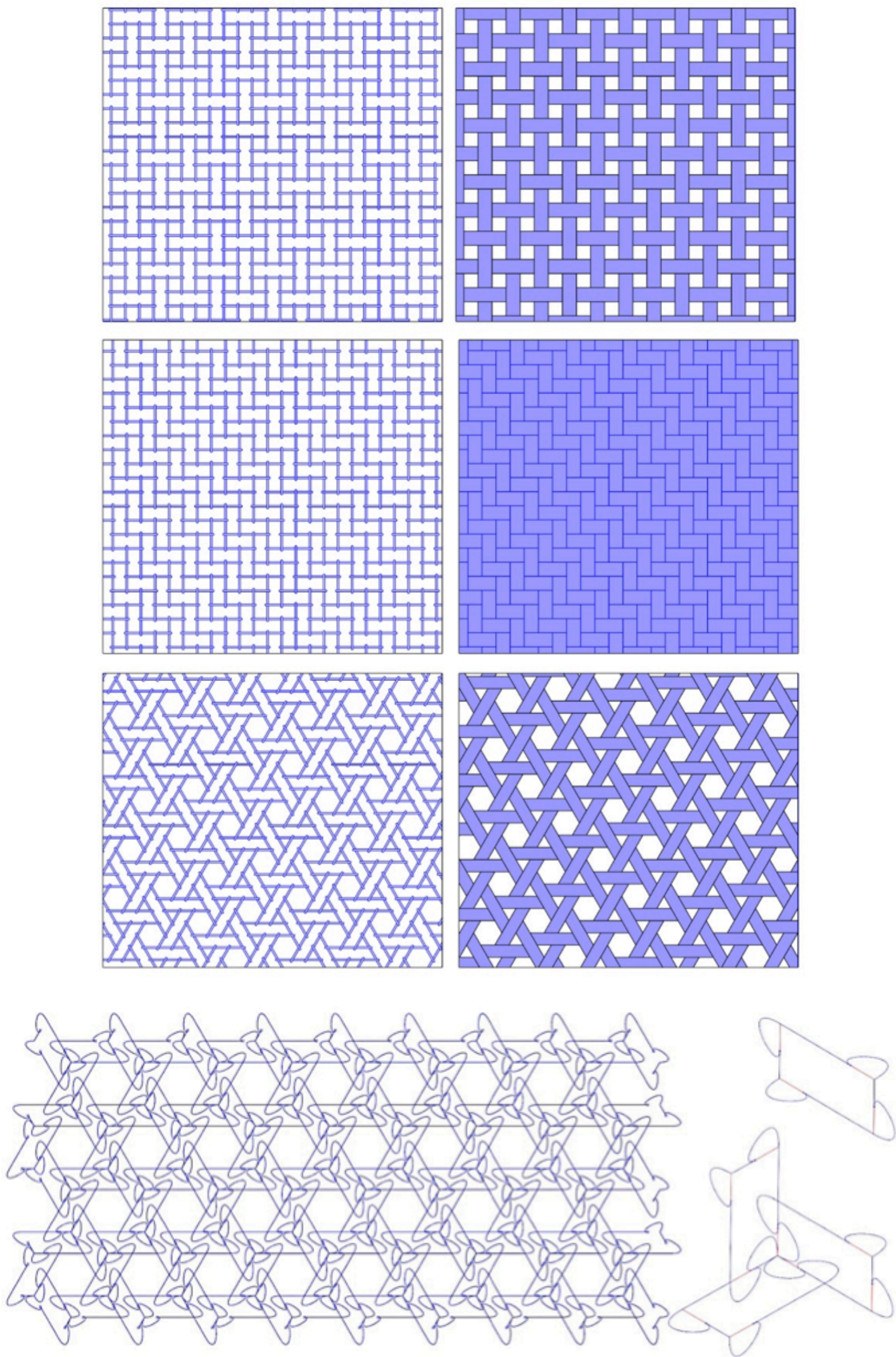


**Figure 5:** *above) Examples of three reciprocal models associated to textile patterns; below) the construction of the third textile pattern shown in detail.*

The first logical generation is the construction of a planar surface that may be used as a guide to construct more sophisticated geometries. In this section we will show planar constructions and the cylinders and cones associated to them. From our experiments we have succeeded in generating twelve planar patterns, all related to square or hexagonal grids. However, one of them is non-periodic and cannot be transformed into a cylinder. Therefore, only eleven regular patterns may be converted into cylinders. Obviously, by extending the planar surface along the y-axis direction we extend the length of the cylinder and by extending it along the x-axis we generate cylinders with longer radii.

Just as a planar pattern can be bent into a cylinder, a pattern constructed with the shape of a circular sector may be warped into a conical shape. However, it is not easy to assemble both straight lines of the circular sector because they will fit correctly only when selected angles are used. We illustrate in this paper only one pattern, producing five cones depending on the angle used in the circular sector. Fig. 6b generate five cones each with angles in the circular sector of 60°, 120°, 180°, 240° and 300°. Note that in some cases, when the apex of the cone is slimmer than the bending allowed by the sheet, the vertex may not be completed, creating a kind of oculus.

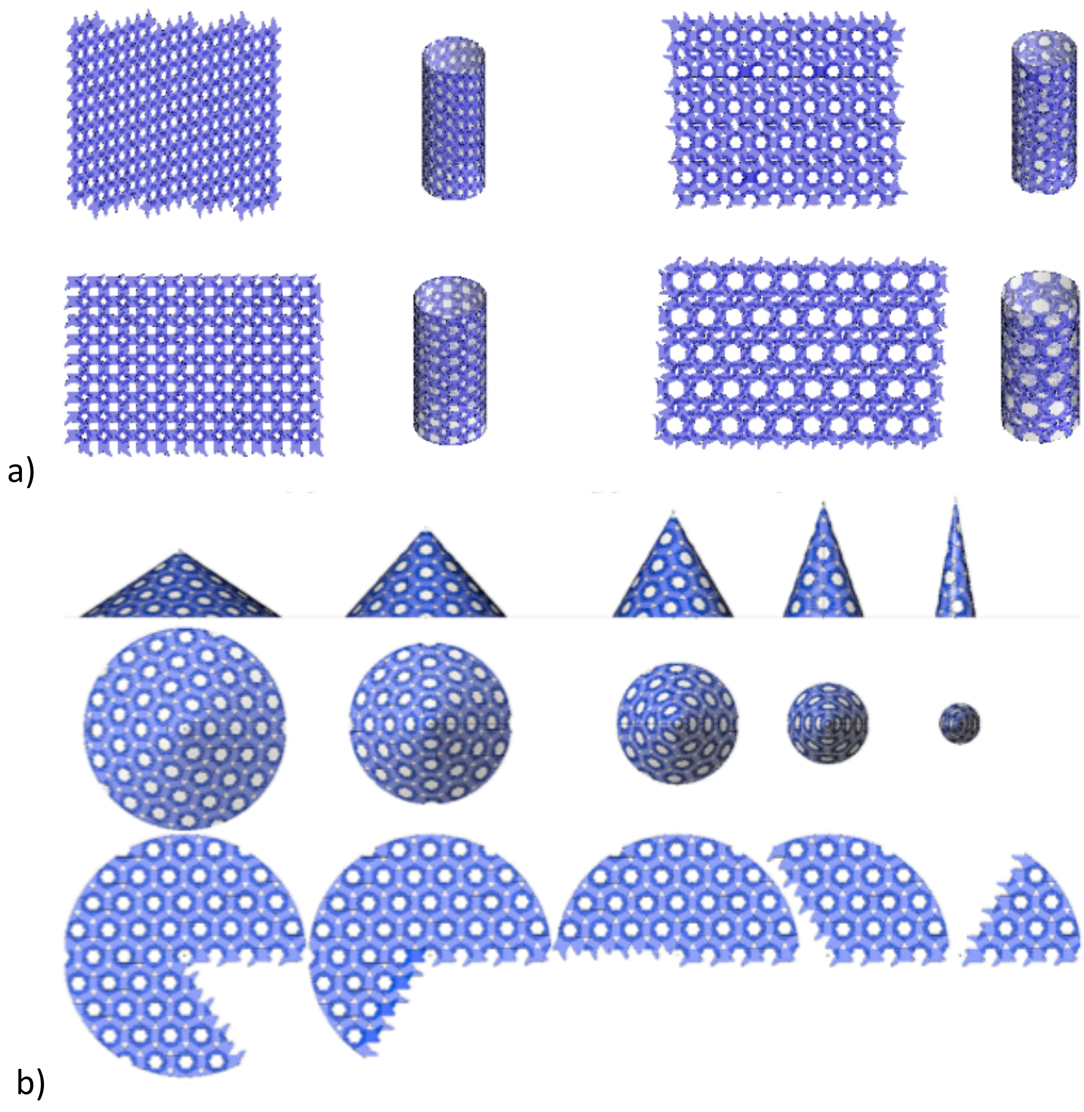



**Figure 6:** *a) Four examples of the eleven possible planar patterns and the regular cylinders associated to them. b) An example of five cones generated using the same planar pattern.*

## Platonic Solids

Polyhedra are always a good test for three-dimensional constructions. In this case, by weaving our sample piece, we can produce easily all the Platonic and Archimedean solids, that is, all regular and semi-regular convex polyhedra. Every polyhedron has several associated laminar reciprocal patterns, so they can be constructed in different ways, depending in how pieces are assembled. The constructive material used is smooth and bends nicely so the pieces do not need any closing system apart from the bending-frictional mechanism.

Note that all the polyhedra constructed in this section have two different shapes of perforations. One is the perforation corresponding to the regular polygon that generates the polyhedra (one type of polygon for regular polyhedra and two or three polygons for semi-regular polyhedra). The second perforation corresponds to the gaps in the union of the tabs, which leaves empty spaces with the polygonal shapes corresponding to the dual polyhedron. The rhomboid pieces are the edges; the tabs situated in the shorter sides define the apex of the polyhedron. The apexes generate empty spaces that cannot be considered faces; the empty spaces around the edges are the faces of the polyhedron.

Thus, the cube in Figure 7a has six squares as generative polygons for a cube plus eight triangles as gaps in the union of the tabs. The octahedron in Figure 7c (dual of the cube) has eight triangles as generative polygons and six squares in the joints. Dodecahedra (Fig. 7d), composed of twelve pentagons and Icosahedra (Fig. 7e), dual of dodecahedra and composed of twenty triangles, also present this duality in the number of polygons and the number of gaps created in the assembly of the tabs.

This generation study starting from a single piece shows several examples but there are many more than the ones shown here. The polyhedra can grow by adding pieces and these may also be duplicated, triplicated or quadrupled. The cylinders can be skewed, the cones can change their apex, flat patterns can be curved on any developable surface and the width of the tabs may be modified in order to decrease or increase the gaps between pieces.

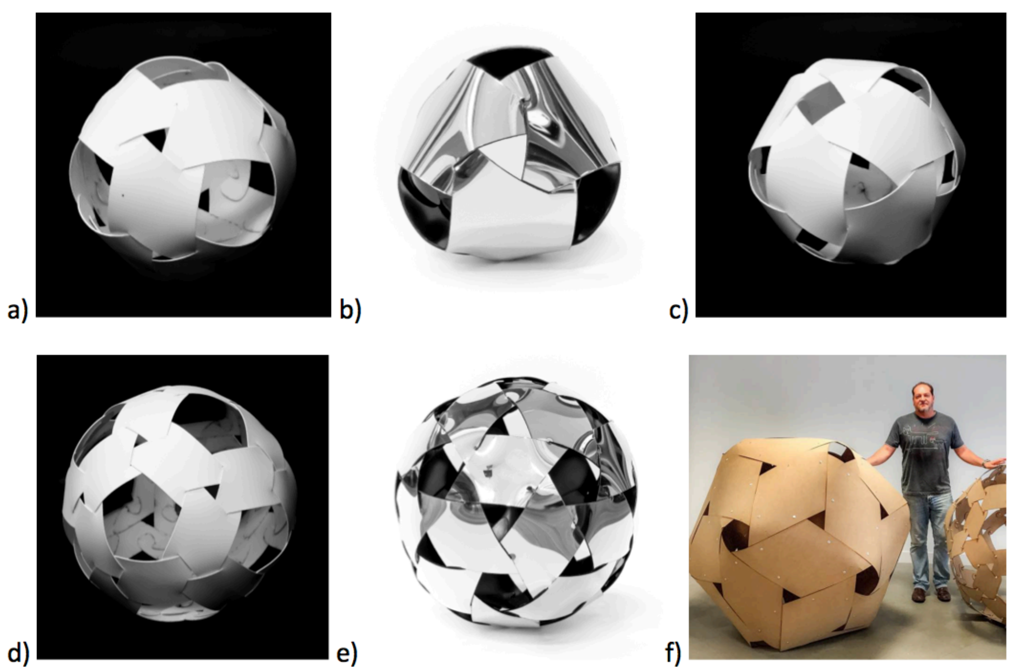



**Figure 7:** *Platonic Solids: a) Cube; b) Tetrahedron; c) Octahedron; d) Dodecahedron; e) Icosahedron; f) Big scale Icosahedron, constructed using medium density fibreboard with a height of 150 centimetres. Although it is stable and self-supporting, all the laminar panels were secured with rivets as a precaution.*

## Conclusions and Future Works

The geometry of linear reciprocal structures is highly conditioned by the thickness and curvature of its structural elements. The freedom of design is much greater in laminar reciprocal structures by flexion, since the thickness does not condition the form. When the geometry is complex, the difference between both types of structure is further accentuated and the scope of application is wider: separating panels, light facades, ephemeral constructions... and the architectural possibilities are more interesting.

The generated surface acts as a sheet with perforations. The pieces can be glued to form a unitary element, avoiding shear stresses in the overlaps and reinforcing the joints. Waterproofing, the great problem of reciprocal structures, is more easily achieved than in linear reciprocal structures, as there is more continuity with less protrusion. The shapes generated with simple curvature (cones, cylinders, convolutes...) can have a textile or laminate coating without facings or cuts,

It is important to highlight that there are many types of surfaces that may be covered using this kind of sheet. At this point it is possible to anticipate that any kind of developable surface can be built using sheet patterns. Warped and double-curvature surfaces may be adapted successfully and the possibilities for further works are really enormous. The application of this type of reciprocal structures on developable surfaces and their adaptation to complex double-curvature forms is demonstrated in [4].

Planar reciprocal structures avoid the complex connections that are generated between the bars organized in reciprocal patterns where solutions of difficult stereotomy appear. The main goal of our research has been to study and document the transition from reciprocal frame structures into closed reciprocal laminar structures without rigid elements. A future objective is not only to design and construct new models, but also to analyze in depth the behavior of the sheets in these constructions.

The two-dimensional character of the sheets makes them very suitable for the construction of bending reciprocal structures. The elements that compose the structure tend to tighten their overlaps and the stresses at the edges generate a surface with more continuity that typical structures working just under gravity. While the rods forming a structure maintain their rigidity, sheets tend to bend if the material has sufficient elasticity. For these reasons, laminar reciprocal structures may be useful in the future of construction because they provide a smart adaption into modern buildings construction techniques.

## References


[1] O. Baverel and O. Popovic. "A Review of Woven Structures with Focus on Reciprocal Systems–Nexorades." *International Journal of Space Structures*, vol. 26, no.4, 2011, pp. 281–288.

[2] O. Baverel and A. Pugnale. 2014. "Reciprocal Systems Based on Planar Elements: Morphology and Design Explorations." *Nexus Network Journal*, vol. 16, no. 1, 2014, pp. 179–189.

[3] S. Duvernoy. "An Introduction to Leonardo's Lattices." *Nexus Network Journal*, vol. 10, no.1, 2008, pp. 5–12.

[4] P. González. "Adaptación de Superficies de Doble Curvatura Mediante Superficies Desarrollables." *EGA: Revista de Expresión Gráfica Arquitectónica*, vol. 21, no. 27, 2016, pp. 210–219.

[5] O. Popovic. *Reciprocal Frame Architecture*. Elsevier Science & Technology, 2007.

[6] A. Pugnale, D. Parigi, P. H. Kirkegaard and M. Sassone. "The Principle of Structural Reciprocity: History, Properties and Design Issues." *Proceedings of the IABSE-IASS Symposium 2011: "Taller, Longer, Lighter"*, 2011.

[7] R. Roelofs. "Two- and Three-Dimensional Constructions Based on Leonardo Grids." *Nexus Network Journal*, vol. 10, no.1, 2008, pp. 17–26.

[8] A. Siza, E. Souto de Moura and C. Balmond. *Serpentine Gallery Pavilion 2005*. Trolley Books, 2005.

[9] K. Williams. "Transcription and Translation of Codex Atlanticus, fol. 899v." *Nexus Network Journal*, vol. 10, no.1, 2008, pp. 13–16.