Marjorie Rice and Her Pentagonal Tilings

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Abstract

The large pentagonal tiles that pave the way to the entrance of the Tekniska Museet in Stockholm may appear to be randomly placed. But they are not; they follow a definite but unusual pattern. Also unusual is the history of how the tiling came to be, and who discovered it. Marjorie Rice, who was not a professional mathematician, discovered this tiling while pursuing for years her self-assigned task of finding all types of convex pentagons that can tile the plane.

The Search for Tiling Pentagons

Which convex polygons can tile the plane? That question, addressed by Martin Gardner in his July 1975 "Mathematical Games" column in *Scientific American*, had far-reaching consequences. Any triangle or quadrilateral (even non-convex) can tile the plane by rotating it repeatedly about the midpoints of its sides. No convex polygon of seven or more sides can tile the plane. One hundred years ago, K. Reinhardt characterized five types of convex pentagons and three types of convex hexagons that could tile the plane. Types were characterized by conditions on the sides and angles of the tiles; each type could represent an infinite number of shapes. The problem then lay dormant for 50 years. In 1968, R. Kershner published his discovery of three more types of convex pentagons that could tile the plane; this was the impetus for Gardner's column. Kershner also claimed that there were no other tiling pentagons.

Photo courtesy of Lars Paulsson

Figure 1: *The entryway to the Tekniska Museet (Technical Museum) in Stockholm, Sweden.*

Within a month, Gardner received an example of a new convex pentagon that could tile; its discoverer was R. James, a computer scientist. Gardner published it in his December 1975 column. In February 1976

Gardner received yet another example of a tiling pentagon, this time from Marjorie Rice, a 52-year old California homemaker with no mathematical training beyond a general math course in high school. Gardner sent me her example to verify it was new, and asked that I correspond directly with her. Her example was indeed new, and thus began a mathematical friendship that lasted for over 30 years.

With no geometry background, Marjorie made up her own symbolic system of labeling the known pentagon types, indicating with arcs inside stylized pentagons how angles would meet in a tiling. She made tables of symbolic labeled pentagons, and pursued a methodical combinatorial search for new possibilities. Each time a symbolic labeling seemed promising, she would sketch (by hand) a sample tiling by pentagons that fulfilled the conditions. Her sketches grew in number, and by December 1976 she had discovered another new type of tiling pentagon; a year later she discovered yet two more new types. During this time, and in the years that followed, I provided her with some reprints and other sources that might be helpful—she devoured technical information from illustrations and turned it into fodder for her investigations. All this was done without her family's knowledge, in the spare moments she had alone at home when her husband and children were at work and school.

She learned about isohedral and block-transitive tilings and set out to find all 2-block and 3-block tilings by pentagons. There were intervals when travels and family events caused her to lay aside her work. She also got interested in aperiodic tilings and tilings by nonconvex pentagons and pursued these interests, producing many examples. In the early 1990s, she was still pursuing the goal of finding all 3-block pentagon tilings. I sent her some printed combinatorial output of 2-block and 3-block tilings from a computer program by D. Huson. She then analysed (by hand) all of this output (several thousand cases) and discovered many new tilings by congruent convex pentagons. Among these was a particularly beautiful tiling by a pentagon that she called a "versatile" because it could combine in so many ways. This tiling was chosen in 1999 by the Mathematical Association of America to grace the foyer of its headquarters in Washington, DC; its glazed ceramic tiles are all ivory. Eighteen years later, the tiling was chosen and installed with three colors of stone tiles to pave the way to the Tekniska Museet entrance (Figure 1).

Marjorie, an admirer of M.C. Escher, used several of her pentagonal tilings to provide an underlying grid for some beautiful tessellations by flowers, shells, fish, and bees. Her artistic work, as well as details of her life and her remarkable investigations can be found in the references.

Acknowledgement

I am grateful for the opportunity to tell Marjorie's extraordinary story in the special issue of the *JMA* dedicated to the memory of Reza Sarhangi [5], and in the Bridges 2018 Reza Sarhangi Memorial Lecture. Reza loved to study and write about tilings, and was a master at encouraging non-professionals to pursue their mathematical and artistic interests in their own way.

References

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