Bridges Waterloo

Mathematics, Art, Music, Architecture, Education, Culture

Conference Proceedings









Celebrating the 20th Annual Bridges Conference at the University of Waterloo Waterloo, Ontario, Canada



Proceedings 2017

David Swart, Carlo Séquin, and Kristóf Fenyvesi, Editors

Tessellations Publishing, Phoenix, Arizona

Editors

Program Chair

David Swart

Waterloo, Ontario, Canada

Short Papers Chair

Carlo H. Séquin

Computer Science Division University of California

Berkeley, USA

Workshop Papers Chair

Kristóf Fenyvesi

Department of Music, Art and Culture Studies University of Jyväskylä Jyväskylä, Finland **Production Chair**

Craig S. Kaplan

Cheriton School of Computer Science University of Waterloo Waterloo, Ontario, Canada

Bridges Waterloo Conference Proceedings (www.bridgesmathart.org). All rights reserved. General permission is granted to the public for non-commercial reproduction, in limited quantities, of individual articles, provided authorization is obtained from individual authors and a complete reference is given for the source. All copyrights and responsibilities for individual articles in the 2017 Conference Proceedings remain under the control of the original authors.

ISBN: 978-1-938664-22-9

ISSN: 1099-6702

Published by Tessellations Publishing, Phoenix, Arizona, USA (© 2017 Tessellations) Distributed by *MathArtFun.com* (mathartfun.com).

Proceedings and Catalog Cover Design: Sara Leblanc, University of Waterloo

Bridges Board of Directors

Kristóf Fenyvesi

Department of Music, Art and Culture Studies University of Jyväskylä Jyväskylä, Finland

Cheriton School of Computer Science University of Waterloo Waterloo, Ontario, Canada

Craig S. Kaplan

Sujan Shrestha

Division of Science, Information Arts and Technologies University of Baltimore

Eve Torrence

George W. Hart

Stony Brook University

Stony Brook, NY, USA

Carlo H. Séquin

Computer Science Division

University of California, Berkeley, USA

Department of Mathematics Randolph-Macon College Ashland, Virginia, USA

Conference Organization

Amy Aldous

Faculty of Mathematics University of Waterloo Waterloo, Ontario, Canada

Anna Beard

Faculty of Mathematics University of Waterloo Waterloo, Ontario, Canada

Brittany Ottewill

Faculty of Mathematics University of Waterloo Waterloo, Ontario, Canada

Artistic and Scientific Committee Members and Coordinators

Robert Fathauer

Tessellations Phoenix, Arizona, USA Art Exhibition

Kristóf Fenyvesi

Department of Music, Art and Culture Studies University of Jyväskylä Jyväskylä, Finland Family Day

Sarah Glaz

University of Connecticut Storrs, Connecticut, USA Poetry Reading

Katie McCallum

University of Brighton Brighton, England, UK Art Exhibition

Mark Vuorinen

Conrad Grebel University College University of Waterloo Waterloo, Ontario, Canada Music Night

Tiffany Inglis

D2L Waterloo, Ontario, Canada Technical Support

Nathan Selikoff

Digital Awakening Studios Orlando, Florida, USA Technical Support

Bianca Violet

IMAGINARY Berlin, Germany Short Film Festival

University of Waterloo Advisory Committee

Linda Carson

Department of Fine Arts University of Waterloo Waterloo, Ontario, Canada

Rob Gorbet

Department of Knowledge Integration University of Waterloo Waterloo, Ontario, Canada

Ruxandra Moraru

Department of Pure Mathematics University of Waterloo Waterloo, Ontario, Canada

Benoit Charbonneau

Department of Pure Mathematics University of Waterloo Waterloo, Ontario, Canada

Rick Haldenby

School of Architecture University of Waterloo Waterloo, Ontario, Canada

J. P. Pretti

Faculty of Mathematics University of Waterloo Waterloo, Ontario, Canada

Daniel Vogel

Cheriton School of Computer Science University of Waterloo Waterloo, Ontario, Canada

Proceedings Program Committee

Abdalla G. M. Ahmed

Khartoum, Sudan

Mara Alagic

Wichita State University Kansas, USA

Ellie Baker

Lexington, Massachusetts, USA

Javier Barrallo

The University of the Basque Country UPV/EHU Spain

Robert Bosch

Oberlin College Ohio, USA **Christopher Carlson**

Wolfram Research Champaign, Illinois, USA

Andrew Cooper

North Carolina State University USA

Kelly Delp

Cornell University New York, USA

Neil Dodgson

Victoria University of Wellington New Zealand **Doug Dunham**

University of Minnesota
Duluth USA

Robert Fathauer

Tessellations Phoenix, Arizona, USA Kristóf Fenyvesi

University of Jyväskylä Jyväskylä, Finland

Paul Gailiunas

Newcastle, England, UK

Susan Gerofsky

University of British Columbia Canada

Sarah Glaz

The University of Connecticut Storrs, Connecticut, USA Susan Goldstine

St. Mary's College of Maryland USA

Gary Greenfield

University of Richmond Virginia, USA

Andrea Hawksley

eleVR, HARC, YCR San Francisco, California, USA

Patrick Honner

Brooklyn Technical High School New York City, NY, USA

Veronika Irvine

University of Waterloo Ontario, Canada

Peter J. Lu

Harvard University Massachusetts, USA

Vince Matsko

University of San Francisco USA

Kerry Mitchell

Phoenix, Arizona, USA

Mike Naylor

Matematikkbølgen Math Creativity and Competency Center Vanvikan, Norway

George Hart

Stony Brook University Stony Brook, New York, USA

Judy Holdener

Kenyon College Ohio, USA

Tiffany Inglis

D2L Waterloo, Ontario, Canada

Darci Kracht

Kent State University Ohio, USA

Penousal Machado

University of Coimbra Portugal

Doug McKenna

Mathemaesthetics, Inc. Boulder, Colorado, USA

Teresa Moore

Ithaca College
Ithaca, New York, USA

Osmo Pekonen

University of Jyväskylä Jyväskylä, Finland

Rinus Roelofs

Hengelo, The Netherlands

Karl Schaffer

De Anza College and MoveSpeakSpin Scotts Valley, California

Carlo H. Séquin

University of California, Berkeley USA

Donald Spector

Hobart & William Smith Colleges NY, USA

David Swart

Waterloo, Ontario, Canada

Bruce Torrence

Randolph-Macon College Ashland, Virginia, USA

Tom Verhoeff

Eindhoven University of Technology
The Netherlands

Radmila Sazdanovic

North Carolina State University USA

Henry Segerman

Oklahoma State University USA

Sujan Shrestha

University of Baltimore Maryland, USA

John Sullivan

Technische Universität Berlin Berlin, Germany

Briony Thomas

University of Leeds Leeds, England

Eve Torrence

Randolph-Macon College Ashland, Virginia, USA

Luke Wolcott

University of Southern California Los Angeles, California, USA

Carolyn Yackel

Mercer University Georgia, USA

Art Exhibition and Catalog Program Committee

Robert Fathauer

Tessellations Company Phoenix, Arizona, USA **Conan Chadbourne**

San Antonio, Texas, USA

Katie McCallum

Brighton, England, UK

Nathan Selikoff

Digital Awakening Studios Orlando, Florida, USA

Karl Kattchee

Department of Mathematics University of Wisconsin-LaCrosse Wisconsin, USA Krista Blake

Perimeter Institute Waterloo, Ontario, Canada

Martin Levin

Portland, Oregon, USA

Short Film Festival Program Committee

Aubin Arroyo Camacho

National Autonomous University of Mexico Mexico City, Mexico **Ana Cristina Oliveira**

Atractor Porto, Portugal

Kim Davidson

SideFX Toronto, Canada **Chantal Landry**

Zurich University of Applied Sciences Zurich, Switzerland

Jos Leys

Antwerp, Belgium

Bianca Violet IMAGINARY

Berlin, Germany

Contents

Preface
Regular Papers
Numbers with Personality
HyperRogue: Playing with Hyperbolic Geometry
Crooked Houses: Visualizing the Polychora with Hyperbolic Patchwork
Sculptural Forms Based on Radially-developing Fractal Curves
Non-euclidean Virtual Reality I: Explorations of H ³
Non-euclidean Virtual Reality II: Explorations of $H^2 \times E$
Invertible Infinity: A Toroidal Fashion Statement
DNA-inspired Basketmaking: Scaffold-Strand Construction of Wireframe Sculptures 57 James Mallos
Making Math Visible
Interwoven Islamic Geometric Patterns
Magnetic Sphere Constructions
An Algorithmic Approach to Obtain Generalized 2D Meander-Patterns

Fun with Integer Sequences	95
A Survey of Symmetry Samplers	103
New Kinds of Fractal Patterns	111
Homage to Eva Hild	117
Artwork Inspired by Dual Dodecahedra and Icosahedra	125
Natural Color Symmetry	131
Let the Numbers Do the Walking: Generating Turtle Dances on the Plane from Integer Sequences	139
Obtaining the H and T Honeycomb from a Cross-Section of the 16-cell Honeycomb Hideki Tsuiki	147
Art of Infinity	153
A Geometrical Representation and Visualization of Möbius Transformation Groups Kento Nakamura and Kazushi Ahara	159
Geometric Factors and the Well Dressed Solids of Archimedes	167
A General Method for Building Topological Models of Polyhedra	175
A Peg Solitaire Font	183
Modelling Seashells Shapes and Pigmentation Patterns: Experiments with 3D Printing	189

Hopeless Love and Other Lattice Walks Tom Verhoeff and Koos Verhoeff	197
Aspects of Symmetry in Bobbin Lace	205
Hidden Beauty in Penrose Tiling: Weavings & Lace Douglas Burkholder	213
Versatile Genius: A Case Study Intersecting Math, Science, Art, and California's National Parks	221
Tuti Inter-Weaving	229
Flowsnake Earth	237
Inter-transformability II	245
Kissing Rings, Bracelets, Roses and Canadian Magnetic Coins: Circle Packing with Ferrite Block Magnets and Magnetic Sheet	253
A Mathematics and Digital Art Course	261
Conics from Polygons: The Chord Ratio Construction	269
Zometool Tribute to Fabien Vienne at Bridges Finland 2016	277
The Artful Kaleidoscopes of the Circular and Spherical Bells	283
Dichromatic Dances	291
The Golden Ratio: How Close Is Close Enough?	299

Seeing and Hearing the Eigenvectors of a Fluid	. 305
On Infinite Kepler-Poinsot Polyhedra	.313
Combinatorics in the Art of the Twentieth Century	. 321
Morphing TSP Art	.329
Short Papers	
3D Printed Tours	.335
Great Books, Poetry and Mathematics	. 339
Hyparhedra Revisited	. 343
Topological Images with Modular Block Print Tiles	. 347
3D Printing in the Secondary Mathematics Classroom	. 351
Math Creations - A Math-Art Competition	. 355
The Poetics of a Cyclic Directed Graph	.359
A Temari Permutation Sampler	. 363
The English Translation of the 1652 Edition of J-F Niceron's Perspective Curieuse	. 367
Triskelion Block Families	. 371

Cut Colored Paper Sculptures of 3D Contour Plots of the Real and Imaginary Parts of Complex Functions	375
Caroline Bowen	
Ballistic Deposition and Aesthetic Patterns	379
Applying Helical Triangle Tessellations in Folded Light Art	383
Émilie, an Opera about Love, Death, and Mathematics	387
Cartesian Lace Drawings	391
Constructing Mini-tools for Tessellations	395
The Sound of Space-Filling Curves	399
A Cellular Automaton for Pied-de-poule (Houndstooth)	403
How to Draw a Line	407
Quilting the Klein Quartic	411
Turing-Like Patterns Revisited: A Peek Into The Third Dimension	415
Algorithmic Aesthetics: Redefining Traditional Islamic Art	419
On the Enumeration of Chequered Tilings in Polygons	423
Designing Modular Sculpture Systems	427

Nebula: Live Dynamic Projection Mapping via Object Saliency	.431
Transforming Squares to Strips in Expanded Polyhedral Forms	. 435
Polyhedra: Eye Candy to Feed the Mind	. 439
Listening to the Logistic Map Andrea Capozucca, Marco Fermani and Simone Giorgini	. 443
Artistic Rendering of Curves via Lattice Paths	.447
3D Printable Golden Sponges	. 451
Using African Designs in Virtual Manipulatives for Geometrical Concept Development	. 455
Constructing Deltahedra from Recycled Plastic Bottles	459
The Complexity of Braids, Cables, and Weaves Modeled with Stranded Cellular Automata	.463
Inversive Diversions and Diversive Inversions	. 467
Shaping Poems – with Visual Forms and Counting	. 471
The Interval Dissonance Rate: An Analytical Look into Chromaticism of Chopin's Op. 10 No. 2 and Scriabin's Op. 11 No. 2	. 475
Obtaining Four Main Animation Cycles Using an Extremely Limited Set of Poses Ergun Akleman, Derya Akleman, Ioannis Pavlidis and Pradeep Buddharaju	. 479
Designing Skeletal Polyhedral Sculptures Inspired by Octet-Truss Systems and Structural Inorganic Chemistry with Bugle Beads	. 483

Surfaces Foliated by Planar Geodesics: A Model for Curved Wood Design	3 7
Tiling Notation as Design Tool for Textile Knotting) 1
Visualizing Math Art Activities at the GameLab)5
The Surprising Versatility of Edge-Matching Tiles)9
Triaxial Weaving for Complex Repeat Patterns and Tessellations)3
Symmetric Binary Trees with Branching Ratios Larger than 1)7
Introducing the Kasparian Constructions	1
Non-Octave Guitar and Keyboard Designs for Ervin M. Wilson	5
The Discovery and Application of the Protogon's Spiral	9
Workshop Papers	
Dancing Rope and Braid Into Being: Whole-body Learning in Creating Mathematical/ Architectural Structures	23
Collaboration in Creating The Mathematical Poem	31
From Rabbit Ears to Origami Flowers: Triangle Centers and the Concept of Function	33
Omnidirectional Robot Construction at the Math Class	39

Author Index	589
Playing in the Lux Dimension	583
Thinking Visually: Triangles as Units of Area	579
Folding the Dragon Curve Fractal	573
How to Use Vector Theory to Write a Story	571
The Aesthetics of Colour in Mathematical Diagramming Eva Knoll, Tara Taylor, Wendy Landry, Paul Carreiro, Katie Puxley and Karyn Harrison	563
Star Origami	557
The Magic of Anamorphosis in Elementary and Middle School	553
Creating Polyhedra with Snapology	547

Preface

Welcome to the 20th Bridges conference! This year the conference is being held at the University of Waterloo in Waterloo, Ontario, Canada. It is an auspicious year to visit Waterloo: 2017 marks the 50th anniversary of the School of Computer Science (which began life as the Department of Applied Analysis and Computer Science) and of the Faculty of Mathematics. It is also the 60th anniversary of the university, not to mention the 150th anniversary of the country. Bridges 2017 marks our third time in Canada and our first visit to Eastern Canada, after the 2005 and 2009 conferences in Banff, Alberta. We are grateful that the Faculty of Mathematics has chosen to host this conference, and we hope that you will have an opportunity to soak up some of the anniversary spirit in the air on campus.

This year's Bridges Program Chair is David Swart. He coordinated an international Program Committee of over 50 experts who provided extensive reviews and editorial comments on submissions. He also served as chair of the regular paper track. Carlo Séquin chaired the short papers track with much help from Douglas M. McKenna. Kristóf Fenyvesi chaired the workshop submissions. Many thanks go to the members of the Program Committee who reviewed the large number of papers received. Special thanks go to Bianca Violet, our Short Film Festival Chair; Aubin Arroyo Comacho, Ana Cristina Oliveira, Kim Davidson, Chantal Landry, and Jos Leys, who served as the jury for the movie festival; Sarah Glaz, the Poetry Reading Chair; and to Kristóf Fenyvesi for organizing the public Family Day.

The 2017 edition of the Bridges proceedings includes 44 regular papers, 47 short papers, and 12 workshop papers. A wide range of topics are explored in this publication: you will find new work on visual art: both 2D (tiling, polygons, space-filling curves, cellular automata, fractals, knots) and 3D (polyhedra, surface modelling, magnetic packing) using a variety of media (including digital media, paper constructions, block printing, fiber arts, 3D printing, and weaving). There are papers that make connections between mathematics and literature, poetry, music, dance, opera, and acrobatic performance. There are discussions on mathematical art, artists, exhibits and competitions. And of course there are topics in mathematical visualization and in mathematics and art in education. Thank you to all of the authors and reviewers for their generous contributions to this year's proceedings, with special thanks to Eve and Bruce Torrence, Craig Kaplan, Carlo Séquin, and Kristóf Fenyvesi for their guidance and advice. We would also like to acknowledge the efforts of George Hart and Douglas M. McKenna and who provided extra support.

This year we were able to offer \$10,000 USD in student travel scholarships to 13 students from around the world who authored accepted papers and created mathematical artworks. We are very grateful to Jade Vinson for his generous contribution, which made this program possible, and to Sujan Shrestha who managed the program.

An exhibition of mathematical art has been an annual feature of Bridges since 2001. This year's exhibition sets a new record for the number of participating artists, over 170. Artists from

Europe, Africa, East Asia, Australia, and North and South America will be represented. A wide variety of artistic media are included in the exhibition, including 2D and 3D digital prints, painting, beadwork, ceramics, wood, metal, quilting, clothing, and paper folding. Artists drew inspiration from the mathematics of fractals, polyhedra, non-Euclidean and four-dimensional geometry, tiling, knot theory, number theory, and more. This year Katie McCallum and Robert Fathauer served as co-curators of the exhibition. Katie was joined by Krista Blake, Conan Chadbourne, Karl Kattchee and Marty Levin to make up the jury. The print catalog was prepared by Conan Chadbourne, and the art submission website was created and administered by Nathan Selikoff. Amy Aldous and Craig Kaplan were the local coordinators in Waterloo for the art exhibition.

Many people contributed to the organization and development of this year's conference. Thanks go first to Kelly Booth and Vic DiCiccio, who initially proposed (in 2013!) that Bridges 2017 be held in Waterloo. Thanks to Ian Goulden, the former Dean of Mathematics, for enthusiastically agreeing to support this project, and to Stephen Watt, the current Dean, and Mark Giesbrecht, Director of Computer Science, for their continuing support. We received a great deal of helpful advice and input from our campus-spanning Advisory Committee, consisting of Professors Linda Carson, Benoit Charbonneau, Rob Gorbet, Rick Haldenby, Ruxandra Moraru, J.P. Pretti, and Dan Vogel. Thanks also to Andrew Houston, Ivan Jurakic, Doug Kirton, and Mark Vuorinen for dedicating time and resources to offer on-campus partner events. We also appreciate the many productive discussions with Andrew Bennett, Krista Blake, Luisa D'Amato, Greg Dick, Bob Egan, Tom Reitz, Brent Wettlaufer, and Graham Whiting about ways to integrate Bridges into the Kitchener-Waterloo community. We are grateful for the fundraising support provided by Ingrid Town, Mark Womack, and especially the tireless and enthusiastic Cynthia Fobert. Sara Leblanc was responsible for the lovely graphic design behind Bridges 2017, including our logo, postcards, posters, and book covers. Finally, our heartfelt thanks go to the staff who provided monumental organizational support and marshalled university resources in support of the conference: Anna Beard and Brittany Ottewill, and most especially the amazing Amy Aldous.

We gratefully acknowledge the support of our sponsors in making Bridges 2017 possible. Thanks to the City of Waterloo, the Fields Institute, and the Musagetes Fund for their generosity in helping to make the conference an exciting and memorable experience for all.

The past year has been a difficult and sometimes painful period of transition, as we have adjusted to the loss of our founder and leader, Reza Sarhangi. We miss his infectious enthusiasm, and recognize more than ever the superhuman effort he put into organizing a successful conference year after year. Reza's spirit lives on within Bridges; indeed, when making executive decisions, the board was often guided by a simple principle: "What would Reza do?". We believe that he would be delighted with this year's program, and we are committed to preserving his legacy by offering an inspiring and exciting conference for many years to come.

The Bridges Organization Board of Directors www.bridgesmathart.org