From Rabbit Ears to Origami Flowers: Triangle Centers and the Concept of Function

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Abstract

Origami, the Japanese art of paper folding, seems to naturally fall at the intersection of art and mathematics. This workshop combines basic folding techniques in an exploration of triangle centers with two traditional origami models used to investigate the concept of function. Participants will learn to fold perpendicular bisectors and angle bisectors in service of triangle centers. We will also learn two traditional origami flowers modified for use with high school students to study the concept of function. Aside from printable materials, participants in the workshop will get to brainstorm further uses of these notions in their own classrooms and in service of their own goals.

Introduction

Oops, not Eureka. Occasionally we find that inspiration falls out of the sky in some type of Eureka moment. More often, however, we make mistakes and, in trying to understand the outcome, we gain deeper insight. This investigation started as a mathematical investigation of various triangles combined with an origami lesson featuring a model based on an equilateral triangle. Apparently I was unclear that the students should begin with an equilateral triangle. The results led to a nice investigation of the concept of function. The line of reasoning that we developed seems to tell a solid story and has led to this workshop and this article.

Triangle Center Activity

Origami, Mathematics, and Education. There has been a long tradition of using origami in education to illustrate and explore mathematical concepts. Investigators and educators like Boakes [1], Cipoletti and Wilson [2], and Gross [3] have explored the deep intersection of origami, geometry, and education. My hope is that this investigation builds off those strong foundations and serves to expand origami into the service of algebra and the concept of function.

Common Core. The result of these explorations is an activity which can be completed in a single 50-minute session. The activity addresses Common Core Math Standards [4] by focusing on standard HSG-C.A.3 and also exploring 8.F. A.1. From HSG-C.A.3, students engaged in this activity have the chance to show they can "construct the inscribed and circumscribed circles of a triangle" as they identify both the circumcenter and incenter of triangles. From 8.F. A.1, this activity promotes the ability to "understand that a function is a rule that assigns to each input exactly one output". The activity also encourages the Standards of Mathematical Practice as students "make sense of problems and persevere in solving them". Through discussions of students reasoning, we reinforce the student's ability to "construct viable arguments and critique the reasoning of others".

Outdoor Portion. As quickly as possible I move the students outside. I choose two trees that are relatively near one another and ask one student to stand so that they are the same distance from each of the trees. I then ask another student to stand equidistant from both trees. Finally, I ask the entire group to stand so that they are the same distance between the two trees. I use group discussion to get the students to realize we have created a perpendicular bisector.

I then move the group to any place where two walkways intersect to form an angle. If you can find an intersection that is not a right angle, the discussion seems more productive. Again I ask one student to stand so that they are the same distance from both walkways. When the entire group places themselves equidistant from both walkways, our discussion shows that we have created an angle bisector.

Indoor Portion. When we have completed the activity outside, I give the students a sheet of paper with two dots representing the trees we encountered outside. They are asked to draw the perpendicular bisector to the best of their ability. Then I ask them to create a fold that represents the line they just drew. Quickly, the group decides that we will get the desired line by folding one “tree” onto the other. In this way, we make the simple shift into origami.

I then give the students sheets of paper with a variety of acute, right, and obtuse triangles and ask them to fold corner to corner creating all three perpendicular bisectors. The results, shown in Figure 1, show that the point of concurrency they have created sometimes falls within the triangle, on its edge, or completely off the triangle. This point of concurrency is the circumcenter for the given triangle.

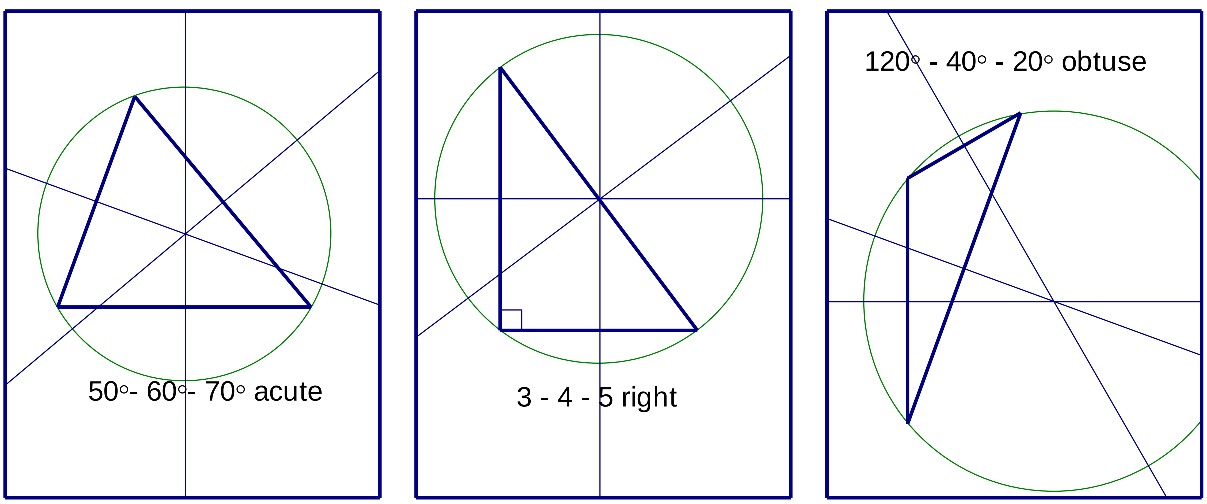


Figure 1: *Perpendicular bisectors meet at the circumcenter.*

I then give the students a sheet of paper with two line segments representing the walkways we encountered outside. They are asked to draw the angle bisector to the best of their ability. Then I ask them to create a fold that represents the line they just drew. Quickly, the group decides that we will get the desired line by folding one “walkway” onto the other.

I again give the students sheets of paper with a variety of acute, right, and obtuse triangles and ask them to fold edge to edge creating all three angle bisectors. The results, shown in Figure 2, show that the point of concurrency they have created always falls within the triangle. This point of concurrency is the incenter for the given triangle.

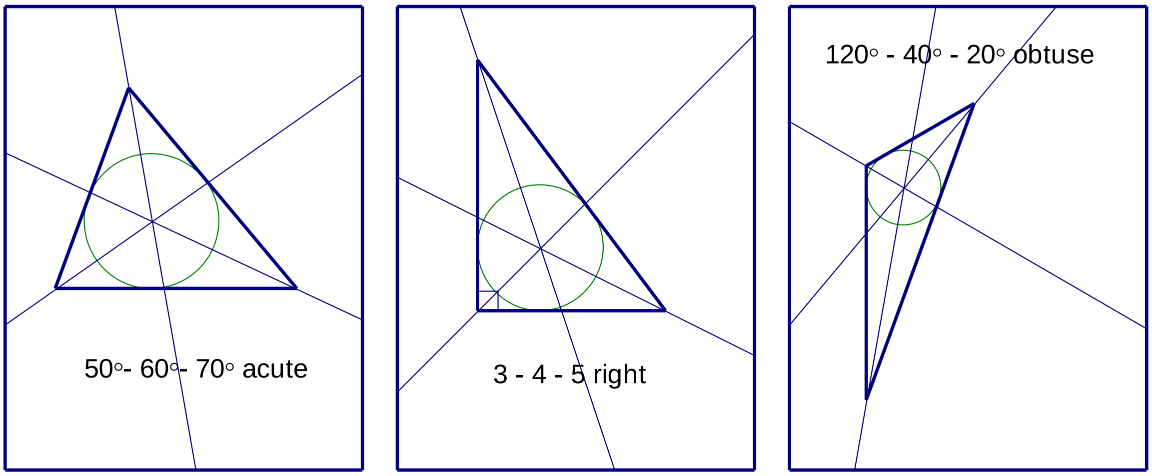


Figure 2: *Angle bisectors meet at the incenter.*

The Rabbit Ear. For those who practice origami, the rabbit ear fold is a collapsing fold which is made by folding all three angle bisectors at the same time. Because the incenter always falls within the body of the triangle, the rabbit ear fold can be made using any triangular piece of paper. We simply fold all three angle bisectors simultaneously. In Figure 3, the dashed lines are valley folds while the dot-dash line to the left becomes a mountain fold.

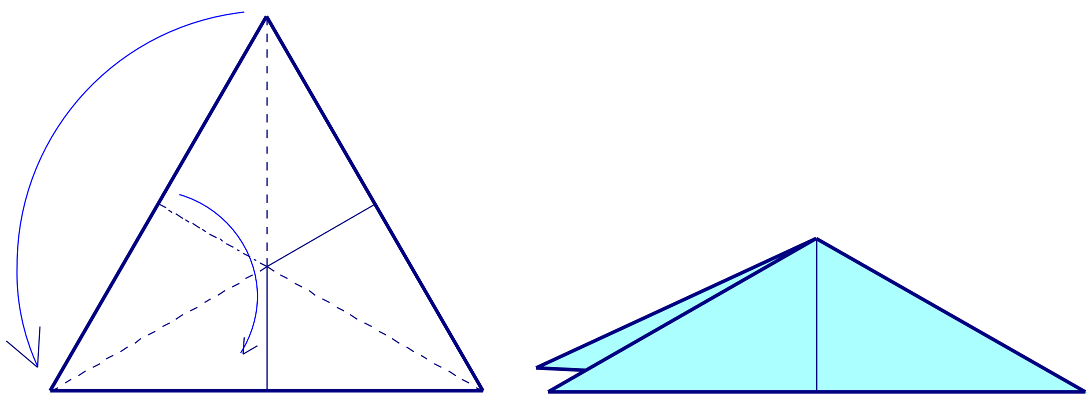


Figure 3: *Collapsing the rabbit ear fold.*

Students then take various acute, right, and obtuse triangles and complete the rabbit ear fold.

Folding Isao Honda's Wild Goose. One of my favorite childhood models is the Wild Goose from Isao Honda's *Joy of Origami*[5]. The model starts with an equilateral triangle and the first step is to collapse the triangle using the rabbit ear fold.

Workshop participants will fold the Wild Goose as part of understanding the rabbit ear fold for its mathematics value. The completed Wild Goose, shown in Figure 4, is accessible to beginning folders. In an odd twist of fate, several students were not quite listening when I asked that they start with an equilateral triangle. This led to the non-standard Geese also shown in Figure 4. Since the rabbit ear works on any triangle, it was quite simple to follow the folding pattern on a different triangle.

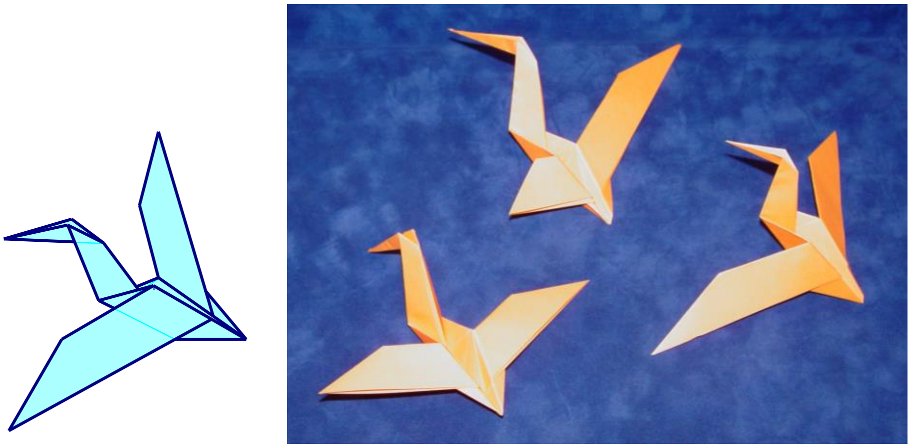


Figure 4: *Wild Geese from Equilateral, Isosceles, and Scale Triangles.*

The notion that a folding pattern could be carried out on any triangle led me to explore the concept of function through paperfolding.

Function Activity

Three Petal Flower. I had already planned to take the group of Governor's School students through Isao Honda's Iris Blossom. It is another piece that begins with an equilateral triangle. Since we had applied the Wild Goose to three different starting triangles, I decided to try the Iris on various triangles as well. Figure 5 shows the results of this investigation.

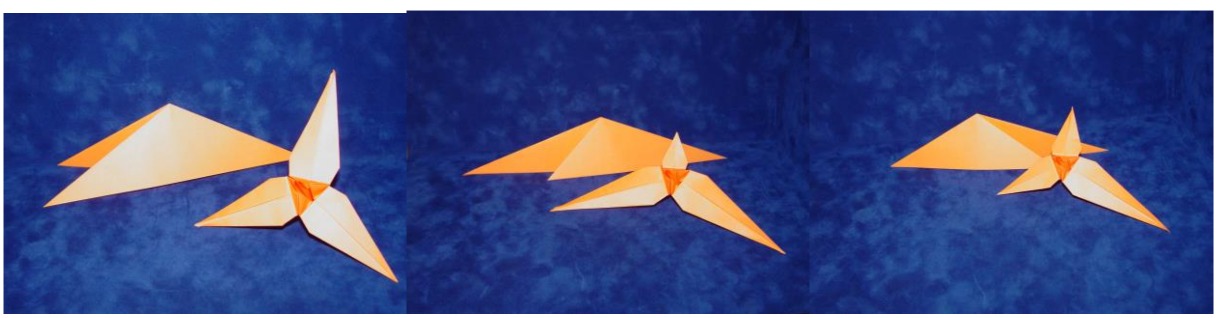
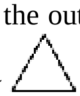
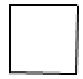


Figure 5: *Iris Blossoms from Equilateral, Isosceles, and Scale Triangles.*

To make the association even more explicit, I wrote a simple function on the board. I asked the students to consider f(x) = 3x + 5. Our notation means that we will take a given input, multiply it by 3, and then add 5 to produce our function output. We created f(0) = 5, f(1) = 8, and f(-3) = -4 just to make sure that the students understood function as a set of instructions that gives a unique output for any given input.

Then we chose to use our set of folding instructions as the function. The input was a specific paper triangle and the output was an Iris. I wrote function notation for each of the flowers in Figure 5. We had created Iris (), Iris (), and Iris() respectively.

Four Petal Flower. Honda also includes a Lily in his book. The folding pattern is almost identical to the Iris except the starting piece of paper is a square rather than a triangle. Again I revisited the notion of origami as function. We decided that Iris () = Lily. You can imagine from Figure 6 how the same folding sequence leads to both models.

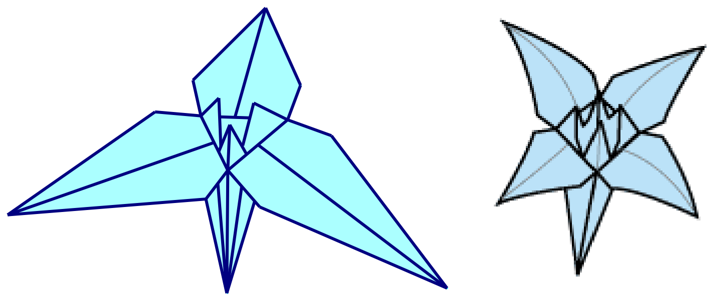


Figure 6: *The Iris Blossom and Lily for comparison*

I have had the chance to fold flowers from regular pentagons, hexagons, and octagons. With each additional side, the bowl of the flower becomes larger and the petals shrink. With the regular octagon, the flower turned out to look much more like a vase with fringe at the top. To combat the shortening of leaves, I have begun to manipulate the starting figure. Rather than a full square, I noticed an interesting pattern in the folds of the origami Bird Base. You can imagine in Figure 7 that the portion outlined in red will reduce the bulk of the petals while increasing their length. The completed Lily models are shown in Figure 8.

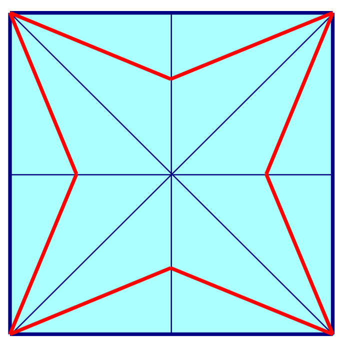


Figure 7: *A Non-regular polygon for investigation*

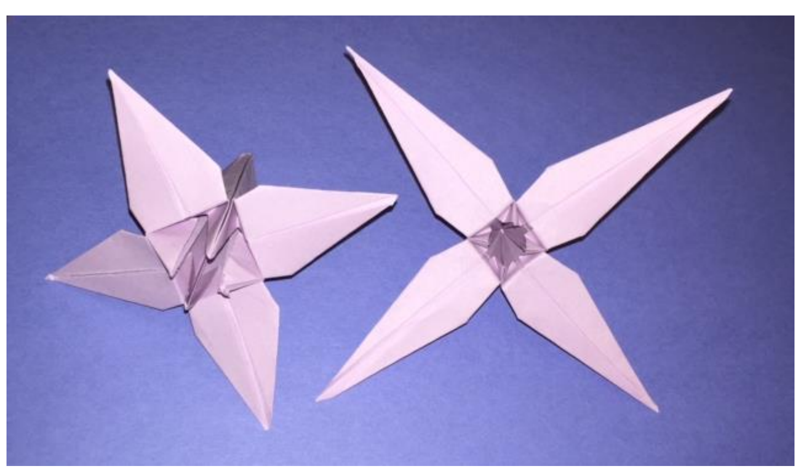


Figure 8: *Regular Lily versus Non-regular Lily*

Extensions. For those students who may finish early or those who would like more origami, I would recommend having a variety of different models available. Traditional models such as the "Star Box" are relatively easy models which can be adapted to triangles, squares, and pentagons.

I have begun to take the concept of folding instructions as function as a challenge in my own folding. If I find a model which begins with a non-square piece of paper, I try to fold the same model using a square. This has resulted in origami ostriches with extra legs or origami fish with extra fins. In contrast, I often take models which begin from a square piece of paper and try the folding patterns out on triangles. This has resulted in origami cranes with no tail or a single wing as seen in Figure 9.

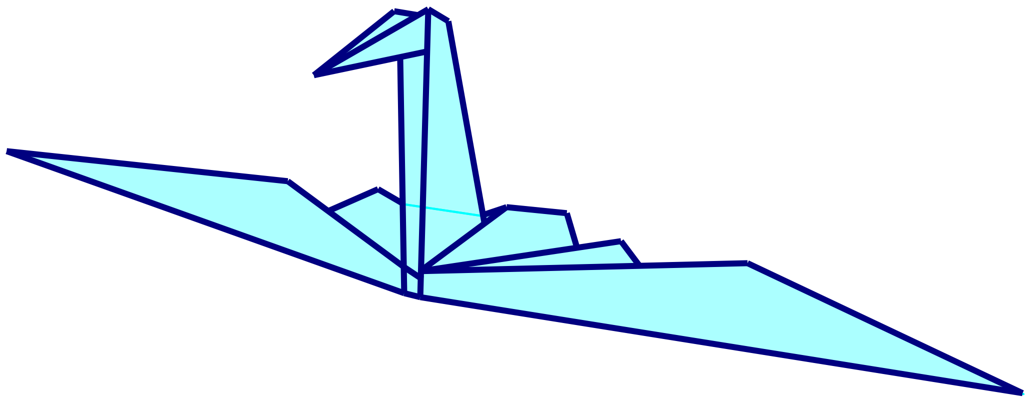


Figure 9: *An origami crane out of a triangular piece of paper*.

Conclusion. Teaching is often inspired by "Oops!" rather than "Eureka!". I began this activity with a group of high school students in an attempt to build knowledge of triangle centers, circumscribed, and inscribed circles. Thanks to a mistake of students beginning with the wrong piece of paper, I have actually found a nice way to reinforce the concept of function. Plus, I have been given a new line of investigation for my art. I hope you will consider trying some of these origami "functions" on different starting shapes.

References

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[5] I. Honda, *Joy of Origami*, Japan Publications (USA), 1979.