The Discovery and Application of the Protogon's Spiral

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Abstract

This paper attempts to appeal to laypersons as well as artists and mathematicians, by trying to put a human face on geometric art. It unveils an artist's motivation through the efflorescence of a distinct discrete spiral he created.

Introduction

This paper is not about spirals, generally, but about one spiral in particular. As well as being about its mathematical construction and graphic description, the paper offers a glimpse into the personal side of its evolution. The inciting incident began atop a California mountain, following a personal tragedy, as an artist viewed the land and sea below, connecting locations he visited with stars.

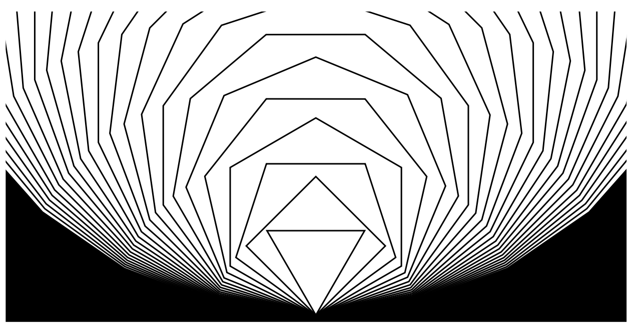

Figure 1: *The design, seminal to this body of artwork: a portion of an infinite number of polygons with equal-length sides sharing an apex, opening to an infinitely large black circle.*

Authors, such as Keith Critchlow [1], and possibly others before him, hinted at how such a configuration could be devised. Reconsidering my original arrangement of polygons (Fig. 1), I next repositioned them, so they had one side parallel. As an artist, I equated these two options to musical "sharps" and "flats", akin to a "gender" difference, and continued to apply them to subsequent variations. I eventually reproduced a 1969 photo collage made with my first "flat" design in 1990 [2].

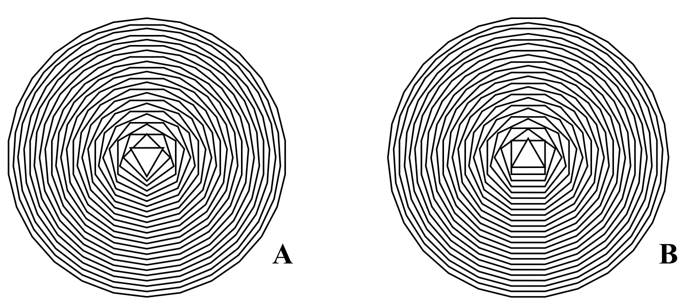

Figure 2: *24 concentric polygons with equal-length sides. Each with apexes aligned "sharp"* A*; each with sides parallel "flat"* B.

Birth of the Protogon's Spiral

The Protogon (Fig. 3), first drawn in 1968, merged the "sharps" and "flats" by using polygons with equal sides, so each successive polygon was rotated to overlap its adjacent neighbour, sharing a subsequent side. By 1980, I had made pencil drawings of nearly 300 other topological variations of the seminal design.

The distance between the windings of the Protogon can be seen to increase (Fig. 3*A*). When attributing a fill colour to alternating polygons (so that the even-number sided polygons are black), a concave polygon—one composed of a continuously connected, simple shape—results (Fig. 3*B*). A spiral progresses through the Protogon design, overlapping alternating white and grey shapes (Fig. 3*C*).

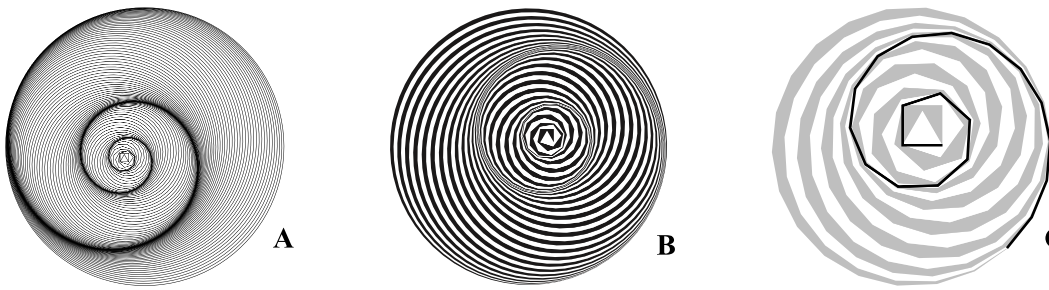

Figure 3: *A 100 polygon Protogon* A*; a 60 polygon Protogon shape* B; *a 20 polygon Protogon shape and a 20 shared-sides Protogon Spiral combined* C.

Influence. My urge to synthesize "sharps" and "flats" was borne out of rotating polygons (Fig. 4). The first version had them all share an apex and have one side parallel [2]. My Protogon, with equal-length sides, evolved from this rotational group (I first believed it was inconsequential).

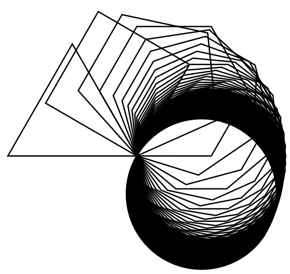

Figure 4: *Polygons rotated on apex (triangle opposite circle).*

Exhibitions. The first display of the Protogon was not until my pop-up show in Cambridge, Ontario in 2014. I had shown prints of it years before in the U. of W. School of Architecture print studio, in the office of the Cambridge Galleries, and recently at the 2016 JMM Exhibition [3].

An "asymptote" (Fig. 5) represents the interior angle of an infinite-sided polygon approaching but never reaching, 180°. The interior angle of an n-sided polygon is (180° / n) · (n - 2). Inherent in the Protogon is a spiral composed of equal-length lines. The interior angle between its first two lines forms a right angle (90°). The spiral's interior angles approach 180° at a rate expressed by $\theta_i = 180° \cdot (i + 1) / i + 3$. Currently, you can view an interactive version of the unfolding Protogon at the Maplesoft internet site [4].

Theta. When theta (θ) is the interior angle, and x and y are the coordinates of the end point, the rotation of each line about the origin is determined by:

$$x' = x \cdot \cos(\theta) - y \cdot \sin(\theta)$$
$$y' = x \cdot \sin(\theta) + y \cdot \cos(\theta)$$

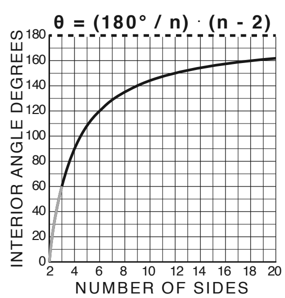

Figure 5: *An asymptote of 3 - 20 sided polygons.*

Rotation. When x_1 and y_1 are the coordinates of the start point of the previous line, the formula to rotate around the end of the previous line is:

$$x' = x + (x_1 - x) \cdot \cos(\theta) - (y_1 - y) \cdot \sin(\theta)$$
$$y' = y + (x_1 - x) \cdot \sin(\theta) + (y_1 - y) \cdot \cos(\theta)$$

Comparing the Protogon's Spiral

I found a distinct discrete spiral in my Protogon with equal-length lines in 2005; a mathematics teacher of Plato, Theodorus of Cyrene, also created a distinct discrete spiral with equal-length lines 2400 years ago.

Theodorus. Both the Protogon's Spiral and Spiral of Theodorus are composed of equal-length lines; both start out with a right-angle turn, and each owes its distinctiveness to a remarkably unique geometric construction. An adjacent diagram (Fig. 6) overlays the two, contrasting their distinct configurations.

Max Bill. In 2005, I faced a design by Max Bill challenging the uniqueness of mine; both are made of equal-length lines (Fig. 7*A*, 7*B*). I was thrilled to find a spiral in my design; his shared sides were a step apart, but mine were adjacent and thus joined.

He devised his "Metagon" (Fig. 7*B*) in 1938. His approach was to unfold the side of each polygon to open into the next polygon [5], resulting in what I call a "two step-rotation". However, this approach does not yield a distinct pattern of nested polygons. Nor does a curved spiral inscribed within it prove to be more distinct from any other one inscribed in the infinite number of subsequently rotated steps (Fig. 7*C* - 7*D*). Even still, and in deference to him, I named my design the Protogon—after his Metagon.

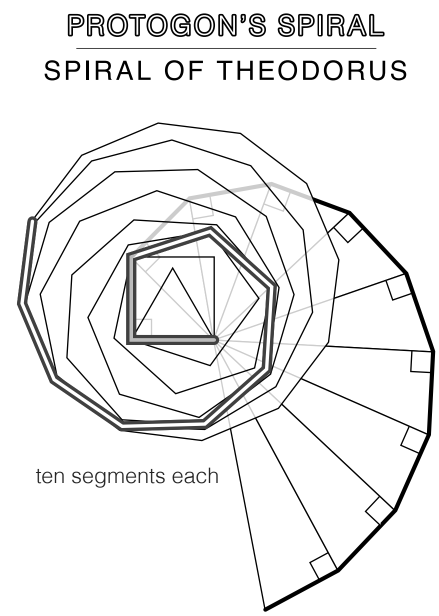

Figure 6: *Comparison between the Protogon's Spiral and Spiral of Theodorus.*

Attribution. Recently, I was disappointed to find Mathematica [6] inadvertently used my Protogon to help explain their software capabilities, citing Max Bill as having created a picture showing a similar design (sans spiral). Moreover, Mathworld, an online encyclopaedia, followed their lead by describing a "Bill picture" as "a sequence of nested regular polygons in which subsequent polygons are each rotated so that they begin one vertex further", describing my particular arrangement but attributing it to Max Bill.

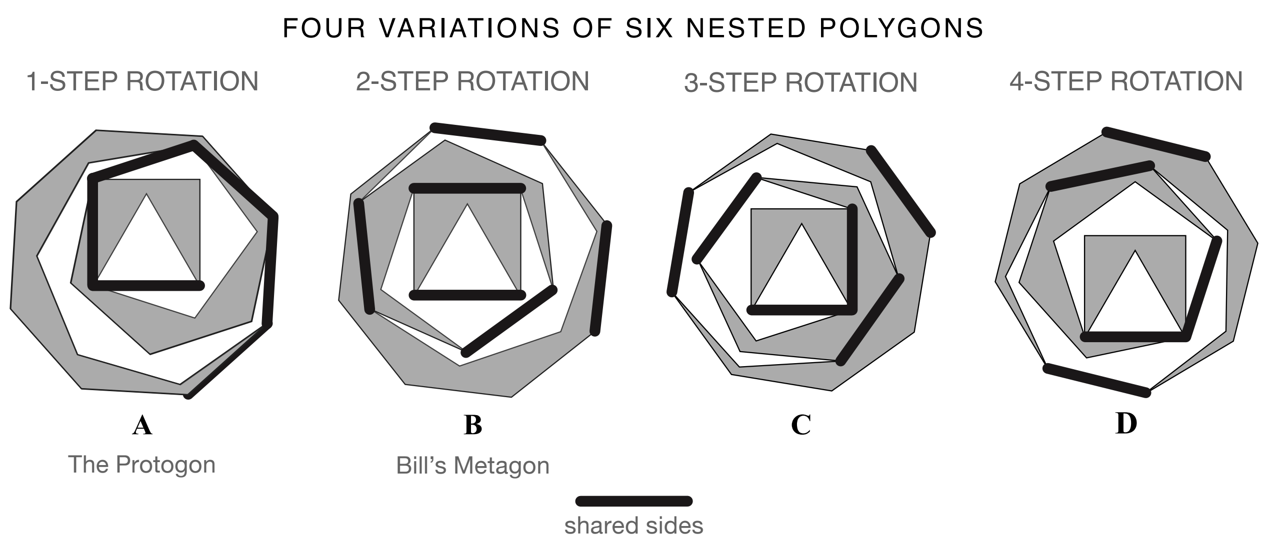

Figure 7: *The Protogon's Spiral, composed of shared sides, dissolves upon further polygon rotation.*

Beyond the Protogon

I have applied the Protogon and its spiral to many of my artworks. Though, the spiral tends to vanish in most cases when I rearrange the polygon components of the Protogon. Influenced by my association with the University of Waterloo School of Architecture, I again began to think more sculpturally, as I had in my youth. I extruded the Protogon's layout into the three-dimensional wood version of the "Protomid" (Fig. 8), displayed at the 2016 JMM Exhibition of Mathematical Art, which they depicted in their catalogue [3].

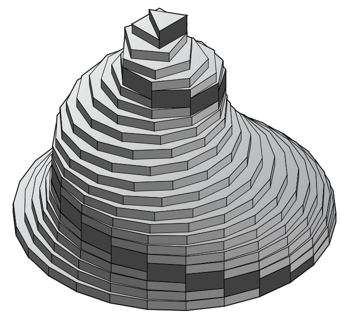

Figure 8: *A digital "Protomid".*

An odd phenomenon occurs viewing my large painting of the Protomid with its realignments (Fig. 9). The central (spiral-based) image, inexplicably, looks like a moving tunnel to a viewer walking towards it.

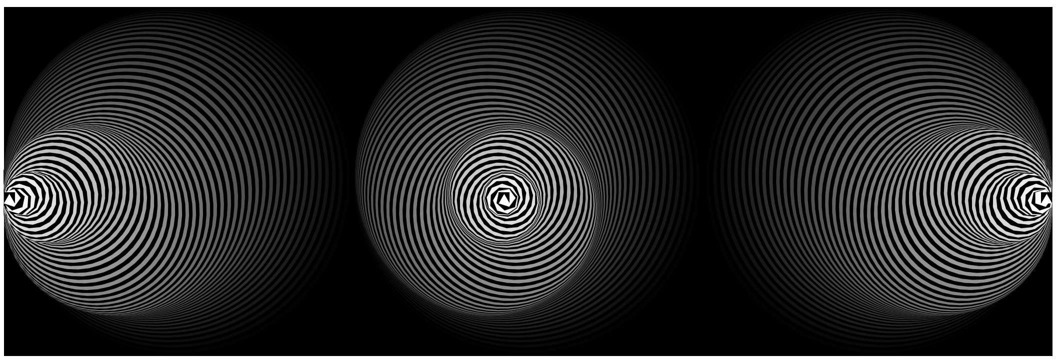

Figure 9: *Study for "Protogon Shift" (triptych), 2014, 7 x 21 feet, acrylic paint on stretched linen.*

Conclusions

The subject of this paper was a spiral, inherent in an arrangement of polygons, I fashioned in 1968. A career detour swayed me from discovering the spiral and whatever value it possessed. Besides describing its physical properties, this paper revealed the manifold fluctuations of the creative process. I overcame any pathos resurrected to explain the spiral's evolution during its artistic application—my Fourth Act.

Acknowledgments

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