Non-Octave Guitar and Keyboard Designs for Ervin M. Wilson

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Abstract

In 1966, Ervin M. Wilson invented his first combination product set – a rational, non-centric structure that musicians can use as a scale. In 2005, David C. Keenan invented a guitar fretboard that can play all twenty notes from a particular combination product set. I propose a Halberstadt keyboard design that makes use of a seventy-note combination product set. The combination product sets used by the two instruments are not reduced to the octave, and they do not repeat at the octave.

Introduction

Ervin M. Wilson (1928-2016) was a Mexican-American music theorist, a high-lysine corn farmer and a veteran of the United States Air Force. At Brigham Young University, Wilson briefly studied music theory. He left frustrated, for his professors were dismissive of unconventional tunings. Wilson did not submit articles to academic journals (aside from the little-known *Xenharmonikôn*); hence few people from Music and Mathematics departments are familiar with his work. Since Wilson did not write in an explicatory style, his exegetes have plenty of work to do.

In 1966, Wilson invented the first combination product set (CPS) that musicians can use as a scale – a novel approach to rational tuning or "just intonation". As John H. Chalmers [1] explained, no note from a CPS "scale" should be considered the tonic. Combination product sets are "harmonically symmetrical, polytonal sets with virtual or implicit tonics which are not [notes] of the scale". Although familiar-sounding triads can sometimes be formed from the notes of a CPS, "the global 1:1 for the whole set is not a note of the scale". It is absent. A CPS is thus a "non-centric musical structure". Why would anyone invent such a thing? According to Kraig Grady [3], the early space and moon expeditions inspired Wilson to investigate rational tunings that lacked "a central point of gravity or tonality".

Diatonic Theory is of Little Relevance

The hexany is the best-known CPS. As the name suggests, a hexany consists of six notes. Four factors (A, B, C and D), taken two at a time, produce the six notes (AB, AC, AD, BC, BD and CD). A musician is not obliged to choose any particular factors. A hexany's number of dimensions is determined by the number of prime factors. If each of a hexany's factors are prime numbers, then the hexany has four dimensions. There is, however, a caveat. If one voluntarily retains the conventional theory of octave-equivalence and thereby treats multiples of 2 as pitch-class identities, then the prime number 2 never adds a dimension. If one were to choose composite numbers or irrational numbers as factors, this would also affect a hexany's number of dimensions.

What can a musician do with a hexany's products? Typically, he/she will treat the products as note-frequencies and modify some of them in order to construct a scale that has six notes within a single octave. This process makes an octave-reduced CPS.

A = 2	AB = 2 x 3 = 6	First note: AB = 6 Hz
B = 3	AC = 2 x 5 = 10	Second note: AD ÷ 2 = 7 Hz
C = 5	AD = 2 x 7 = 14	Third note: BC ÷ 2 = 7.5 Hz
D = 7	BC = 3 x 5 = 15	Fourth note: CD ÷ 4 = 8.75 Hz
	BD = 3 x 7 = 21	Fifth note: AC = 10 Hz
	CD = 5 x 7 = 35	Sixth note: BD ÷ 2 = 10.5 Hz

For people that are familiar with "just intonation" theory, should one describe a CPS as a sequence of frequency-ratios (starting from the lowest note designated "1:1")? No. This would overlook the aforementioned fact that there is no 1:1 or privileged "tonic" in a CPS. One should not rush to make comparisons with conventional scales. Too often, this results in confusion. In the hexany above, BD and AD happen to make the ratio 3:2, which is a familiar "perfect fifth". Is this familiarity of any significance whatsoever? No. If one chooses to make a hexany with different factors, the 3:2 ratio might disappear. Consider the second hexany example below. It does not involve a 3:2 ratio.

A = 3	AB = 3 x 5 = 15	First note: AB = 15 Hz
B = 5	AC = 3 x 7 = 21	Second note: AD ÷ 2 = 16.5 Hz
C = 7	AD = 3 x 11 = 33	Third note: BC ÷ 2 = 17.5 Hz
D = 11	BC = 5 x 7 = 35	Fourth note: CD ÷ 4 = 19.25 Hz
	BD = 5 x 11 = 55	Fifth note: AC = 21 Hz
	CD = 7 x 11 = 77	Sixth note: BD ÷ 2 = 27.5 Hz

The note-frequencies specified above are not of practical use. For an octave-reduced hexany to become useful, it needs to repeat at each octave, just like a conventional scale. Alternatively, one can make a twenty-note CPS or a seventy-note CPS with a non-octave-reduced frequency range that happens to be useful for musicians. Such a CPS does not need to repeat at the octave (or at any other interval).
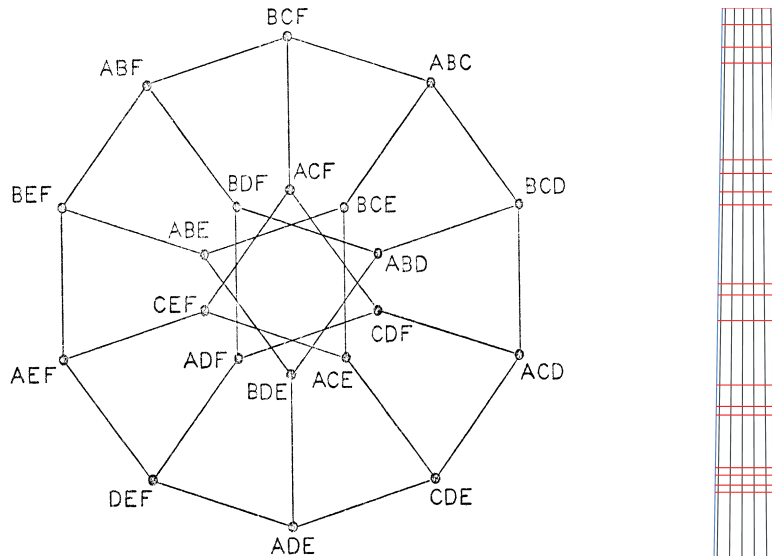
Eikosany Guitar

The eikosany (from the Greek *eikosi*, "twenty") is a twenty-note CPS. Its notes are produced by six factors (A, B, C, D, E and F), taken three at a time. Wilson made his first eikosany diagram some time between 1966 and 1968. Figure 1, derived in part from [4], illustrates the eikosany's general construction. Note that there is no central point marked "1:1" and, for simplicity's sake, the diagram is reduced to two dimensions. Each line in the diagram connects two products that differ by just one factor. If the diagram were exact, there would be three times more of these lines. Wilson presumably limited the number of lines so that his diagram would not look messy and ridiculous. (I am not able to comprehend all of Wilson's decisions or determine exactly what is missing from his many CPS diagrams. This work must be left to others. See [4,8,9].)

Although it is not common, it is possible to use a CPS that is not reduced to the octave, does not repeat at the octave and does not retain the theory of octave-based pitch-classes. See [2,6]. In such a CPS, even if some notes are involved in a 2:1 rapport, the 2:1 rapport has no special significance; it is merely incidental. In 2005, the Australian music theorist David C. Keenan [7] designed a guitar that can play all the notes from a particular, non-octave-reduced eikosany. Keenan's eikosany is arguably easier for a guitarist to play than a set of twenty notes crammed within an octave. Using standard, straight-across frets, the guitar's maximum error with respect to each note of the eikosany is a mere 3.57 cents. (A cent, as the name implies, is one-hundredth of an equal-tempered semitone. The latter is defined as $\sqrt[12]{2}$, which means that a cent is $\sqrt[1200]{2}$.)

For his eikosany, Keenan chose the following six factors: 4, 5, 6, 7, 9 and 11. Because these factors involve the prime numbers 2, 3 and 5, Keenan's guitar sometimes plays familiar interval-ratios such as

3:2 (perfect fifth), 4:3 (perfect fourth), 5:4 (major third) and 6:5 (minor third). Other times, it produces harmonies that involve the prime numbers 7 and/or 11. Guitarists cannot make harmonies like this if they use twelve-tone equal temperament. Twelve-tone equal temperament approximates just one septimal interval (the tritone, 7:5), and it does not approximate any intervals that involve the prime number 11. Keenan's design thus allows guitarists to experiment with exotic, rational harmonies, and it does so without adding an ostentatious number of frets.

Figure 1: *Wilson's eikosany diagram and Keenan's fretboard design.*

Keenan's guitar design cannot be reduced to the following: "one fret directly pertains to one note from the eikosany". It is more sophisticated. For exact specifications, see Keenan's spreadsheet files and my didactic video, stored in the Bridges Archive.

Hebdomekontany Keyboard

The hebdomekontany (from the Koine Greek *hebdomékonta*, "seventy") is a seventy-note CPS. Its notes are produced by eight factors (A, B, C, D, E, F, G and H), taken four at a time. The familiar, Halberstadt keyboard layout – seven white keys plus five black keys – is confusing if one works with anything other than twelve divisions of the octave. On the other hand, electric Halberstadt keyboards with weighted, pressure-sensitive keys afford performers superior dynamic control compared to non-standard controllers with crude *on/off* buttons. Consider, for example, musicians pressing buttons on primitive touch-screens that are not pressure-sensitive. The performance's dynamic range is flat. For this reason, I mapped a particular hebdomekontany's notes to a conventional Halberstadt keyboard. This is intended as a stopgap until non-Halberstadt keyboards that are both sophisticated and affordable become available.

Seventy notes obviously will not fit within the octave of a Halberstadt keyboard. A non-octave-reduced hebdomekontany, however, can be mapped to a Halberstadt keyboard. To ensure that the hebdomekontany's frequency range is of practical use, I chose the following eight factors: 1, 2, 3, 4, 5, 7, 9 and 11. The resultant range is 24 Hz to 3465 Hz. (The lowest note is this: ABCD = 1 x 2 x 3 x 4 = 24 Hz. The highest note is this: EFGH = 5 x 7 x 9 x 11 = 3465 Hz.) This is not far from a conventional grand piano's range (27.5 Hz to 4186 Hz).

In [9], Wilson constructed a hebdomekontany with the factors 1, 3, 5, 7, 9, 11, 13 and 15. If one treats it as a non-octave-reduced hebdomekontany, its highest note-frequency is this: EFGH = 9 x 11 x 13 x 15 = 19305 Hz. This is not of practical use. If, as I suggest, one divides every note-frequency in the hebdomekontany from [9] by four, then the highest note-frequency (EFGH) becomes 4826.25 Hz, and the lowest note-frequency (ABCD) becomes 26.25 Hz. Keyboard players can then use this hebdomekontany, too; but I suspect that many of them will prefer my hebdomekontany, since it can generate both familiar harmonies and strange harmonies. For musicians that want to study odd-order harmonics, the hebdomekontany from [9] is superb; but to conservative audiences, its harmonies will sound too alien. Aside from 7:5 (tritone) and 5:3 (major sixth), it does not involve any familiar interval-ratios.

Conclusion

Keenan's non-octave eikosany and my non-octave hebdomekontany are useful for guitarists and keyboard players. Preliminary experiments suggest that they each have musical potential. As expected, harmonies that involve the prime numbers 2, 3 and 5 sound familiar; harmonies that involve the prime numbers 7 and/or 11 are comparatively strange. For further information about prime numbers and musical harmonies, I recommend that musicians consult Ben Johnston's theoretical work [5]. As for mathematicians, I hope that they examine Wilson's diagrammatic work and develop it further for audiences with formal, scientific training.

Supplements and Acknowledgements

The spreadsheets that I uploaded to the Bridges Archive specify the exact fret-positions and open-string frequencies from Keenan's eikosany guitar plus each of the note-frequencies from my hebdomekontany keyboard design. I also uploaded a didactic video that demonstrates each note from Keenan's eikosany. I thank Keenan for allowing the Bridges Archive to store his spreadsheet files. I also thank him for his comments pertaining to Wilson's eikosany diagram. I thank Grady, too, for granting me permission to use Wilson's eikosany diagram.

References

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