Tiling Notation as Design Tool for Textile Knotting

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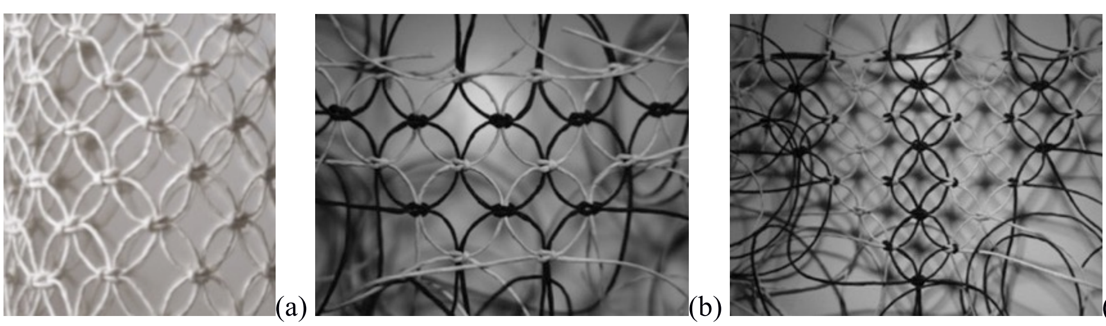
Abstract

A textile practitioner and two mathematicians with an interest in textile knotting carried out collaborative work to analyze textile knot practice and create novel knot pattern designs using mathematical tiling methods, in particular the Wang tiles. This paper shows how tiling notation was used to design knot diagrams and visualize novel patterns and structures.

Introduction

Collaboration between a textile practitioner (Nimkulrat) and a mathematician (Matthews) has demonstrated the use of mathematical knot diagrams as a tool for design development and prediction of textile knot patterns [2]. While a textile knot is a fastening made by looping and tightening linear material, a mathematical knot is a closed curve in three-dimensional space. Although the textile technique used here can be called macramé, Nimkulrat's textile practice originated from an exploration of connected reef knots learned in childhood. The knot diagrams Matthews used to communicate mathematical insights to Nimkulrat became, in themselves, a tool. Although novel colored knot designs were successfully created, the underlying structure remained unchanged (Figure 1). This led to the following questions 1) whether other opportunities, e.g., new knotted structures, might arise from the use of mathematics in textile knot practice, and 2) what might be other mathematical concepts that can be used as design tools.



(a) (b) (c)

Figure 1: *Single colored knot pattern (a) and two-colored knot patterns (b and c). All patterns have the same knot structure – reef knots, four strands, even spacing .*

Having considered the potential of mathematical tiling for exploring novel knot structures, Nimkulrat and Matthews sought a mathematician who specializes in mathematical tiling and found Nurmi who presented his paper about an aperiodic set of Wang tiles with colored corners [3] at Bridges Jyväskylä 2016.

The Work

Wang tiles are topological unit squares with colored edges. They are placed in a square grid and two tiles are allowed to be adjacent if colors on their touching sides match [1] [3]. This is a common method in computer graphics and applicable to the design of knotted textiles. Together with Nurmi, we started exploring new textile knot patterns and structures using methods presented in [3].

Based on the two-tone knot patterns in [2], we identified the reef knot pattern with four strands as a unit cell. Sixteen different variations were identified (Figure 2). The list represents a binary coding of such knots with two colors and is exhaustive as there are two colors and four incoming and exiting strands, always in the same order. Variations 0, 3, 6, 9, C, and F have been used to create Nimkulrat's circle and stripe patterns (Figures 1b & 1c). There are ten other variations (1, 2, 4, 5, 7, 8, A, B, D, and E) that to-date have not been employed in her textile knot practice. We believed these might be tiled together to create further novel knot patterns and structures. To verify this hypothesis, we made various versions of the Wang tiles with colored edges to study whether the patterns work topologically, and images of tightened knots to visualize the eventual outcome.

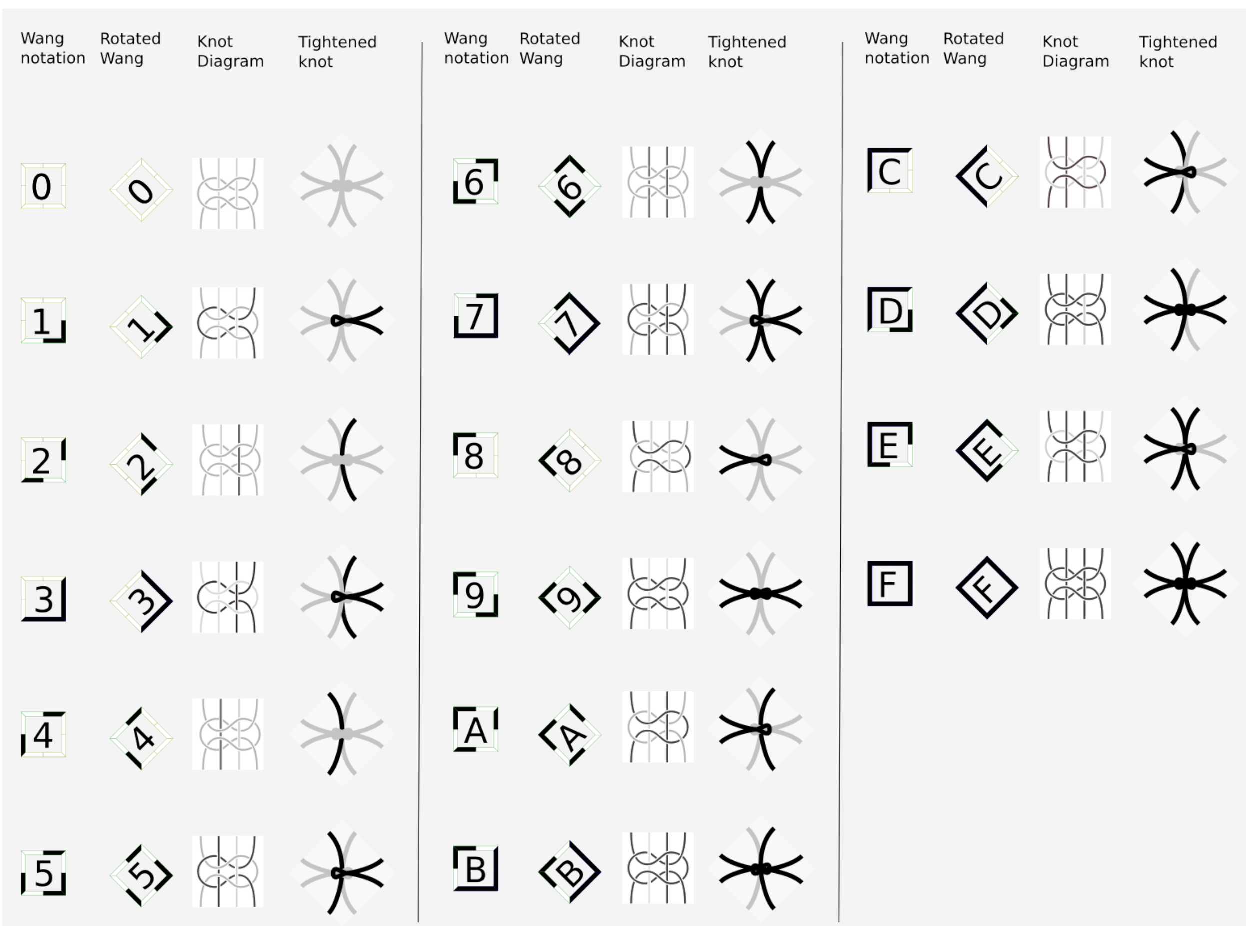


Figure 2: *The 16 knot units identified.*

First we reverse engineered the original knot work (Figure 1b) and created the knotting diagram using the tiling based method (Figure 3).

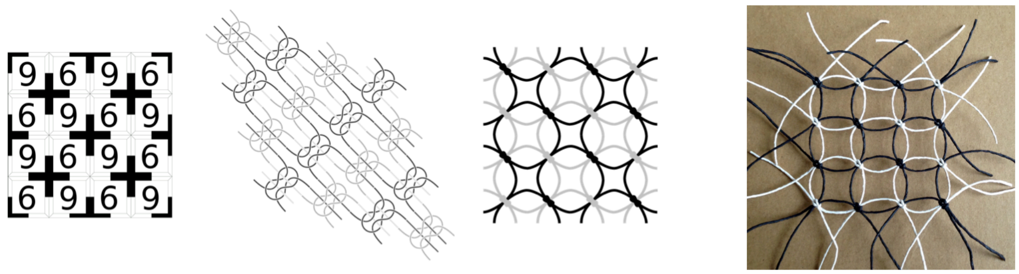


Figure 3: *From left: pattern as Wang tiles (6 and 9 in combination), generated knot diagram, predicted, and actual outcome.*

Next we explored new possibilities with the same six tile variations (0, 3, 6, 9, C, and F). Variations of the pattern in 1c with different stripe widths (Figure 4) take us beyond the checkerboard pattern.

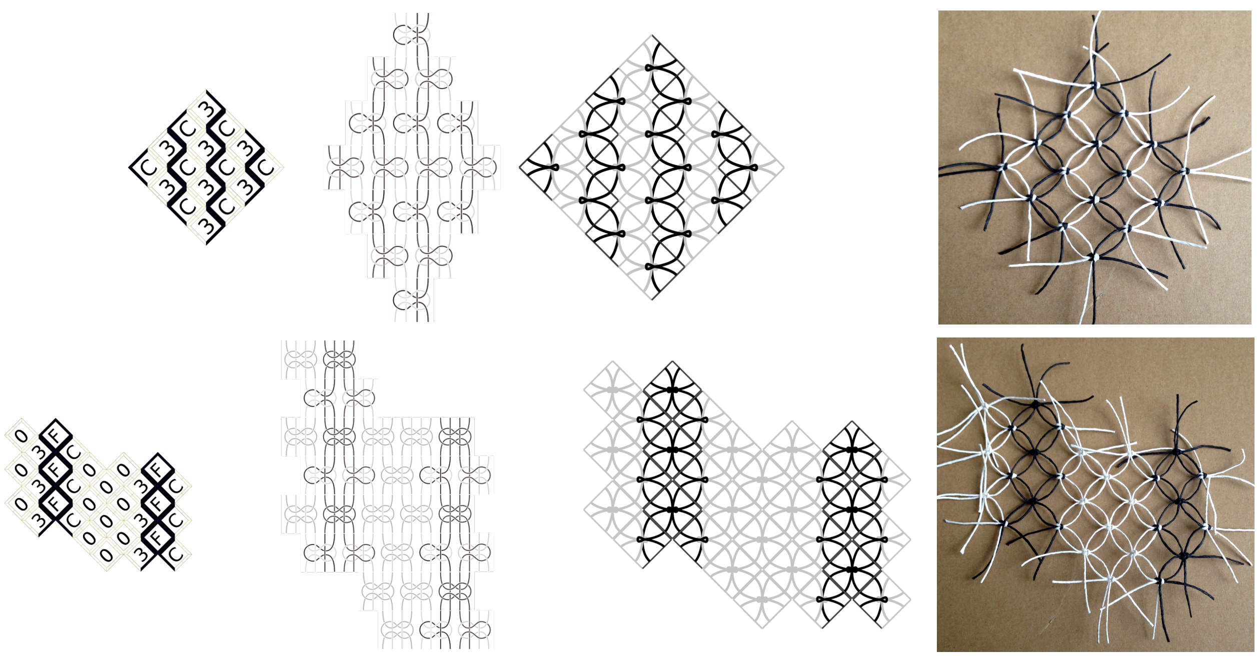


Figure 4: *Tiling based designs and sample knots of two stripe patterns.*

The colors on the Wang tile edges simply code what color strand enters and exits the tile, and in which position. Thus the Wang tile matching rules provide a safe way to ensure the continuity in designs. How about if we work the other way around? Ensure continuity in design, and then see if we can produce a valid (non-Wang) tiling for it? It would then automatically produce a valid knot diagram. To test this we abandoned the square grid topology and explored half-step patterns. In the tile space, continuity is enforced by matching individual colors, which code the strands. The pattern in Figure 5 only requires strands to be continuous. This approach radically alters the structural symmetry of the pattern. It is also very different to physically knot as the active and passive strands do not swap regularly, a characteristic observed in previous pattern analysis [2]. Three-dimensional patterns (Figure 6) are another application for tiling based design. Here we mostly retain the common Wang topology (square lattice), but mark some seams with lines. Note how the triangle shape transforms a flat piece into one with a three-dimensional form.

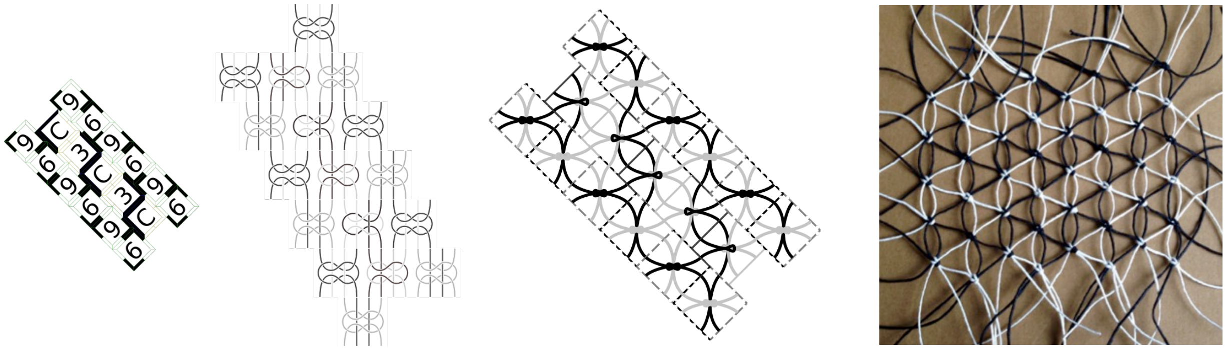


Figure 5: *Tiling based design that discards the Wang square grid topology.*

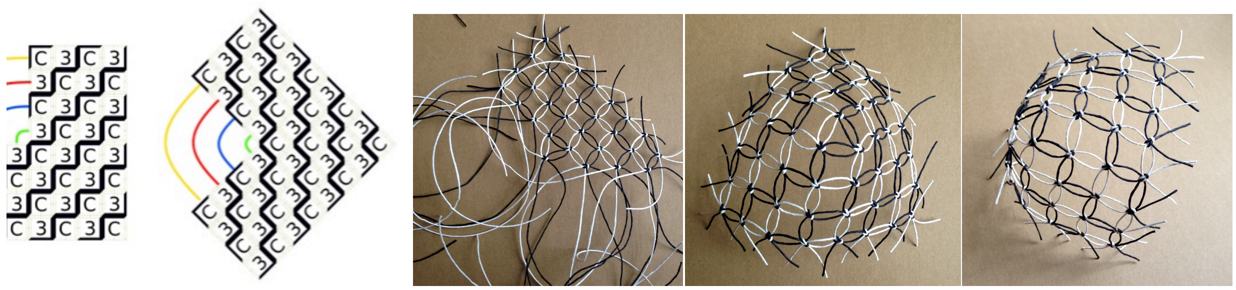


Figure 6: *Three-dimensional knot structure based on color-coded tiling design*

Conclusion

This paper has demonstrated how tiling notation can be used as a design tool in a textile knot practice. We have used this method to design several novel two-tone patterns. There is much potential for further work. For instance, we will explore all 16 knot tiles to design and implement novel structures and patterns. The tile set can easily be adapted to design complex braid-type patterns. The underlying tiling may easily be adapted from simple square grid into the rhombille or the aperiodic Penrose P3 tiling. Knots beyond the basic reef knot and knots that utilize a different number of strands (e.g., 3 or 5) could be used. Knots implementing permutations along with braid type designs have a connection to reversible cellular automata (RCA). The tiles could be used to model such RCA and design knot patterns that visualize how the CA operates.

References

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