The Interval Dissonance Rate: An Analytical Look into Chromaticism of Chopin's Op. 10 No. 2 and Scriabin's Op. 11 No. 2

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Abstract

The concept of dissonance and consonance in music is established from two or more simultaneous notes. There are multiple approaches into dissecting this concept; some of these are acoustical, psychological, and mathematical. We developed an Interval Dissonance Rate (IDR) – an innovative tool that integrates musical and mathematical analyses in non-monophonic Western music, using interval vectors and the frequency of recurrent pitches in the vectors to determine the percentage of dissonant and consonant intervals. Two tonal pieces with musical chromaticism by Frédéric Chopin and Alexander Scriabin are used for IDR analysis. According to results, the IDR of Chopin's *Étude Op. 10 No. 2* is equal to 17.65%, while the IDR of Scriabin's *Prelude Op. 11 No. 2* is equal to 29.37%. The IDR also allows one to look for intervallic patterns of various composers and in music of specific genres.

Introduction

The type of sound that is created between two or more simultaneous notes provides a given work with a sense of cohesiveness. The intervallic makeup of a composition defines characteristics of its style and genre. The tension created between two notes in a musical set depends on pitch, register, intensity of the sound, and texture of the harmony [1]. The function for evaluating the relative dissonance of two tones is based on the roughness of their frequencies [2]. The series of harmonic intervals throughout a piece of music create either consonance or dissonance. There are multiple approaches into dissecting this concept; some of these are acoustical, psychological, and mathematical. This study will present a new approach based on mathematical method. The tradition of Pythagoras is considered to be the first step taken in research to point out that the more consonant tones are the ones, whose frequencies are related by the simple ratios [3]. In the early part of the 6th century, Boethius characterized consonance as "suaviter et uniformiter", or gentle and pleasant sound, and dissonance as "aspra et iniucunda", or harsh and unpleasant sound [4]. In 1638, Galileo states that "agreeable consonances are pairs of tones which strike the ear with a certain regularity; this regularity consists in the fact that the pulses delivered by the two tones, in the same interval of time, shall be commensurable in number, so as not to keep the eardrum in perpetual torment" [5]. The aspects of consonance and dissonance depend on the fundamental frequency ratio between two different tones of a dyad, ranging within an octave from 1:1 to 2:1 [6].

It is possible to generalize the amount of dissonance in a musical piece mathematically. We developed an Interval Dissonance Rate (IDR) – an innovative tool that can be used to determine the percentage of consonant and dissonant intervals in non-monophonic Western music, by using interval vectors and the frequency of recurrent pitches in these vectors. This tool will provide a percentage of dissonant intervals, as compared to all possible intervals in a composition. This research uses pitch class notation, such as {0, 1, …, 10, 11} to define the pitches of a musical scale, where pitch C is represented as 0, pitch C♯ / D♭ is represented as 1, pitch D is represented as 2, and so on. The intervallic distance between two pitches in IDR will be measured in number of semitones (*t*). In modern (Western) music theory, it is accepted that intervals, such as perfect unison (P1), minor third (m3), major third (M3), perfect fourth (P4), perfect fifth (P5), minor sixth (m6), major sixth (M6), and perfect octave (P8) are consonant, while intervals, such as minor second (m2), major second (M2), tritone (A4), minor seventh (m7), and major seventh (M7) are dissonant. Therefore, consonance is defined by the set {0*t*, 3*t*, 4*t*, 5*t*, 7*t*, 8*t*, 9*t*, 12*t*}, where 0*t* = 12*t*, 3*t* = 9*t*, 4*t* = 8*t*, and 5*t* = 7*t*, while dissonance is defined by set {1*t*, 2*t*, 6*t*, 10*t*, 11*t*}, where 1*t* = 11*t* and 2*t* = 10*t*. We will define the set of consonant intervals (*CI*) and dissonant intervals (*DI*) as follows, where the addition of two elements in each c_X and d_X is equivalent to 12 semitones:

$CI = \{c_1, c_2, c_3, c_4\}$, where $c_1 = 0t+12t$, $c_2 = 3t+9t$, $c_3 = 4t+8t$, and $c_4 = 5t+7t$

$DI = \{d_1, d_2, d_3\}$, where $d_1 = 1t+11t$, $d_2 = 2t+10t$, and $d_3 = 6t$

The concept of enharmonic equivalence is applied to diminished and augmented intervals in this study. Therefore, an augmented third will be tantamount to the perfect fourth – a c_4 interval with the distance of 5*t*. Also, if an interval undergoes an inversion, the quality remains the same. For instance, an inverted M2 is m7, where both intervals are dissonant; an inverted P4 is P5, where both intervals are consonant. Similarly, the quality of the interval does not change if such interval undergoes an expansion via the means of octave equivalence [7]. Consequently, P4 and P11 are consonant, while m7 and m14 are dissonant.

Analysis

IDR allows one to take a musical composition and analyze it for dissonance. This is done by analyzing the interval class vector and the frequency of repeated pitches of every harmonic unit of a certain set. An interval class vector (ICV) is a series of six digits, where each digit represents the amount of specific intervals. The first number represents the total number of minor seconds and major sevenths in a set, the second number represents the total number of major seconds and minor sevenths in a set, the third number – the total number of minor thirds and major sixths in a set, the fourth number – major thirds and minor sixths, the fifth number – perfect fourths and perfect fifths, and the sixth number represents the amount of tritones in a set [8].

For instance, the ICV of pitches D, A, and B♭ played simultaneously is ⟨100110⟩, where there are three intervals of (D&A), (D&B♭), and (A&B♭); these correspond to two consonant intervals of (P5) and (m6), as well as a dissonant interval of (m2) with IDR of 33.3%. Interval vector can be expressed by c_X and d_X, such as $[d_1, d_2, c_2, c_3, c_4, d_3]$. As seen, the interval vector does not consider the c_1 consonance. IDR analysis, on the other hand, does this by taking the total number of dissonant intervals (*DI*) and dividing it by the total amount of all possible intervals (*TI*). The following formula is used: *IDR* (%) = *DI* / *TI*. There are four primary triads in Western classical music. The major triad (ICV: ⟨001110⟩), the minor triad (ICV: ⟨001110⟩), and the augmented triad (ICV: ⟨000300⟩) have 0% dissonance, since d_X does not exist in these structures, while the diminished triad (ICV: ⟨002001⟩) has 33% dissonance, due to a tritone.

This paper compares the IDR of Frédéric Chopin's *Étude Op. 10 No. 2 in A minor* (1833) and Alexander Scriabin's *Prelude Op. 11 No. 2 in A minor* (1895). Frédéric Chopin, one of the most significant composers in the history of Romantic music, has composed primarily for piano. The second piano étude of Op. 10 is written in 4/4 meter and contains 49 bars. Chopin's music provides a great deal of influence for Scriabin's early compositional output. Scriabin's Op. 11 is a noteworthy set of composition, as this is the first set of preludes by a Russian composer in all major and minor keys [9]. The second prelude of Scriabin's Op. 11 is reminiscent of Chopin's style of composition. The prelude is in 3/4 meter and contains 68 bars. Other than for identical key, both works share two significant unique traits: the form and traces of chromaticism.

Chromaticism is a musical pattern, in which a musical melody ascends or descends by 1 semitone. By using chromaticism, composers look for ways to move between chordal structures contrapuntally [10]. Chopin's approach to chromaticism reveals its non-harmonic function and covers over the primary musical material [11]. In bars 1-2, the right hand constantly follows a chromatic motion, as the melody ascends from pitch A towards the apogee of the phrase – pitch F, before descending downward, all while supported by the harmony found in the bass accompaniment and the inner voices, as seen in Figure 1 [12]. According to Schenkerian analysis, the pitch A is prolonged through the two bars and is a part of a bigger theoretical structure. However, this idea differs from performer's perspective, as there is a rest separating every musical structure found in the left hand. The IDR looks solely at the intervals, created by notes played at the same time. Since the third and the fourth pitches (B and C) in the right hand's chromatic run are played separately and are not part of any harmonic intervals, these pitches are neither consonant nor dissonant.

Figure 1: *Bars 1-2 of Chopin's Étude Op. 10 No. 2.*

The chromaticism in Scriabin's prelude is hidden and covered over by the primary melody. In bars 7-9, the left hand's chromatic motion descends from pitch E towards C-sharp, overshadowed by the main melody in the right hand, as seen in Figure 2 [13]. Figure 3 shows the data extracted from IDR analysis of both works. The total number of intervals, the number of consonant and dissonant intervals, as well as the IDR value are provided for both pieces of music. Additionally, their intervallic breakdown can be seen based on the amount of c_X and d_X, as well as their respective percentage. Such intervallic breakdown can likewise be used as a separate method for analysis.

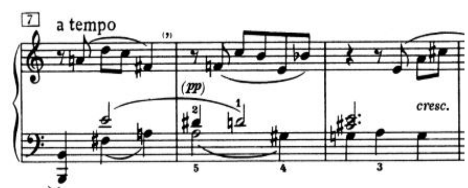

Figure 2: *Bars 7-9 of Scriabin's Op. 11 No. 2.*

Conclusion

The IDR of Chopin's *Étude Op. 10 No. 2* is 17.65%. The work contains a total of 3196 intervals, with 564 intervals being dissonant. The c_1 and c_4 intervals are most common in this work, which include all of the perfect intervals in music. These intervals occur 1409 times, which is 44% of total intervals. The interval of a tritone is only seen 134 times in the étude, which accounts for merely 4% of total intervals. In Western classical music, the tritone is considered the most unstable and most 'tonally ambiguous' interval, which therefore requires special resolutions [14]. While tritones are essential parts of certain seventh chords, Chopin nevertheless attempts to avoid the use of this interval in his composition. The IDR of *Prelude Op. 11 No. 2* is 29.37%. This work contains a total of 1522 intervals, where 447 intervals turned out to be dissonant. Unlike Chopin's étude, Scriabin's prelude is a slower and a less technically challenging work, which explains why the sixty-eight bars of the prelude produce half the number of intervals as the forty-nine bars of the étude. Similarly to the étude, the prelude is dominated with perfect intervals; 448 account for 30% of all intervals. However, the most common interval in the prelude is c_2. 346 minor thirds and major sixths account for 22.73% of the work. The most dissonant interval is d_2, as Scriabin's prelude contains only 161 tritones, or 10.58% of all intervals.

	Chopin Op. 10 No. 2	Scriabin Op. 11 No. 2
Total # of intervals	3196	1522
# of consonant intervals	2632	1075
# of dissonant intervals	564	447
IDR%	17.65%	29.37%
c1	690 (21.59%)	138 (9.06%)
c2	586 (18.34%)	346 (22.73%)
c3	637 (19.93%)	281 (18.46)
c4	719 (22.49%)	310 (20.37%)
d1	210 (6.57%)	35 (2.3%)
d2	220 (6.88%)	251 (16.5%)
d3	134 (4.2%)	161 (10.58%)

Figure 3: *IDR data for Chopin's Étude Op 10 No 2 and Scriabin's Prelude Op 11 No 2*

Western classical music contains a repertoire of multiple styles and various theoretical analyses have been applied to music literature of different epochs. One of the ways to subdivide multiple centuries of compositions is by the concept of tonality. Roman numeral analysis and Schenkerian analysis, which is based on the principles of harmony and counterpoint, are two of the common methods to look at music that contains one (or multiple) tonal centers [15]. To dissect and understand the structure of 20th century music that deviates from tonal center, one must use alternative analytical techniques, such as axial symmetry, or twelve-tone row [7]. The IDR can be used to look for patterns of consonance or dissonance in the music of various composers or even specific musical genres. The IDR and the overall fraction intervallic analysis may provide a tool to categorize or identify composers and genres of music. The Interval Dissonance Rate allows one to analyze both tonal and atonal music, providing necessary statistical information on how mathematically dissonant a piece of music is to Western ears.

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