Cartesian Lace Drawings

Susan Happersett
Beacon, NY
E-mail: fibonaccisusan@yahoo.com
Website : www.happersett.net

Abstract

The author explores the genesis of her new series of mathematically inspired drawings. Based on mappings between sets of points on the Cartesian coordinate system, and combining multiple elements in a single drawing, the author creates hand-drawn patterns and tessellations with a lace-like quality.

This is the story of how my interest in the visualization of network mapping lead to new art work. Technical diagrams illustrate different types of mapping phenomena. Applying graphing techniques taught in high school math classes I illustrate these basic relationships, then proceed to layer and build lace-like patterns. This work creates a link between technical diagramming techniques like network diagrams, and the traditions of hand work like lace making.

The developmental process for this new type of drawing began with the axes of the Cartesian coordinate system's 2-dimensional plane. I plotted the first few sets of points on both axes bounding quadrant I. Then I proceeded to map a point from the x-axis to a point on the y-axis using a bijective mapping, drawing a line segment to connect the points. "If there exists a bijective function $f : A \to B$, then we say that A and B are in one-to-one correspondence [1]".

In the example shown (Figure 1), I chose to use 8 points numbered 1-8 on each axis. Then, I mapped point n from the x-axis to point $9 - n$ on the y-axis. (1,0) connects to (0,8), (2,0) to (0,7), (3.0) to (0,6), etc. Continuing in a similar bijective mapping process in 4 quadrants, this results in the drawing in Figure 2. These initial patterns are reminiscent of the craft of string art, popular in the 1970's, which produced curves, using only straight lines.

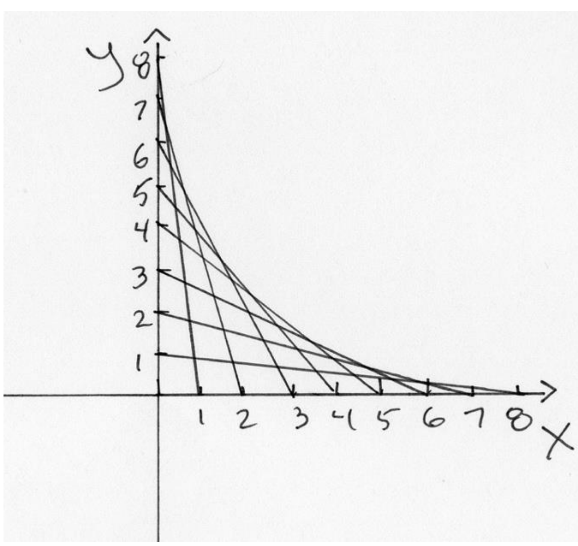

Figure 1: Bijective mapping in quadrant I.

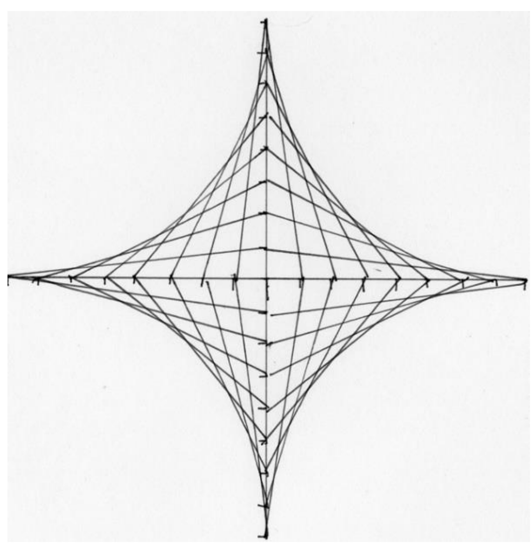

Figure 2: Mapping in 4 quadrants.

By copying and rotating the x and y axes 45 degrees, and repeating the design on these 2 new axes, I create the illusion of space, projecting a 3-D image on a 2-D plane (Figure 3). This 8-pointed star became the building block. I started placing them into a grid formation so that they tile or tessellate the plane (Figure 4).

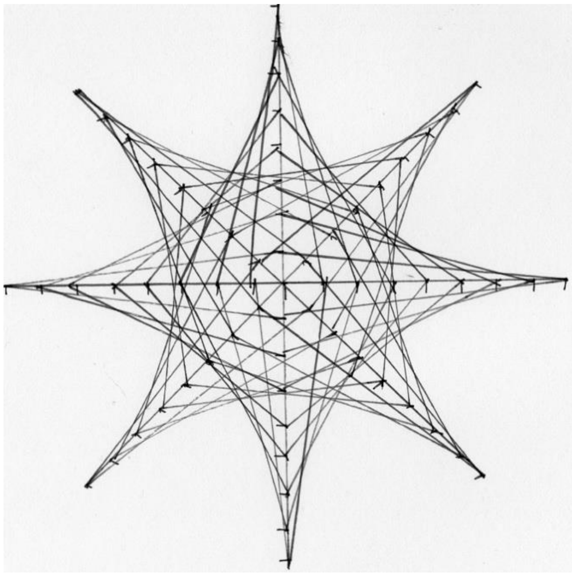

Figure 3: Mapping on 4 axes.

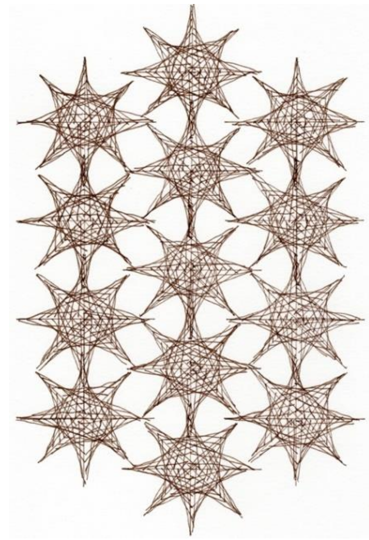

Figure 4: Tessellation of 8-pointed stars.

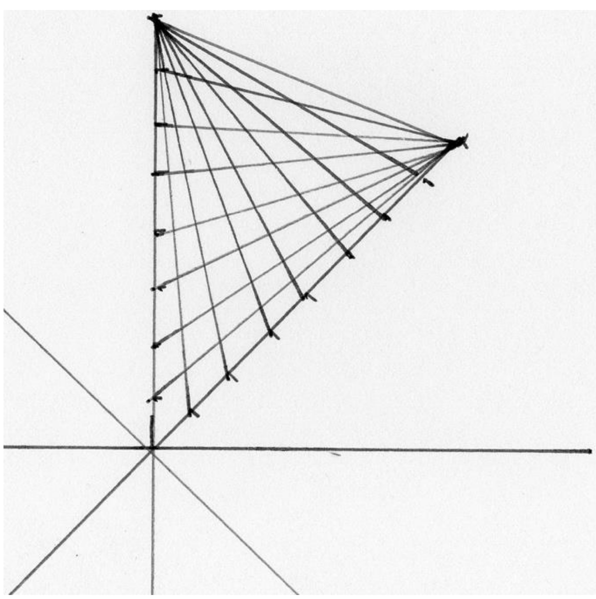

Figure 5: Non-bijective mapping.

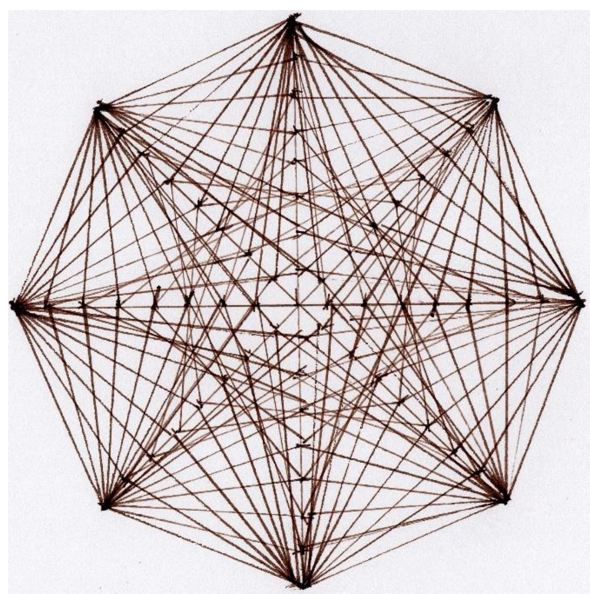

Figure 6: Bijective and non-bijective mapping.

Each individual building block features order-8 rotational symmetry. When the building blocks are placed in columns as in Figure 4, where each subsequent column is shifted vertically by half a cell, a wallpaper pattern of group p4m emerges.

I then started to explore the use of non-bijective mappings by connecting one point on each axis with multiple points on another axis. In the next example (Figure 5), on each axis, the farthest point from the center is mapped to each of the 8 points on both of the adjacent axes. I feel this creates the illusion that the drawing is coming out of the plane.

Once I had two distinct lace patterns to work with, I was able to start drawing both patterns on the same set of axes (Figure 6).

Only my initial drawings were done with a ruler. To create a closer connection to the intense hand work involved in the creation of lace, I practiced these patterns over and over, until I could draw close approximations free-hand. I only draw the basic structure of the axes with tools. The actual connections of the line segments are made without assistance.

Figure 7 is a completed drawing, "Octagon Lace", layering both mapping patterns. By overlapping the octagonal structures, the drawing becomes the basis for a plane-filling tiling. It is not possible to create a non-overlapping plane filling tessellation pattern using only regular octagons. If we consider the area where the two octagonal patterns overlap as separate tesserae, this drawing becomes a periodic tiling made up of quadrilaterals, trapezoids and rhombi. The rhombi have vertices of 45 degrees and 135 degrees, and the trapezoids have vertices of 45, 135, 62.5, and 117.5 degrees. The construction of 8 trapezoids and 4 rhombi possesses an order-4 rotational symmetry.

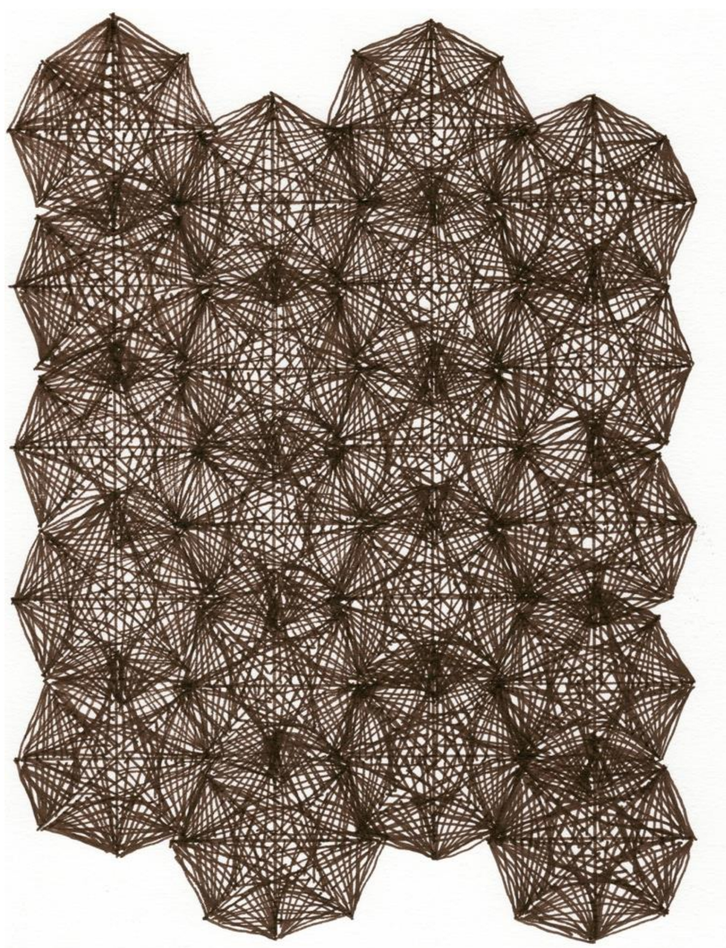

Figure 7: "Octagon Lace", pen on ink, 2017.

Experimentation with scale plays an important role in my artistic practice. I started to change the measurement scales on selected axes. In Figure 8, "Bijective Cartesian Lace", the points I use on the vertical axis for the bijective mapping are 50% farther apart then the points on the other 3 axes.

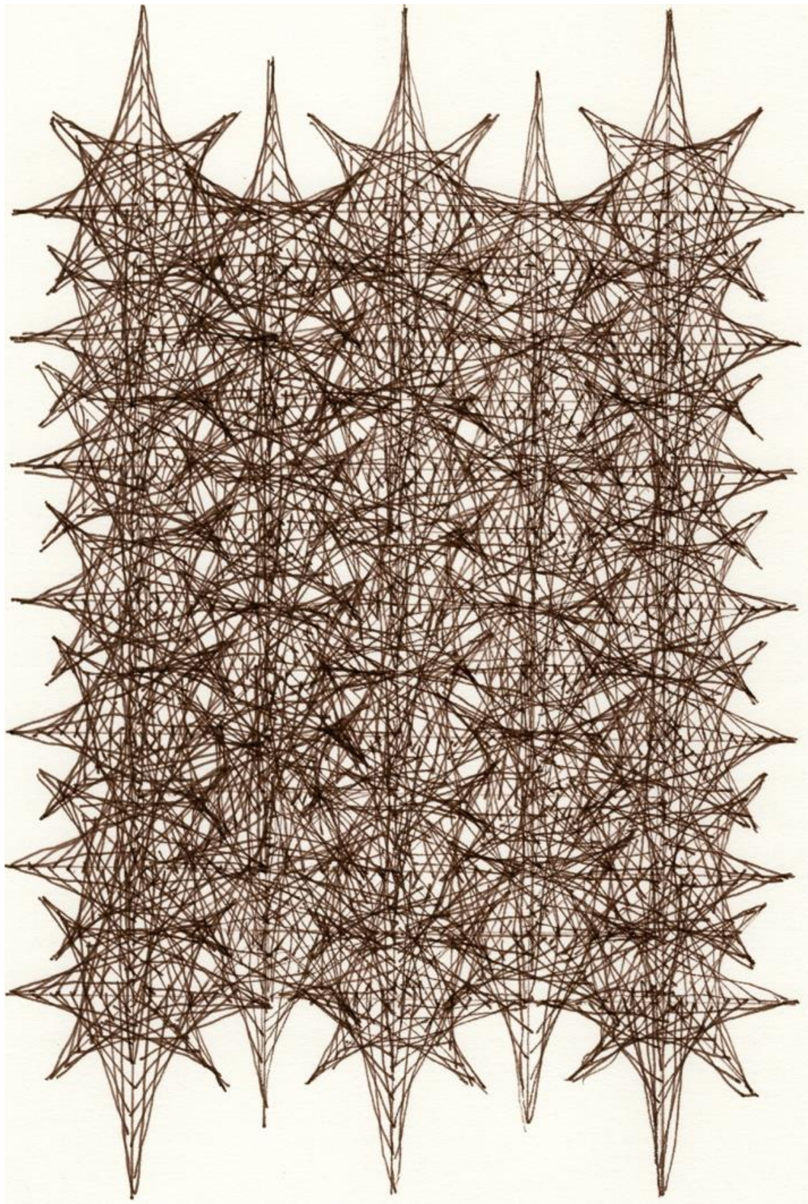

Figure 8: "Bijective Cartesian Lace", pen on ink, 2017.

Once I started to intensify the complexity of the hand drawing, I became increasingly aware that these patterns mirror biological phenomena like neural connections and microbial growth patterns. As with all new types of drawing I develop, I will continue to explore the lace drawings and see where the journey will take me.

References

[1] Pinter, Charles C. (1971), A Book of Set Theory, 1014 ed., Dover Publications, 54