Applying Helical Triangle Tessellations in Folded Light Art

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Abstract

This article describes how I created a collection of lamps made of folded sheets of material using helical triangle tessellations, which are also called Nojima patterns. I started by working with a periodic helical triangle pattern to fold a piece of light art that is shaped in a hexagonal column. I continued by modifying the periodic pattern into a semi-periodic design by adding variations so that the tessellation could be folded into a light art that is shaped in a twisted column. I further developed tessellations that consisted of self-similar helical triangles by using a geometric construction method. These self-similar helical triangles form algorithmic spirals. I folded the tessellation design into a work of light art that is shaped in a conical hexagonal form.

Introduction

In this article, I will describe the process of how I use geometric principles found in a helical triangle tessellation pattern to create a collection that I call Folded Light Art, made up of lamps of folded sheet material. In Figure 1, the larger figures show the three lamp shade designs and the smaller figures show the two-dimensional collapsed forms folded from similar creased patterns used in the lamp shades respectively. I started by working with a periodic helical triangle pattern to fold a piece of light art that is shaped in a hexagonal column (Figure 1a). I continued by modifying the periodic pattern into a semi-periodic design by adding variations so that the tessellation could be folded into a light art that is shaped in a twisted square column (Figure 1b). I further developed tessellations that consists of self-similar helical triangles by using a geometric construction method. These self-similar helical triangles form algorithmic spirals. I folded the tessellation design in a light art that is shaped in conical hexagonal form (Figure 1c).


(a) (b) (c)

Figure 1: *Folded Light Art, Booma Collection. (a) Booma Hexagonal Column. (b) Booma Twist Square. (c) Booma Conical Hexagon.*

Helical Triangle Tessellation

Helical Triangle tessellations, also called Nojima patterns, were first discussed extensively by Taketoshi Nojima, an engineer at Kyoto University. Nojima [1] demonstrated how helical patterns could be folded in the axial direction. He developed his designs based on inspiration from living organisms such as the phyllotaxies of leaves and flowers, insect wings, and DNA and protein structures. Nojima's helical patterns include periodic helical triangle patterns and their variations, as well as other helical patterns such as helical trapezoid patterns. One of the goals of Nojima's study was to use these foldable and deployable models to interpret the mechanics of the unfolding of flower buds and insect wings for potential use in building aerospace structures.

A periodic helical triangle tessellation is made up of repetitive units of triangles of the same size with the valley creases moving in a diagonal upwards direction (Figure 2a). A periodic helical triangle tessellation is flat foldable and deployable. In order for a crease pattern to be locally flat-foldable a necessary and sufficient condition, called Kawasaki's Theorem (see Kawasaki [2]), must be satisfied. Kawasaki's Theorem states that in order for a crease pattern to be locally flat-foldable then the number of lines connecting to a single inner vertex (the points where crease lines meet) must be even and the sums of alternating angles must be 180 degrees. Figure 2b shows an example of a single inner vertex of a periodic Helical Triangle pattern that satisfy the conditions listed in Kawasaki's Theorem. For an entire folded structure to be flat folded, the Kawasaki condition must be able to be applied to all the inner vertices of a crease pattern globally, and in addition, there should be no collision of the parts of the folded structure during assembly. Because of the periodic quality of Helical Triangle tessellation, Kawasaki's Theorem is satisfied at all the inner vertices globally, resulting in a flat-foldable pattern.

Below I will describe how I used these geometric principles that are found in a periodic Helical Triangle tessellation to explore form findings in art and design. The result is a collection of Folded Light Art, which I call Booma Folded Light Art. In this article, I will focus on three pieces of Folded Light Art in the Booma collection: Booma Hexagonal Column, Booma Twisted Square, and Booma Conical Hexagon. The goal of the Booma Collection is to create light art that is ecological using origami-inspired paper craft. Each piece of light art in Booma can be easily deployed into three-dimensions and then can be collapsed back to a two-dimensional flat shape that is much smaller, for ease of shipping and storage. The main material I used for this collection is Hi-tec Kozo paper, stainless steel, and 7mm transparent plastic snap buttons. Stainless plates were laser cut to specific shapes and dimensions to serve as luminary hardware, and plastic snap buttons were used for connecting paper pieces. Hi-tec Kozo paper is a type of tear-free Shoji paper which has a three-layer structure, with eco-friendly polyester film as its core and Kozo Washi on both sides. Kozo Washi is a renewable material that is made from the inner bark of Kozo, a type of mulberry tree. Kozo plants grow more than three meters high in a year and can be sustainably harvested each year. These large sheets of Hi-tec Kozo paper were also laser cut and etched with the crease patterns by using an industrial laser cutter. Furthermore, LED bulbs were used to illuminate Folded Light Art because of their low heat emission and low electricity consumption.

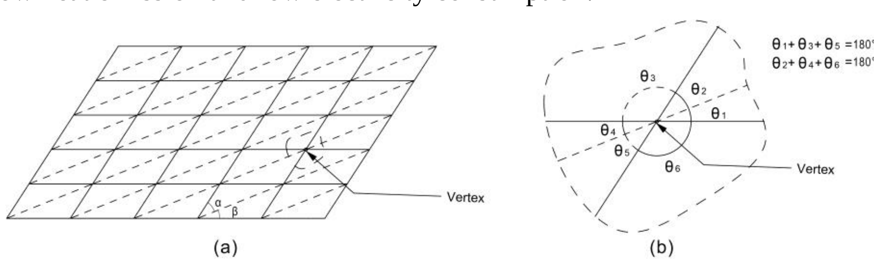

Figure 2: (a) *An example of a periodic Helical Triangle crease pattern. (b) An inner vertex from (a) that satisfies the conditions listed in Kawasaki's Theorem*

Booma Hexagonal Column

To fold a periodic Helical Triangle tessellation into a hexagonal column, as shown in Booma Hexagonal Column (see Figure 1a), crease angles α° (see Figure 2a) needs to be determined. To fold the periodic Helical Triangle tessellation into a regular polygon column with n edges that is flat-foldable, Kawasaki's

Theorem needs to be satisfied. In general, the relationship of α angle to n edges satisfies the following condition while β can be arbitrary:

$$\alpha = 90 - \frac{(n-2)\times 90}{n} \quad (1)$$

Figure 3a shows a special case of a periodic Helical Triangle tessellation in which crease angle $\alpha = \beta$. Figure 3a can be folded into a hexagonal column. I soon discovered that the helical triangles can also be mirrored so that a folded pattern can be more interesting and easier to deploy (see Figure 3b).

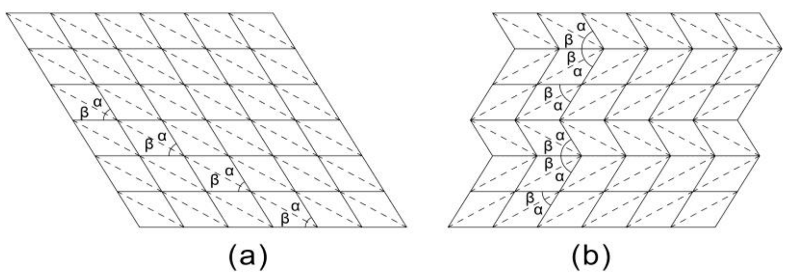

Figure 3: *An example of a periodic Helical Triangle crease pattern and its folded forms.*

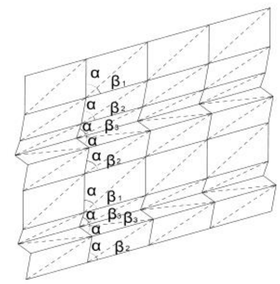

Figure 4: *A semi-periodic flat-foldable Helical Triangle tessellation and its folded forms*

Booma Twisted Square

To achieve the semi-periodic Helical Triangle tessellation that can be folded into Booma Twisted Square (see Figure 1b), I did a few modifications to the original periodic tessellation. Per equation (1), to fold a Helical Triangle tessellation into a square column that is flat foldable and deployable, α needs to be 45°. Since β can be of an arbitrary value, a semi-periodic flat-foldable helical triangle tessellation that satisfies Kawasaki's Theorem can be achieved. Figure 4 shows an example of a semi-periodic flat-foldable helical triangle tessellation with mirrored triangles with α = 45° and β_1, β_2, and β_3 having a different value respectively.

Booma Conical Hexagon

The tessellation found in Booma Conical Hexagon (see Figure 1(c)) has a self-similar quality. Two figures are self-similar if corresponding lengths have the same ratio, that is, if one is either a magnification or a reduction of the other. The common ratio between lengths is referred to as the *magnification* or *growth factor* [3].

Below I will describe my process of creating a tessellation (see Figure 5) to be folded into a hexagonal cone by first using a geometric construction. Though I did this using a CAD program called Rhino, this method could also be accomplished using a straight edge, a compass, and a protractor. In Figure 5a, an arbitrary line AA′ is drawn first. Draw an acute isosceles triangle OAA′ with line AA′ as the base and lines OA and OA′ as the two equal sides. Let δ be ∠ OAA′, let α be any arbitrary angle that is smaller ***δ***. Start the geometric construction by following the steps below:

1. Rotate line AO using point A as the center of rotation in α degree clockwise to get line AM.
2. Rotate line AM using point A as the center of rotation in μ degree clockwise to get line AM′.
3. Rotate line A′O using point A′ as the center of rotation in α degree clockwise to get line AM″.
4. Line AM′ intersects Line A′M″ at B′.
5. Draw an arc using point O as the center of arc that goes through point B′.
6. The arc intersects line AM at point B.
7. Draw Line BB′.

8. Repeat step 1 to step 7 three times to get Figure 5b.
9. Figure 5b can further be tessellated into the final crease diagram in Figure 5c.

Because of its self-similar quality, each interior vertices of Figure 5c satisfy Kawasaki's Theorem, therefore Figure 5c is flat foldable if the interior vertices don't collide. To be folded into a hexagonal cone, the tessellation in Figure 5c must satisfy the following additional conditions:

$$\mu = \boldsymbol{\delta - 60°}; \quad \text{while } \alpha < 60° \;\&\; \boldsymbol{\delta} > 60° \;\&\; \mu > 0° \qquad (2)$$

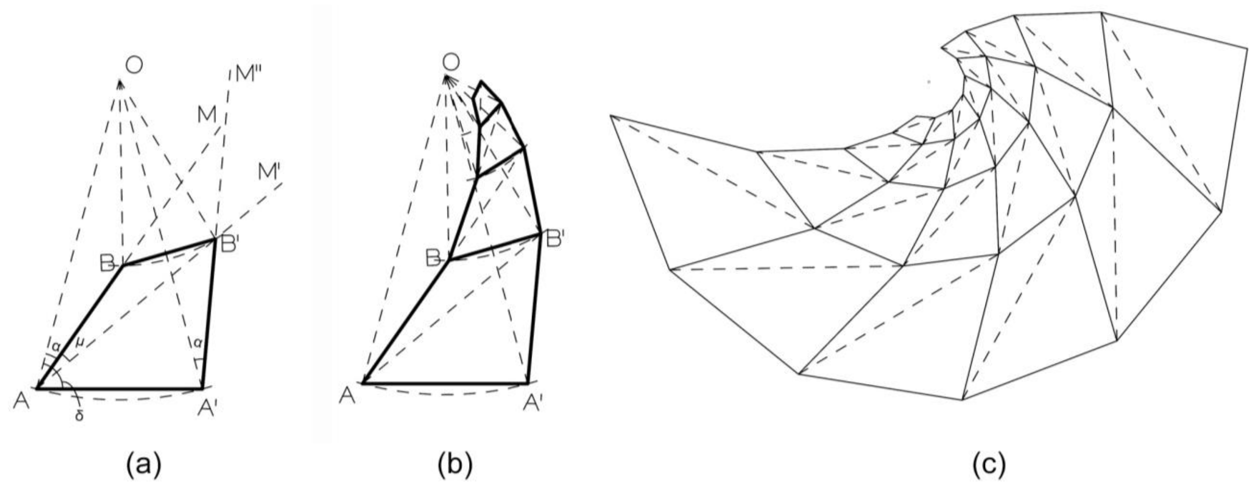

Figure 5: *Geometric construction sequence of a self-similar Helical Triangle tessellation that can be folded into a hexagonal cone*

Booma Conical Hexagon (see Figure 1c), like the other lights in the Booma collection, is made from Hi-tec Kozo and is cladded in stainless plates. In order to conceal the seams where the two cones meet, I changed the way the paper was cut. Instead of folding from two pieces of paper, I folded from six pieces of Hi-tec Kozo instead to get cleaner and more consistent look at the seams. This piece of light art can be collapsed into a two-dimensional hexagonal from.

Conclusion

In this article, I described how I, as an artist, used simple geometric rules and a geometric construction method to create variations of helical triangle tessellations. I then folded these tessellations into a collection entitled Booma Folded Light Art. Though helical triangle tessellations are credited to Taketoshi Nojima, the geometric methods that are presented in this article are the result of my own artistic explorations. It is important to note that more advanced understandings of the helical triangle tessellation by using trigonometry and parametric design are needed into order for artists to further explore the creative potentials of the helical triangle tessellations.

References

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