Ballistic Deposition and Aesthetic Patterns

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Abstract

We explore patterns produced using variations of the ballistic deposition model. Deposition is a process whereby moving particles aggregate according to rules determining where they should adhere. We introduce compositing and rendering techniques for enhancing the aesthetics of the resulting patterns in order to create a series of artworks based on the underlying model.

Introduction

The use of randomness in the visual arts by making use of pseudorandom number generators is central to algorithmic art [9] and evolutionary art [8]. In mathematical art, randomness is usually encountered through the simulation of random walks (see Greenfield [4], Grusky [5], and Matsko [7]) or other random processes (see Dunham and Shier [3] and Kostner et al. [6]).

In the March 2016 issue of the *Notices of the AMS* we were captivated by the cover image and the companion article by Ivan Corwin [2] which, in simple terms, surveyed what is known about the long term growth of certain complex random systems. The genesis for the work herein is Figures 1 and 2 of that article which explain the difference between random deposition and ballistic deposition. The series of artworks we describe arises from variations of the ballistic deposition model.

Random Deposition

To visualize random deposition we imagine blocks being dropped one by one into the tops of randomly chosen columns of a grid and coming to rest when they reach either the bottom of the grid or the most recently dropped block. The process stops the moment one of the columns becomes full. Figure 1 shows three different ways of "randomly" choosing a column as follows. Start with a randomly chosen real number r in the half-open interval $[0, 1)$. Then, using left to right respectively, r, r^2 and $\sqrt{r}$ scale the respective result to the grid width and truncate it to obtain a valid column index. This process is called *binning*. Because we are using a coarse 60×60 grid, we get a rough approximation to the graphs we would expect for each of the three probability distributions.

Ballistic Deposition

The ballistic deposition model is similar to the random deposition model except now the block attaches edge to edge to the first block it encounters. Therefore, if a column to the immediate left or right of the column the block is dropped into has a block that sits higher than the topmost block of the insertion column, the inserted block will adhere to an adjacent column block. Figure 2 shows examples of ballistic deposition at varying grid resolutions when the probability distribution for column indices is uniform. The process is

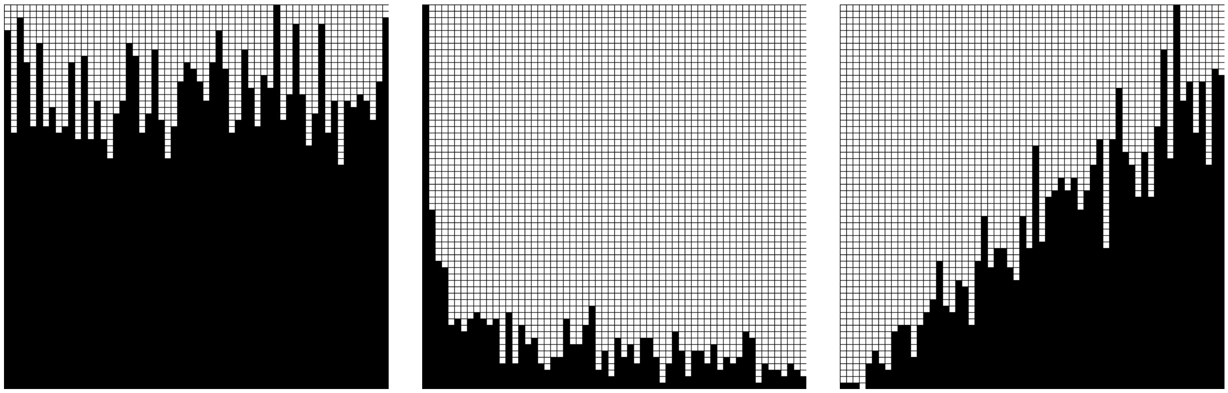

Figure 1 : *The random deposition model where successive blocks are dropped into columns determined by binning (left to right)* r*,* r^2 *and* $\sqrt{r}$ *where* r *is uniformly distributed in the half-open interval* $[0, 1)$.

called ballistic deposition because in many physics applications block motion is assumed to follow a ballistic trajectory. The deposition model for virtual blocks described here can also be viewed as a variant of diffusion limited aggregation.

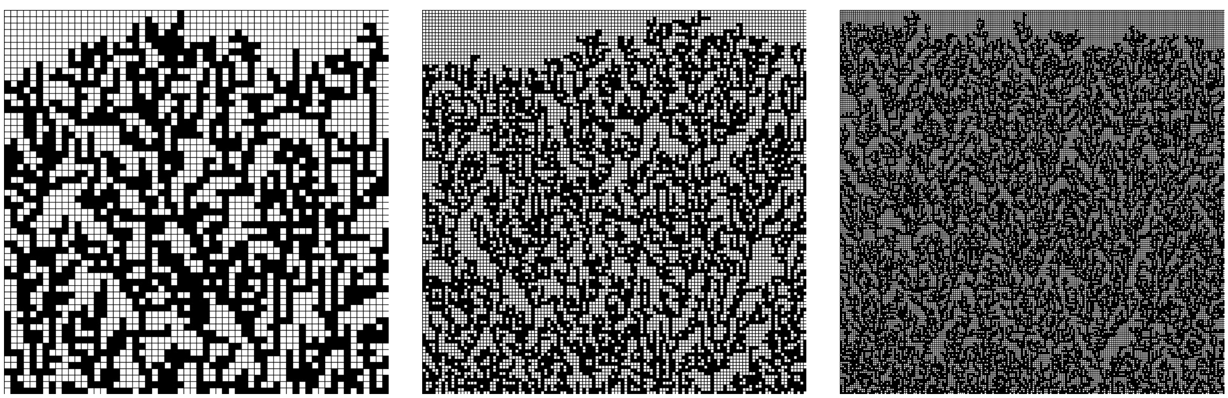

Figure 2 : *The ballistic deposition model at resolution (left to right)* 60×60*,* 120×120*, and* 200×200.

Varying the Rule

Although one can think of any number of ways of modifying the rules for ballistic deposition, the one we wish to consider here is motivated by the concept of remote sensing. That is, rather than a falling block adhering to a higher block in a neighboring column, it ceases to fall if the "neighboring" block is in a column that is $w \geq 0$ columns distant. We use the convention that if w columns distant would cross the boundary the non-existent column has height zero. Clearly, $w = 0$ yields the random deposition model and $w = 1$ gives the ballistic deposition model. Figure 3 shows three examples of this variation.

Rendering Issues

While we have implemented the ballistic model using blocks (i.e., filled rectangles), the cover for the AMS Notices article implements this model using differently colored TETRIS® shapes (i.e., tetrominoes) [1].

Figure 3: *Variations of the ballistic deposition model when the grid resolution is* 60×60 *and where the neighboring columns considered are* w *columns distant when (left to right)* $w = 2$, $w = 5$, *and* $w = 8$.

The article itself visualizes blocks using color ramps. Figure 4 shows how color ramps can enhance aesthetic interest by showing, from left to right, a gray scale color ramp for the filled squares, a monochrome color ramp for the filled squares, and separate color ramps for both filled and unfilled squares. The key thing to notice is that even though the rightmost bi-colored image uses the same hue for filled squares as the central monochrome image, both hues in the bi-colored example are accentuated by ramping in opposite directions to provide additional contrast.

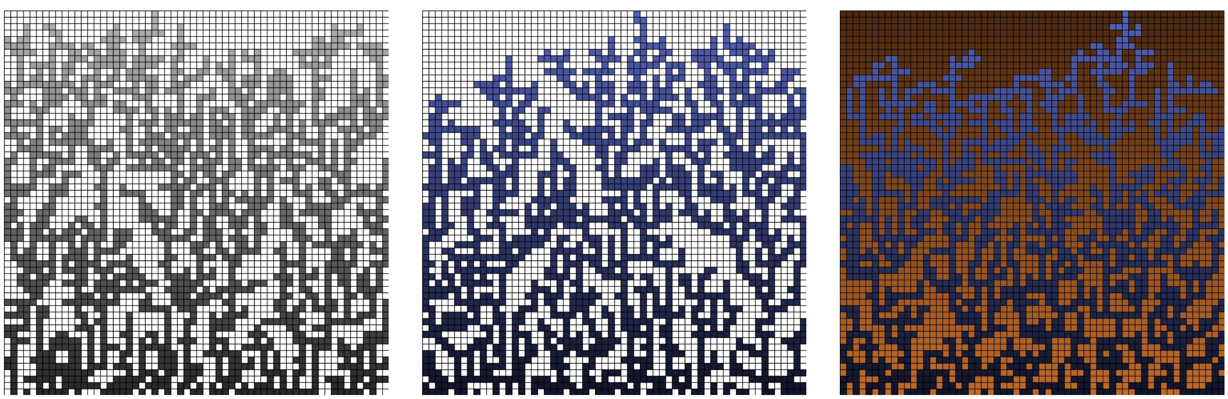

Figure 4: *Three rendering options for the ballistic deposition model: (left to right) gray scale ramp, monochrome color ramp, and ramps for both filled and unfilled rectangles.*

The Ballistic Deposition Series of Aesthetic Patterns

In Figure 5 we show two examples from a series of patterns we generated by combining some of these ideas. To remove the annoying asymmetry that occurs in ballistic deposition we use two simultaneous processes, one with blocks falling down and one with blocks falling up, and we image the overlap between the two patterns. This is equivalent to generating two ballistic deposition patterns, rotating one 180 degrees and then compositing them using a logical and operator as if they were binary digital images. We also adjust the ramping scheme to meet in the middle. At low resolution, the resulting patterns exhibit a sparse, Zen-like quality, while at higher resolutions they are reminiscent of 1960s era tabulating device displays. Either way, the images themselves mask the complex processes and computational effort that underlie their creation.

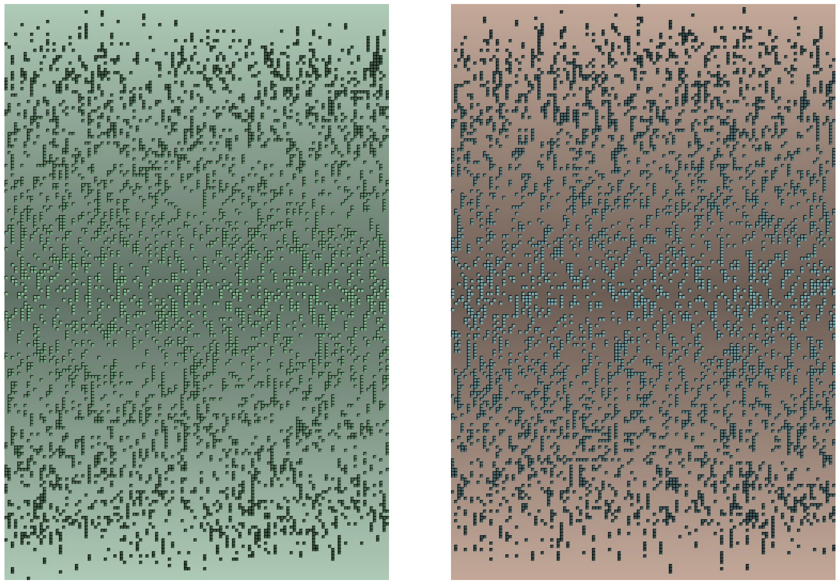

Figure 5 : *Two* 120×180 *ballistic deposition patterns from our series. The one on the left uses* $w = 5$ *and the one on the right uses* $w = 8$.

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