3D Printing in the Secondary Mathematics Classroom

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Abstract

3D printing is rapidly growing in popularity as a teaching and learning tool. While numerous innovators are exploring its use in university and recreational mathematics, the application of 3D printing in K-12 classrooms remains limited. Here I outline some introductory 3D printing projects and their associated mathematical curricula that are suitable for secondary mathematics classrooms.

Introduction

3D printing has quickly established itself as a viable and useful educational technology. An increasing emphasis on STEM (Science, Technology, Engineering, and Mathematics) education has brought with it more makerspaces, innovation labs, and engineering and design programs, all of which benefit from the accessibility and short design-to-production turnaround time of recreational 3D printing.

Mathematics education at the university level is already being impacted by 3D printing. Educators are using the technology to supplement traditional instruction in courses like Calculus, and are also designing new mathematics courses built around the technology itself. The pioneering work of George Hart [1], Laura Taalman [2], and Henry Segerman [3] shows the power and promise of such work.

In secondary mathematics, however, 3D printing is still a classroom technology in its infancy. In this short paper, I outline some introductory 3D printing projects suitable for secondary classrooms that emphasize mathematical ideas, minimize the demand for software expertise, and engage students in art and design.

Technology

There are many approaches to designing objects for 3D printing, and many software packages available for doing so. For the introductory projects outlined below, students exclusively use 3D graphing utilities in the design process. This keeps the focus of the project on mathematical ideas, and minimizes the need for software expertise. Two such graphing utilities that work well and have built-in STL export capabilities are Calc Plot 3D [4] and the Mathpix Grapher [5].

As with any basic graphing calculator, these utilities allow the user to graph a multitude of functions and relations in *xyz*-space. When exported as an STL file, the surfaces are automatically thickened to create a solid whose surface can be triangulated. The resulting STL file can be processed by printer-specific proprietary software and printed. For most shapes, the STL files generated by the above graphing utilities were reliable enough to print with no further editing or clean up.

Mathematical Design Tools

At the heart of the introductory projects outlined below, students use graphs of functions and relations to realize 3D designs. Graphs of functions and relations in two dimensions, their families, and their transformations are an integral part of the secondary mathematics curriculum [6]. The end goal of 3D printing gives students both motivation and context for strengthening familiar knowledge of plane coordinate geometry and extending that knowledge into higher dimensions.

Basic Mathematical Tools

Cylinders of the form $f(x,y)=k$ are a natural starting point, as they leverage student knowledge of basic graphs in the plane and help transfer the concept of *graph of an equation* from two to three dimensions. Surfaces of revolution of the form $x^2+y^2=(f(z))^2$ leverage knowledge of circles, one of the fundamental graphs in secondary mathematics. Both kinds of graphs are accessible to secondary students and provide them an immediate library of design tools. Additionally, the end goal of 3D printing highlights the utility of understanding surfaces through their lower-dimensional cross-sections.

Functions of the form $z=f\left(x^2+y^2\right)$ can help students internalize properties of the polar coordinate system, as well as provide a tool for designing objects with radial symmetry. Implicitly defined relations of the form $f(x,y,z)=0$ together with the associated transformations $f(-x,y,z)=0$, $f(x-a,y,z)=0$, $f(z,y,x)=0$, and so on, give students a powerful set of tools to build objects and design for symmetry while solidifying understanding of the behavior of basic functions and relations.

Some Intermediate Mathematical Tools

Certain graphing techniques not typically included in the standard secondary mathematics curriculum can aid in the design process.

For example, the graph of the product of two relations $f(x,y)\,g(x,y)=0$ yields the union of the graphs $f(x,y)=0$ and $g(x,y)=0$. This was a valuable design tool where the graphing utility limited the number of equations.

Domain restriction was often desirable in design, but not always permitted by the graphing utilities. Mathematical procedures for restricting domain were therefore useful. For example, multiplying the relation $f(x,y,z)=0$ by

$$w(x)=\begin{cases}1 & (x,y,z)\in S\\ \text{undefined} & (x,y,z)\notin S\end{cases}$$

creates a new relation whose graph is the part of the graph of $f(x,y,z)=0$ that intersects S. Thus, the graph of $f(x,y,z)\left(\sqrt{x}/\sqrt{x}\right)=0$ is be the portion of the graph of $f(x,y,z)=0$ that lies on the positive side of the *yz*-plane, and the graph of $f(x,y,z)\left(\sqrt{1-x^2-y^2}\Big/\sqrt{1-x^2-y^2}\right)=0$ is the portion of the graph of $f(x,y,z)=0$ that lies inside the unit cylinder. The design of such "filters" proved themselves to be compelling exercises in geometry, algebra, and domain.

The concept of linear combination also provided a powerful design tool. Equations of the form $a \cdot f(x, y, z) + b \cdot g(x, y, z) = 0$ allowed students to experiment by combining the graphs of $f(x, y, z) = 0$ and $g(x, y, z) = 0$ to create hybrid graphs. This proved particularly relevant when looking at equations of the form $a \cdot f(x, y, z) + (1 - a) \cdot g(x, y, z) = 0$ for $0 \le a \le 1$, as this allowed students to extend basic notions of convexity and midpoint, and to visualize smooth transformation from one shape to another.

Project Ideas

Below are three simple project ideas that allow students to quickly engage in mathematical design for 3D printing.

Replicate an Existing Object

In this project, students choose an existing object and attempt to replicate it using a collection of mathematical functions and relations. Students search for appropriate parent functions to work with, then combine and transform them to meet their design goals. Example student work can be seen in Figures 1 and 2.

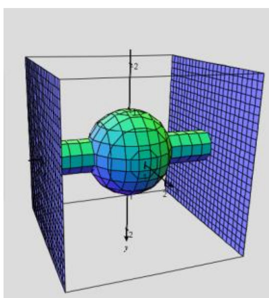

Figure 1: *Tie Fighter.*

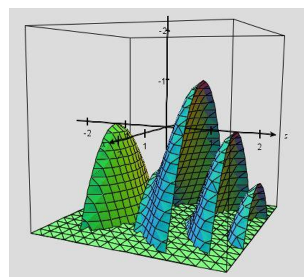

Figure 2: *Sydney Opera House.*

Portray a Mathematical Concept

This project asks students to create an artistic representation of a mathematical concept. Some suggestions were: *symmetry; rotation; inversion; singularity*, but students were free to choose their own topics. Students played around with different graphs and then explored the artistic space of those graphs. Students were tasked not only with producing a final shape, but explaining their choices and comparing various prototypes. Example student work can be seen in Figures 3 and 4.

Figure 3: *Revolution.*

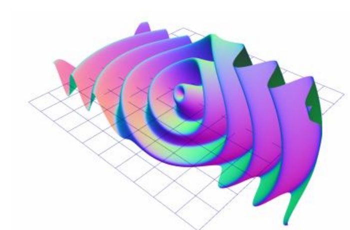

Figure 4: *Rotation.*

Linear Combination

In this project, students explore the space of surfaces generated by linearly combining equations of known surfaces. One approach was to look at the surfaces of the form $a \cdot f(x,y,z) + (1-a) \cdot g(x,y,z) = 0$ and examine the transition from one surface to another. The final product could be either a sequence of prints documenting the linear transformation, as shown in Figure 5, or a single print for a given parameter chosen for aesthetic or mathematical reasons. This project was inspired by an exhibit at the National Museum of Mathematics [7].

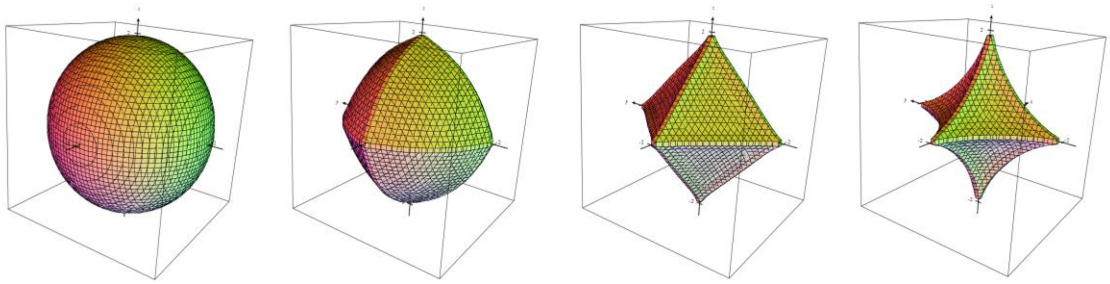

Figure 5: *Sphere to Astroid Transition.*

Conclusion

The above 3D design and printing projects allow students to engage with mathematical ideas in a novel and creative way. This is particularly valuable in classroom settings, as it creates an opportunity for students to reset their relationship with mathematics. In my experience, students who demonstrate reluctance to engage with advanced mathematical ideas still enthusiastically engage in these projects, and can find success realizing their ideas with mathematics. And there is no immediate ceiling for what more comfortable students can accomplish.

This collection of projects served as a successful introduction to 3D printing in my secondary mathematics course. New mathematical ideas were introduced as extensions of familiar ones, 3D printing provided both incentive and context for the development of those ideas, and the initial technology demands on students were minimal. And the 3D printing of the final product made the endeavor more real to students. Many expressed satisfaction and a genuine sense of accomplishment when first holding the object they designed in their hands.

References

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