3D Printed Tours

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Abstract

I discuss ongoing experiments in using 3D printing in TSP Art and other tour-based artwork. My goal is to employ the technology to create sculptures that pull the viewer into the artwork and lead them to make discoveries or to see something in a new light.

Introduction

Figure 1 displays two of my favorite pieces of TSP Art [3,5]. Figure 1(a) shows *Connecting the Dots*, a piece designed to focus attention on the process of making TSP Art (and also to pay homage to Escher). I started with a photograph of my hand, taken while it was tracing a labyrinth. Next, I converted the photo into a stipple drawing, a collection of dots that collectively resemble the photo. I then used the Concorde TSP Solver to find an optimal solution to a 1640-city Traveling Salesman Problem [4] based on the dots. Finally, I created a PostScript file of (most of) the optimal tour. My hope was that by omitting part of the tour, I would induce viewers to try to connect the remaining dots with their eyes, leading them to discover that the completed piece is a large number of dots joined together to form a simple closed curve.

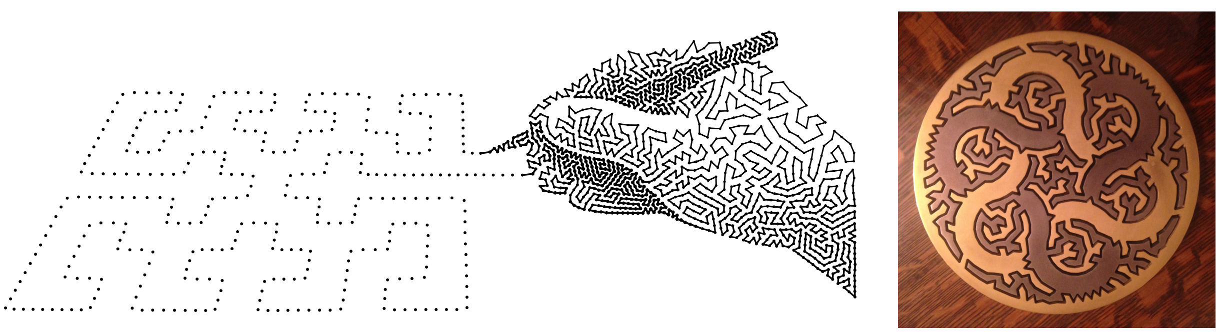

Figure 1 : *(a) Connecting the Dots © RB 2016, (b) Embrace © RB 2009.*

Figure 1(b) is a photograph of *Embrace*, which was awarded First Prize at the 2010 AMS Exhibition of Mathematical Art. Here I started with a drawing of a two-component link. This time, I forced the stipple drawing to be symmetric, and I used a modification of the Dantzig-Fulkerson-Johnson integer programming formulation of the TSP to find a symmetric tour [1]. Instead of drawing the tour, I found someone who had access to a water jet cutter and hired them to cut along the tour through quarter-inch thick, six-inch diameter disks of steel and brass, dividing each disk into an "inside piece" and an "outside piece." By swapping the inside pieces I obtained two sculptures. In the one shown here, steel is inside and brass is outside. *Embrace*'s three elements—the steel inside piece, the brass outside piece, and the gap between them (the tour)—lead the viewer into contemplation of the famous Jordan Curve Theorem: every simple closed curve in the plane is like a cookie cutter in that it divides the dough of the plane into two regions: the part that lies inside the curve (the cookie), and the part that lies outside (the leftover dough that will be made into more cookies).

In the present paper, I describe my efforts to use 3D printing technologies in TSP Art and other TSP-Art-related artwork. As in the two examples described above, my goal is to create pieces that pull the viewer into the artwork and lead them to make discoveries.

3D Printed TSP Art

Figures 2 displays a 3D printed piece of TSP Art based on a section of Leonardo da Vinci's *Mona Lisa*, and Figure 3 displays two closeups. After using Concorde to find an optimal solution to the 1200-city TSP, I created an OpenSCAD model of the tour. To make it easier for the viewer to follow the tour, I extruded it toward a focal point positioned approximately one meter from the picture plane, varying the height of the extrusion—the amount it extends outward from the picture plane—in accordance with the salesman's position on the tour. I then 3D printed the extruded tour with Shapeways and placed it on a light board.

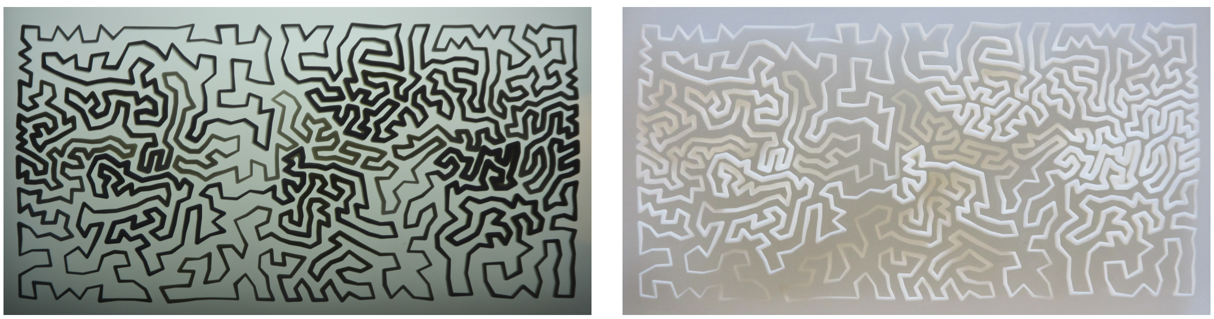

Figure 2 : *Lit and unlit 3D printed TSP Art viewed from the focal point*

When the light board is off, the viewer will notice that the sculpture is like a path that winds its way along the ridges of a mountain range. When traversing this path, the salesman goes uphill then downhill, up then down. When the light board is on, the viewer will need to walk around the sculpture and view it from a variety of positions to discover the optimal viewpoint, from which the sculpture is transformed from abstract to figurative. When backlit, some "higher" sections of the tour appear darker than some "lower" sections.

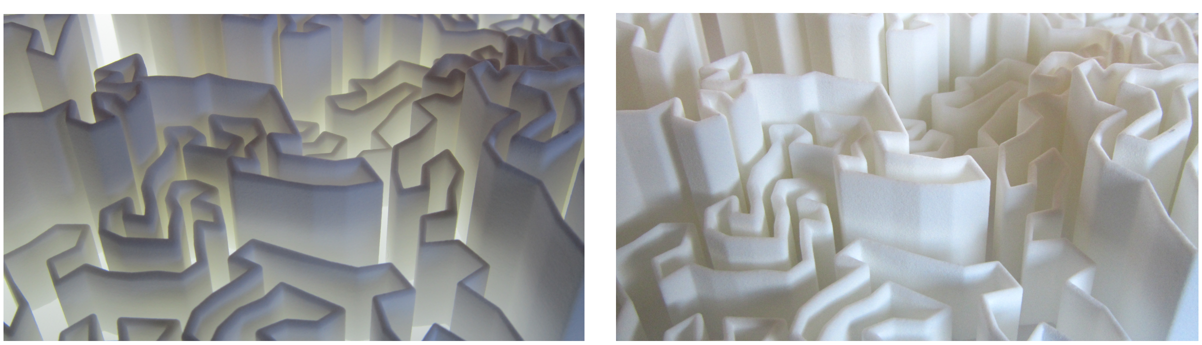

Figure 3 : *Lit and unlit detail of 3D printed TSP Art viewed from an angle.*

As an alternative to a light board, I have experimented with mounting the 3D print on plexiglass and backlighting it with book lights, as shown in Figure 4. Shapeways' "white, strong, and flexible" material is indeed quite flexible, so adhering the print to the plexiglass without deforming the tour requires a great deal of patience and a very steady hand.

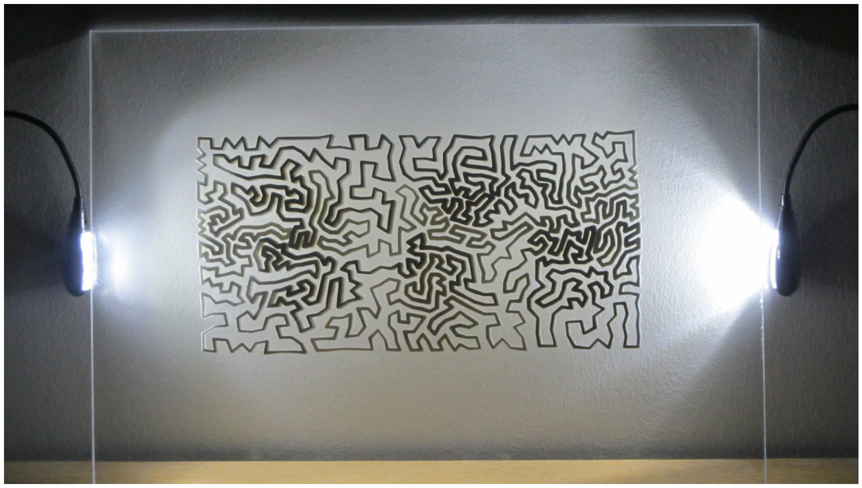

Figure 4 : *3D printed TSP Art mounted on plexiglass and backlit by book lights.*

3D Printed Knight's Tours

There are 13,267,364,410,532 closed undirected knight's tours of a standard 8×8 chessboard, and 608,233 of them have two-fold rotational symmetry [6]. None has four-fold rotational symmetry, and none has any form of mirror symmetry. Due to their numerous edge crossings, all are difficult to follow by eye.

I devised an integer programming (IP) formulation (similar to [2]) for finding knight's tours that are as close as possible to having four-fold rotational symmetry. It turns out that an 8×8 knight's tour can have up to 60 *instances of rotational symmetry*, segments of the tour that have all three of their rotated counterparts in the tour. Figure 5 displays photographs of two 3D printed sculptures based on two of these "almost four-fold" knight's tours. When viewed from the focal point (from one meter away, straight on) they look flat, like traditional drawings of knight's tours. From other viewpoints, they look like architectural models.

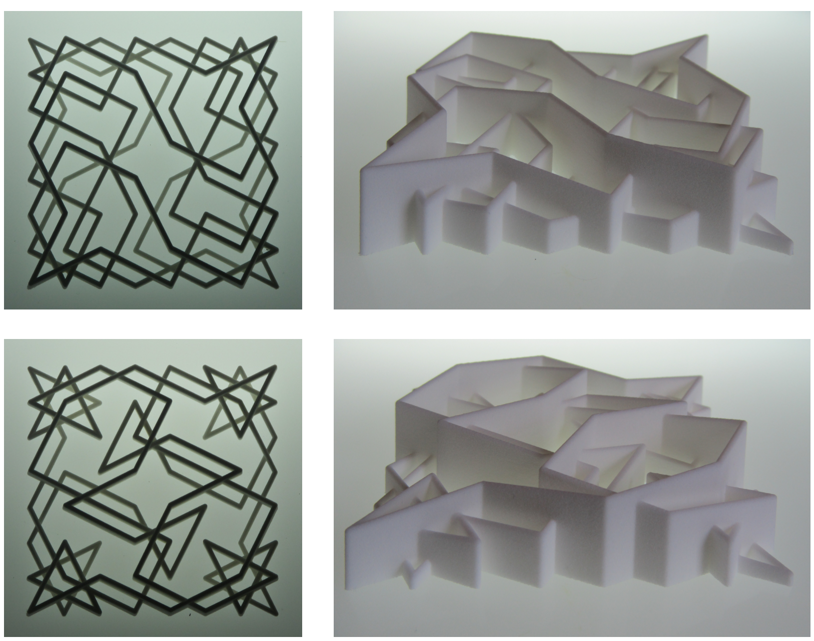

Figure 5 : *Front and offset views of two 3D printed "almost four-fold" knight's tours.*

I created a second IP formulation for finding knight's tours that are as close as possible to having vertical mirror symmetry. It turns out that an 8×8 knight's tour can have up to 56 *instances of vertical symmetry*, segments of the tour that also have their vertical mirror counterpart in the tour. Figure 6 displays photographs of two 3D printed sculptures based on two of these "nearly mirror" knight's tours.

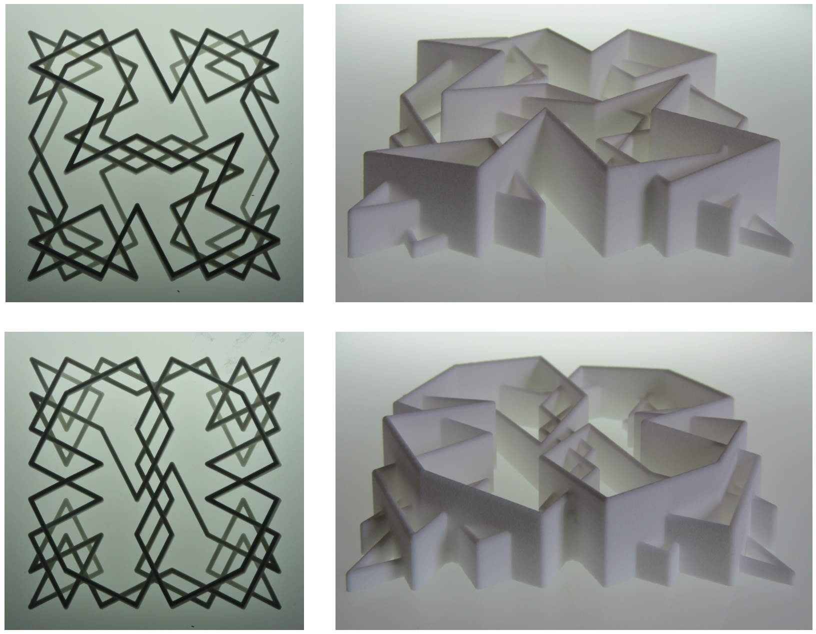

Figure 6 : *Front and offset views of two 3D printed "nearly mirror" knight's tours.*

As with the 3D printed TSP Art, the variation in the amount of extrusion makes it easier for the viewer to follow the tour. (And the sinusoidal up and down "motion" of the tour may remind the viewer of a merry-go-round.) When viewed from an angle, the 3D printed tour segments resemble the walls of a castle.

References

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