# Euclid's Digital Elements:
# Straightedge and Compass Softwares as Aid for Geometrical Design

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## Abstract

In this workshop we propose an activity to explore a geometric drawing software as an easy and quick tool to plan and perform bold and beautiful geometric paterns based on simple well-known constructions with straightedge and compass. The workshop might be of particular interest either to teachers trying to rise interest in Geometry through artistic activities or to both experienced and experiencing mathematical artists involved in plane geometrical drawings. No previous knowledge on the software used, Geogebra, will be required, but the participants are expected to be able to have access to the software, or an equivalent one, during the workshop.

## Introduction

Geometric drawing with straightedge and compass is part of the first contacts of children with geometry in early school, and the discipline's very name "drawing" suggests a conection with the visual arts. Indeed, geometric drawing has been part of the knowledge of painters, sculptors and architects classically as a tool to achieve results, as one can see for example in the description of perspective in the treatise "On Painting", by Leon Battista Alberti [1], or as the artwork *per se*, as in string art, for example, a case that connects well to the kind of designs this workshop is about. A good start point to know string art is, as usual, the Wikipedia's page on the topic, specially the references [4].

Although straightedge and compass open a manifold of possibilities of drawings, the algorithms are very simple, hence they are taught to children and have been coded in a lot of geometric softwares, usually with similar interfaces. In this workshop we use GeoGebra, an open source free mathematical software designed for educational purposes, specially straightedge and compass drawings, to help make experiments with geometric figures with artistic purposes in a much faster pace than actually using paper and pencil. With this tool, not only we can experiment faster, hence more, but also more intricate pictures can be drawn with arbitrary precision, enhancing the possibilities of creation for the artist or student. Geogebra can be obtained for free at its website [2].

## The Activity

But what sort of drawing can be made? The options are vast, and the intention of the workshop is to give the tools one needs to explore the possibilities. But since a choice is necessary for the presentation, I shall focus the initial part of the meeting in presenting constructions that are somehow basic and that have been presented to me by a Maths teacher at the beginning of my "career" as a geometry enthusiast. The aim is to show the basic tools offered by the software from scratch with some interesting constructions. For example, by subdividing a pair of segments and correctly connecting the dots, one can get straight lines that look curved (see figure 1) or, drawing all diagonals in a polygon, one can draw the "super-circle" (see figure 2). The constructions shall use no more than the tools for building or bissecting a segment, building a polygon, drawing a circle or inverting lines, all functions already built-in Geogebra's options.

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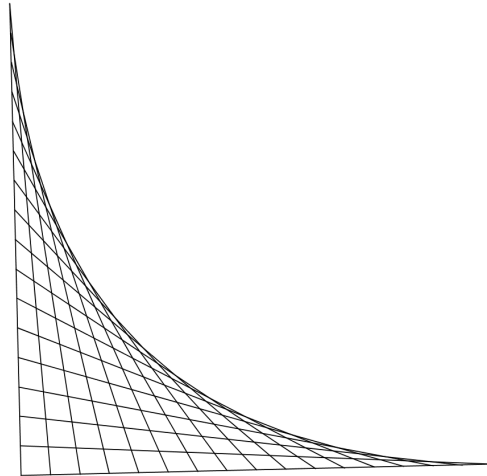
**Figure 1** : *Straight lines that look curved on orthogonal segments.*

Watching how to make these constructions will allow the participants to become acquainted with Geogebra's interface and basic techniques of making the drawings, coloring them and using the drag and zoom tools in order to help with difficult designs.
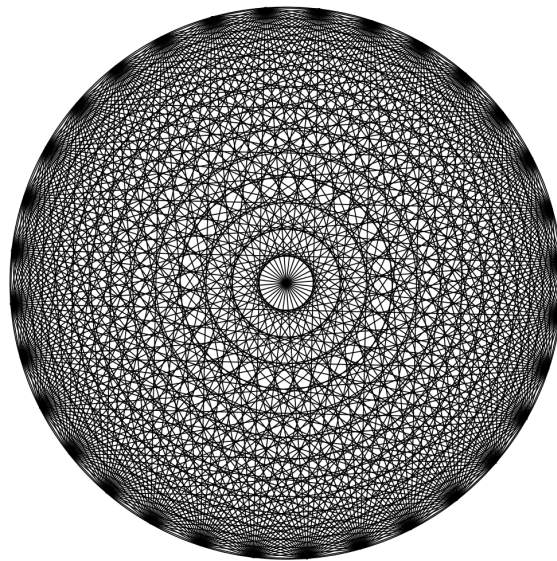


**Figure 2** : *A 32-sided supercircle.*

After this short preamble of maybe half an hour, the idea is that participants get their hands dirty trying to draw something of their own using the software and the tools presented. The point is not to create a masterpiece in forty minutes or so, but to struggle with the drawing in a controlled environment, where you can interact with other participants and solve your questions with others and with the presenter. The goal is to give the participants autonomy in drawing artistically with Geogebra in order to allow further explorations by themselves.

## But... What For?

That is a tricky question concerning mathematical, or artistic, subjects, but it is a good excuse to address some background aspects that surround these drawings. It is important to stress that most of the following considerations are born from my personal experience as a young learner exposed to this sort of activities with paper and pencil, and they should be taken exactly as such.

From an early schooling point of view, the activity has an intrinsic value as a stimulus to practice geometric drawing. Kids like to do these beautiful things, and they probably will strive to do more intricate drawings, with more lines, exploring new shapes, just for the sake of it. They will also probably develop their own ways of managing the drawings, since one does not draw 528 lines simply by grabbing a pencil and doing it. I remember my teacher giving a class on how to draw the straight lines that look curved and, at the end of the class, showing an album with the drawings of some former students of hers. On the next class, a couple of "super-circles" had been spontaneously drawn by a bunch of 11-year-old children.

Besides this challenge aspect, it is interesting to show ways of letting them know their drawings better. Discovering the number of lines in each drawing is a simple, yet very concrete, counting problem, which may pose itself in different ways that depend on the drawing itself. For example, counting the number of lines needed for the "super-circle" of figure 2 is the same thing as finding the number of diagonals of a polygon (including its sides), $\frac{n(n-3)}{2} + n$ for an $n$-sided "super-circle", while the number of lines in a mixed image like figure 3 is much simpler, since it depends only on how many layers of hexagons one decides to draw inwards, but demands a more careful model of the drawing in order to be counted, checking how many lines are really inversed in the drawing and if the repetition of steps can make the counting problem easier - since the drawing on the left is inversed along two circles in order to get the picture on the right, counting the number of lines of the first drawing gives exactly a third of the total number of lines in the final result.

Also, the simplest drawings pose plane geometric questions like the size of the lines, their angles, and some transformation questions, based on the symmetries of the problem, that allow the teacher to explore basic results in euclidean plane geometry concerning lengths and angles.

For more advanced students one can either conceptually or even technically, for calculus learners, the straight lines that seem curved are examples of envelope families of curves in the plane, that is, they are the family of tangents to a curve given by the equation:

$$F(t, x, y) = \partial_t F(t, x, y) = 0$$

with $F(t, x, y)$ the equation of the family of curves indexed by the parameter $t$.

The most common example is the parabola that appears in the simplest stringy drawings, like the one in figure 1, but another curve, the astroid, characterized exactly as the curve which tangents all define segments of the same length in the first quadrant or, in other terms, as the curve enveloped by a family of segments of constant length connecting two orthogonal lines. The astroid is an interesting example because it is defined as a curve enveloped by an easily drawn family of segments. The calculations for both examples can be found in the Wikipedia page for envelope curves [3].

An interesting sidenote is that if one tries to reach both examples, the parabola and the astroid, by an iterative construction, they behave differently: while the envelope of the parabola changes its shape when you increase the number of lines in the drawing, the envelope of the astroid sticks to the curve in all steps.

On the other hand, for an artist, the use of a drawing software provides an invaluable opportunity to explore pictures rapidly, resulting in a more dynamic creative proccess that allows fast improvement. Also, some constructions are too intrincate to be made by hand, but are possible with a computer, specially with its zoom in/out tools and the possibility of saving macros such as inverting. An example is figure 3 (on the right), made directly at Geogebra, with iterated use of the "Inversion" tool on a simple structure of nested

hexagons (on the left). Certainly, I wouldn't be able to invert all those lines by hand keeping the cleaness of the drawing, specially adding color. The proccess was allowed, though, in a couple of hours with Geogebra.
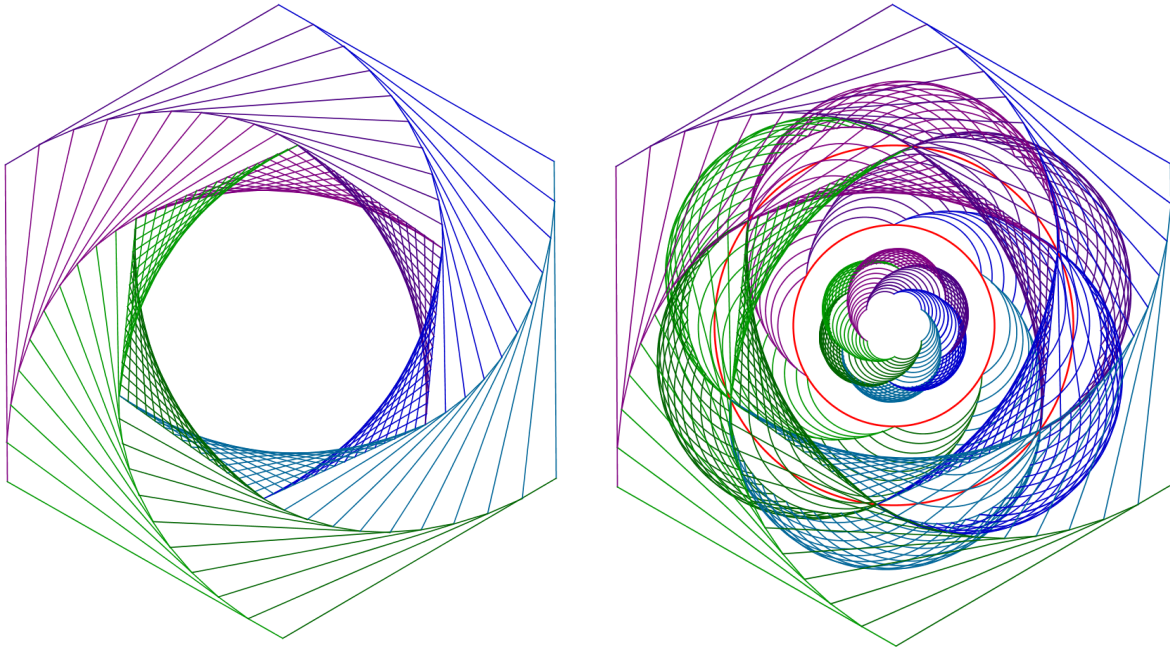


**Figure 3** : *Logo of a minicourse given by me on the relationship between Mathematics and Art (right) obtained by inverting the nested hexagons (left) with respect to a circle.*

## Acknowledgements

This workshop is a direct result from the classes given by Janira Margareth Rodriges Netto, the forementioned Maths teacher that, maybe unaware, put me in the first steps of Euclid's Ladder.

## References

[1] L. B. Alberti, "Da Pintura", translation by Antonio da Silveira Mendona, 2nd ed. (1999), Editora da Unicamp.

[2] Geogebra's website is `http://www.geogebra.org/` (as of Feb. 14, 2016).

[3] Wikipedia entry on "Envelope (Mathematics)" is available at `http://en.wikipedia.org/wiki/Envelope_(mathematics)` (as of Feb. 14, 2016).

[4] Wikipedia entry on "String Art" is available at `http://en.wikipedia.org/wiki/String_art` (as of Feb. 14, 2016).