Spinning Arms in Motion:
Exploring Mathematics within the Art of Figure Skating

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Abstract

In this workshop, participants will explore middle and secondary level mathematics related to a figure skater's arm movements while performing an upright spin. From the bird's eye view, the skater's elbow and hands travel in curves. We provide two dynamic animations of the arms' motions to model and explore the trajectories in which the hands and arms travel. Some of the mathematical topics used in these activities include proportional relationships, regression, circular geometry, and trigonometry.

Mathematical modeling is recognized as an important content area to be learned in school mathematics. Students are expected to apply their knowledge of mathematics to different real-life situations and solve real-life phenomena. Contexts of interest to students include art and sports. Figure skating is a captivating art and fascinating sport. One of the most beautiful required elements in a singles figure skating program is a spin. Being able to perform a spin requires the simultaneous interaction of multiple body parts in order to execute each movement within the spin.

Figure Skating Upright Spins

The upright spin is a basic figure skating spin in which the skater balances the torso upright while standing on one leg. The upright position is also the most efficient spin position, as two Guinness Book of World Records were recorded in the upright position: the longest spin [1] and the fastest spin [2]. During the rotations, the skater's arms and free leg move, while the skating leg rotates on a vertical axis in a stationary location. In the International Judging System, the spin is judged based on the following criteria: "1) good speed or acceleration during the spin, 2) ability to center a spin quickly, 3) balanced rotations in all positions, 4) clearly more than required number of revolutions, 5) good, strong position(s), 6) creativity and originality, 7) good control throughout all phases, and 8) element matched to the musical structure [3, p. 12]." Both physical power and artistic prowess must be combined in order to execute a high-scoring spin.

Usiskin [4] has advocated simplifying complex real-life problems for the purposes of mathematical modeling in the school curriculum. Consistent with this suggestion, we focus on the motion of one particular body part, the arms. Also for simplicity, we ignore the additional distance traveled by the rotation of the skater's body.

In this workshop, the perspective we take on the skater's body is from the bird's eye view. There are four stages to the upright spin: 1) the skater enters the spin on her left foot [and continues using her left leg to spin on until the spin's exit] 2) she begins spinning in a counterclockwise direction with her arms extended, 3) she pulls her arms and legs to accelerate the spin and prolong the rotations, and 4) she exits the spin on her right foot. The activities in this workshop are designed to investigate the position of the arms in the second and third phases of the spin when the arms are at an extended position and the skater bends her elbows and brings her forearms in towards her torso. We assume that the left and right sides of the body are moving symmetrically and simultaneously, at a constant rate, throughout this stage of the spin.

Dynamic Geometry Animation Representations of the Arms in an Upright Spin

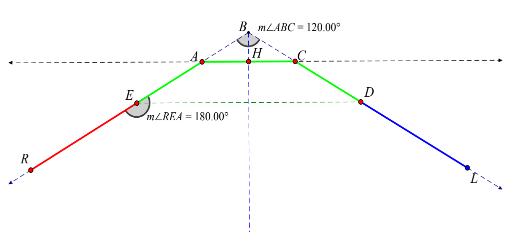
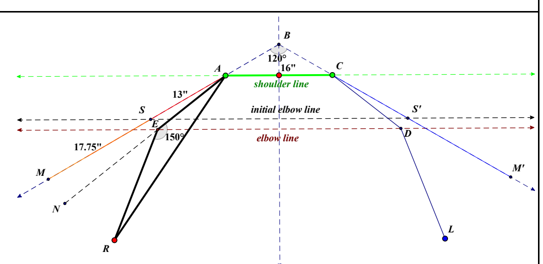
There are two models that we have used with pre-service teachers and high school students, depending on their mathematical backgrounds. With pre-service elementary and middle school teachers, we used a model in which we assume the skater's upper arm remains stationary, with only the lower arm moving. With pre-service secondary teachers, we used a model in which both the upper and lower arms are in motion. In the workshop, we will show both of these models and their corresponding Geometer's Sketchpad software (GSP) files that dynamically depict positions of the upper arms and lower arms as the spin takes place. (The files are available at:
https://sites.google.com/site/mathematicswithinanuprightspin/gsp)

The following points represent the skater's body parts as listed below.
In both models, the following three static points are depicted:

- Point *H*: head
- Point *A*: right shoulder
- Point *R*: dynamic position of right hand
- Point *C*: static position of left shoulder
- Point *D*: dynamic position of left elbow
- Point *E*: right elbow (static in Model 1, dynamic in Model 2)

In Model 2, the additional points are depicted:

- Point *S*: initial position of right elbow
- Point *M*: initial position of right hand
- Point *S′*: initial position of left elbow
- Point *D*: dynamic position of left elbow
- Point *M′*: initial position of left hand

Front View	Bird's Eye view of Arms (Model 1)	Bird's Eye view of Arms (Model 2)
		

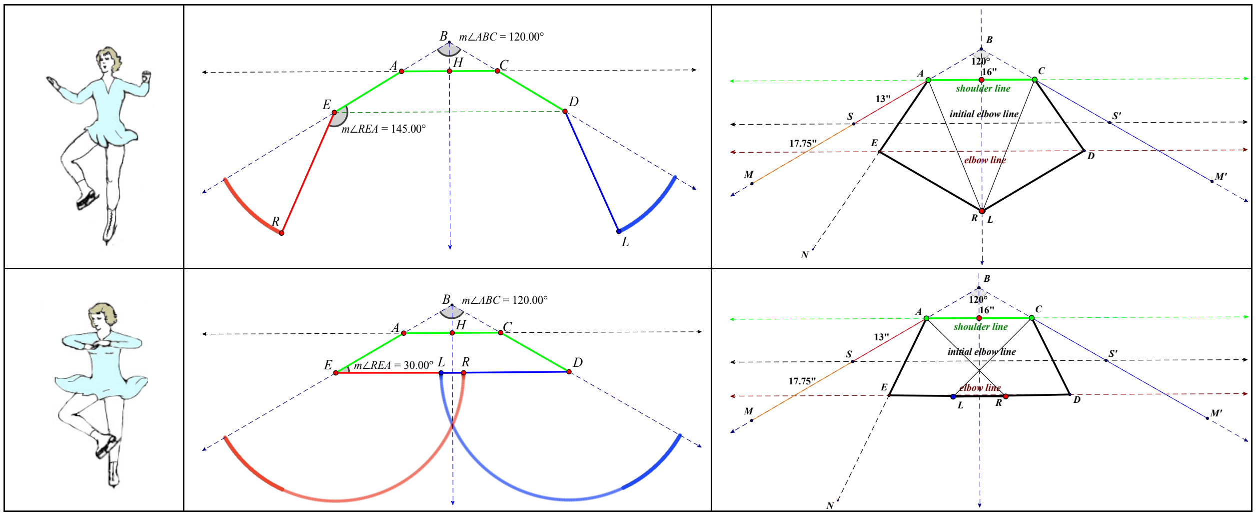


Table 1: *A bird's eye view of arms in the upright spin in three positions: Two models.*

Activities Related to Model 1: Proportional Reasoning & Algebra

In Table 1, the upper arm's length *AE* is 3.66 centimeters and it represents the skater's actual upper arm length of 13 inches. We ask questions related to the scale factor between real-life sizes and the model such as the following:

a) What is a scale factor between the skater's actual body size and the sizes provided in the diagram?
b) The skater's forearm length is 17.75 inches. How long should *ER* be drawn on the diagram? Provide your answer in centimeters.
c) *AC* is the distance between the skater's shoulders. In the diagram, *AC* is 4.5 centimeters. How long is the skater's actual shoulder line (in inches)?

Using the Geometer's Sketchpad file for Model 1, participants will complete a table by recording the distance of *AR* (in centimeters) for given measurements of $\angle REA$. Since a scale factor was used to represent the skater's arms in the animation, participants will also determine the actual distance of *AR*. Based on this collected data, participants will create scatterplots to represent the following: the measure of $\angle REA$ over time, the actual distance *AR* (in inches) over time, and the distance *AR* as a function of the measure of $\angle REA$. Participants will also determine the curve of best fit to describe the relationship between the dependent and independent variables in each graph, and explain how they decided which type of function best describes this relationship.

Activities Related to Model 2: Trigonometry & Geometry

Using the GSP file for Model 2, participants will start animating point *R*, and stop the animation when $\angle REA = 150°$ (at 3.93 seconds) and when $\angle REA = 100°$ (at 11.07 seconds). Participants will answer the following questions:

a) Solve ΔREA completely: determine all of the missing angle measurements and segment lengths.
b) Find the area of ΔREA.
c) What is the difference between *AM* and *AR*?
d) Compare your answers to a-c) for the two different measurements of $\angle REA$. Does the area change? Does the distance *AR* change? Explain why or why not.

Since the skater's upper arm lengths are fixed, one can conclude that points *S* and *E* are points on the same circle with its center at point *A*. Likewise, since the skater's lower arm lengths are fixed, *M* and *N* also are points on the same circle with its center at point *E*. Participants will answer these questions:

a) Find the length of circular arcs *SE* and *MN* at both times: 3.93 seconds and 11.07 seconds.

b) Find the difference between two ratios of *SE/MN* at both times. What does your answer mean?

During the upright spin, skater's hands cross each other. In Figure 1, we model this situation, when the left and right hands first touch each other. Use the GSP file and a stopwatch to determine when it happens as well as the measure of $\angle REA$.

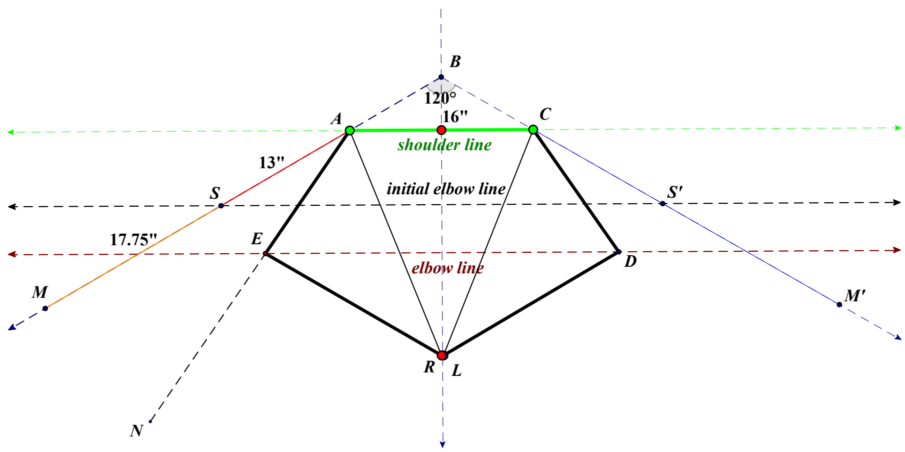


Figure 1: *Pentagonal position of arms, Model 2.*

a) Does pentagon *ACDRE* have symmetry? If so, what type? Explain.

b) Find the measures of all the interior angles of the pentagon.

c) Find the length of diagonal *AR* of the pentagon.

d) Determine the area of the pentagon.

Conclusion

Arm movements contribute important aspects of the spin. Depending upon how the arms are placed, skaters can achieve longer spins or greater rotational speed or more aesthetically pleasing positions. In our workshop, we discuss two models of a particular set of arm motions from the bird's eye view to demonstrate middle and secondary level mathematical activities. We will discuss participants' responses to these questions, pre-service teacher and high school student responses to these questions, as well as any additional questions that arise regarding these two models.

References

[1] Guinness World Records. (2015). Fastest spin – Ice Skating.
http://www.guinnessworldrecords.com/world-records/fastest-spin-ice-skating

[2] Guinness World Records. (2003). Most continuous upright spins - Ice skating.
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[3] International Skating Union. *Communication No. 1944: Single & Pair Skating: Scale of Values, Levels of Difficulty and Guidelines for Marking Grade of Execution, season 2015 - 2016*. 2015.
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[4] Usiskin, Zalman. *Mathematical Modeling and Pure Mathematics.* Mathematics Teaching in the Middle School, Vol 20 (8), pp. 476-482. 2015.