Why Do Mathematical Presentations Sometimes Sound Like Cookery Shows?

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Abstract

I consider what procedural discourse in mathematical presentations suggests about mathematical activity, and why food-based explanations tend to be popular and effective.

Introduction

Often, an outsider to mathematics is able to access the ideas and culture of the discipline only through the mediation of mathematicians' reports about their own experiences, or through metaphorical explanations of mathematical content which, though often illuminating, by their nature must omit or misrepresent as much as they portray. As an artist fascinated by mathematics, determined to find innovative ways for people from my own discipline to overcome this frustrating communication barrier, I am engaged in a research project that focuses on one of the few social and visible aspects of the discipline. The aim is to prioritize observation of communication at mathematics conferences and to see what insights can be gained from this to answer the most pressing questions that outsiders often ask: what is a mathematician really doing, and what does mathematics really consist of? I am using ideas from linguistic pragmatics, the study of how context contributes to utterance interpretation, in an analysis of mathematical presentations. In this paper I present some preliminary observations that focus on the procedural discourse used in presentations and the particular vehicle for popularizations of mathematics that this has inspired, and consider how the experience of mathematics available to the outsider differs from that of the insider.

Procedural Discourse and Recipes

Mathematics presentations are couched in procedural discourse, which is used in instruction manuals and recipes, and often takes the form of chronologically-arranged instructions addressed to a non-specific agent. Much of the language of mathematics presentations is reminiscent of a how-to video or a cookery show. Take this quote and board notes from a talk on Geometric Techniques in Knot Theory by Jessica Purcell:

(1) *"Step two… is you cut the whole thing in half along the projective plane..."* (Fig. 1) [1]

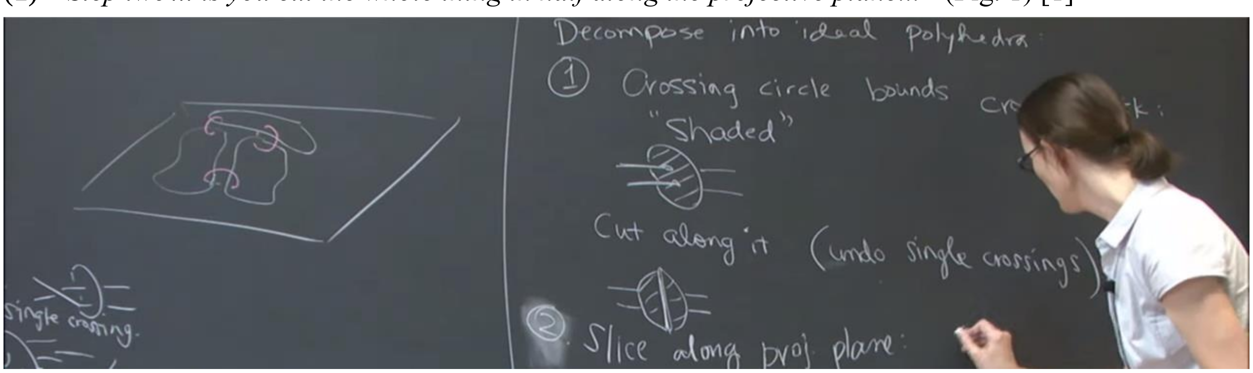

Figure 1. *Board notes: "Slice along proj. plane"*

A paper by pragmatist Tim Wharton examines the language used in recipes and what this reveals about the aims of cookery books. Paraphrasing David Farkas, Wharton describes the aims of a piece of procedural discourse as “first, to sell itself as a procedure—the recipe must try to be convincing and credible and implicitly assure the reader that it comes from a reliable source; and second, to convince the reader that any effort put into following it will be rewarded.” [2] Much of the work of the mathematics paper is to do with the first of these. Convincing the audience of the validity of the various steps is key, and trust in the process of peer review is an essential part of the functioning of such a large and complex discipline. But what might the rewards be for following its various directives? What is the effect of reading a sentence like (1)?

In his *Remarks on the Foundations of Mathematics*, Wittgenstein describes pieces of mathematics as techniques or machinery, saying that proving a new calculus is what allows us to use this new “machine-part.” [3] Interestingly, this type of constructivist description is borne out in some phrasing that can be seen in mathematics presentations, in which the utility of pieces of mathematics as tools or techniques is often referred to:

(2) *“I'm going to introduce a lot of these, uh, a lot of this machinery and a lot of these techniques...”* [1]

For Wittgenstein, a piece of mathematics does not so much reveal properties of an independently existing mathematical object as construct them. We redefine an object as having a certain property by constructing the argument that proves it. If I think hard about the dish I'm reading about, it's true that I can almost taste it, but in mathematics, the learning of the procedure is what itself makes the meal.

Mathematics Explained Using Food

This throws an interesting light on the strong theme of food metaphors and analogies in mathematics popularizations. Two books that used such a device came out in the spring of last year: Eugenia Cheng's *How to Bake Pi* uses culinary metaphors as a vehicle for tricky concepts such as axiomatization and category theory, and Jim Henle's *The Proof and the Pudding: What Mathematicians, Cooks, and You Have in Common* highlights common themes of pleasure and experimentation in mathematics and cooking. Food itself often crops up as an explanatory device in conference presentations when a visual referent is needed, as below:

(3) *“So just—you can think of this, this is a full slice of pita bread and it's—I've sliced it down the middle. So it's open inside, so you can kind of push it apart from itself. When you do this, after you've cut along it, notice that you can undo single crossings.”* (Fig 2.) [1]

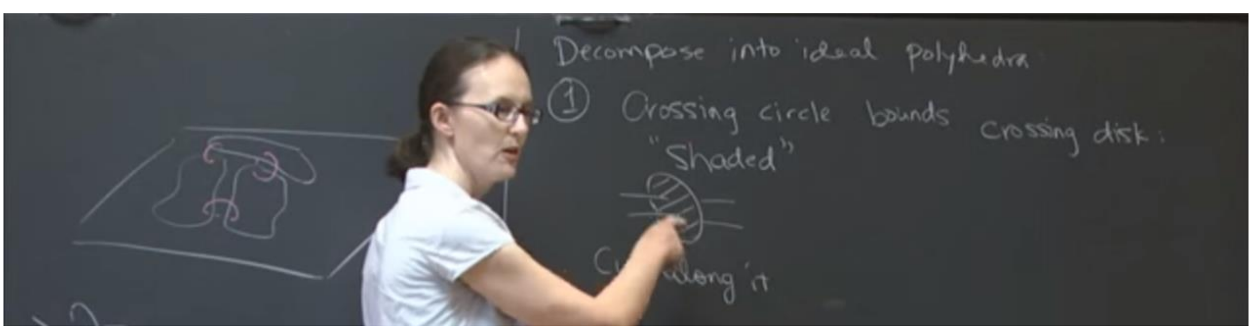

Figure 2. *Pita bread diagram*

Many metaphorical explanations of mathematics for non-mathematical audiences work with food. Eugenia Cheng has a YouTube channel exclusively using food to explain mathematics, and culinary materials show up in the videos of both Vi and George Hart [4]. Here's an excerpt from a colloquial explanation that F. Luke Wolcott wrote of his work:

(4) *"Sometimes I think of the metaphor: rings are like fruit, and derived categories are like pies made from those fruit. Apples yield apple pie; peaches yield peach pie. We can understand a fruit by studying the type of pie it yields."* [5]

In his paper, Wharton refers to *Relevance Theory*, the work of Dan Sperber and Deirdre Wilson, which is a theory of utterance interpretation that "... is defined in terms of costs and rewards: the more positive effects gained, and the less processing effort expended in gaining those effects, the greater the relevance of the input to the individual who processes it.*"* [6] The most relevant interpretation is the one that gives the most benefit for the least cognitive processing, and the results have to justify the effort that the processing requires. This benefit comes in the form of "…some contextual effect, in the form of an erasure of some assumptions from the context, a modification of the strength of some assumptions in the context, or the derivation of contextual implications." [7] Using food gives the non-mathematical viewer a concrete context: a substance that we are accustomed to using, manipulating and transforming until it becomes something different. As non-mathematicians we are able to get more out of this, because the materials are a lot more familiar to us. However, the contextual effects are pretty much limited to the scope of the metaphor – beyond that our ability to register effects dissolves. We might therefore learn something locally interesting, but not be able to see where else it might lead. A more interesting outcome is that we might perhaps alter our understanding of what it is that mathematicians do all day.

This use of food as the subject of such speech provides a concrete substance to replace that which is acted upon when an experienced mathematician interacts with a new piece of mathematics, so the next question to ask is what exactly that might be. Understanding a piece of mathematics requires considerable effort, for example in accessing the contextual knowledge that is assumed to be shared by the speaker and the audience in order to understand what a polyhedron is. The outcomes then must similarly be more substantial. Considering the contextual effects described by Sperber and Wilson, the effects of following a piece of mathematics might be to modify the audience member's knowledge and understanding of polyhedra. One could say, then, that that which is being acted upon is the contextual knowledge provided by the mathematician, consisting of all of the related mathematics that have previously followed and which give the current work its meaning. In that case, the outcome is an enhanced construction which can, importantly, then be used as the substance of the next piece of mathematics. The payoff for watching a mathematics talk could then be said to be something similar to the learning of a new technique, such as how to sauté potatoes, except that at the same time we're actually building the potatoes. If we are mathematicians, then, we can take these sautéed potatoes and make them into much more exotic dishes.

Tunafish Sandwiches

Thinking about the content of a presentation not as clarifying our picture of an independently existing mathematical object but as an act of mathematics in itself renders that presentation as a performance piece. This is similar to the shift away from object and toward process in the Process Art that began in the 1960s, with Jackson Pollock's drip paintings, and Joseph Beuys' wild performances and lectures. For Beuys, his performances or "actions" were rituals, designed to induce new ways of perceiving; by performing with a dead hare and some gold leaf he saw himself as a shaman, the performance itself bringing about a change. The written form has a parallel in Yoko Ono's event scores such as in her artist's book *Grapefruit*, which is full of instructions that a reader can choose whether or not to enact [8].

TUNAFISH SANDWICH PIECE

Imagine one thousand suns in the
sky at the same time.
Let them shine for one hour.
Then, let them gradually melt
into the sky.
Make one tunafish sandwich and eat.

1964 Spring

The first few lines simply ask the reader to imagine, an instruction which could arguably be partially fulfilled simply in the act of reading and comprehending the text. The last instruction does call for action, but it does not need to be literally carried out to have an effect on the reader, and there is undoubtedly a significant payoff simply to reading the instruction and considering what it implies. We don't need to actually make and eat the sandwich for the relationship in our minds between the cosmic and the domestic, and indeed our ideas about tunafish sandwiches, to be strangely and subtly altered, perhaps permanently. Similarly, though an instruction like (1) might not be followed in any observable way, it might reconstruct a part of the audience member's understanding in slightly different terms, and this will affect the approach taken in the future and so the next wave of mathematics that builds upon these results.

Conclusion

The difference between the background knowledge wielded by the mathematician and by the non-expert is a considerable barrier when it comes to opening up mathematics to a wider audience. It is interesting to unpick exactly what it is that is gained and also what is lost by some of the innovative solutions that have been developed to address this problem, as any novel way of presenting an idea highlights some aspects of a problem and obscures others. The question of what exactly mathematics consists of is as difficult as it is intriguing to mathematicians and non-mathematicians alike. A consideration of some of the ways that its ideas are conveyed to different audiences offers a way to approach these questions with reference to observable phenomena, which may be an opportunity to make such questions more visible.

References

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