Classifying Hexagonal Tilings in Islamic Architecture with a Single Numerical Parameter

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Abstract

Islamic designers drafted a large variety of hexagonal patterns in various media, including stone, tile, wood and cloth, throughout the medieval Islamic world. To highlight the geometric similarities among these patterns, we present a simple scheme to classify different types of hexagonal tilings in Islamic architecture, using a single numerical parameter.

Islamic artisans and designers exploited geometry to create a wide variety of intricate patterns in a range of media, including brick, stone, ceramic tile, wood and cloth. In many cases, the underlying structure of these patterns was derived from compass and straightedge techniques [1]; on other cases, particularly decagonal and pentagonal geometry, a set of prototiles known as "girih tiles" was used to lay out geometric patterns [2]. One of the simplest periodic structures is based on tiling the plane with hexagons, forming a honeycomb lattice.

In this paper, we illustrate how this simple hexagonal honeycomb structure was elaborated to form a range of line patterns that appear throughout Islamic art and architecture; these patterns are expressed in a number of different media: brick, stone, ceramic and cloth. Although these patterns may appear at first glance to be quite different, we show that in fact these patterns all share not only the same symmetry, but also the same underlying structure: hexagons placed at the vertices of a tessellation of hexagons of a different size. We present a number of patterns that can be classified in part by a single dimensionless numerical parameter: the ratio of the length of the hexagons that appear in the final pattern relative to that of the underlying honeycomb lattice, with only some minor variations. This simple scheme allows a large number of patterns to be compared with only a single number, facilitating comparison and contrast among this collection of historical Islamic art and architecture.

The drafting of a hexagonal honeycomb grid using a compass and straightedge was well understood both by the ancient Greeks and by medieval Islamic architects and designers. The specific sequence of steps using the compass and straightedge to create the honeycomb tessellation of hexagons has been well illustrated in a number of publications [1, 3]. Line patterns with underlying hexagonal geometry have appeared throughout Islamic art and architecture. Perhaps the simplest way to decorate this grid is to place a motif at the vertex of each hexagon in the grid; one common motif is another hexagon, which appears in a large number of patterns throughout Islamic history. Examples are expressed in a wide variety of materials, including brick, stone, ceramic and cloth, and cover a range of dates and locations throughout the Islamic world, as shown in Fig. 1, where the underlying hexagonal honeycomb grid is delineated with black dotted lines, and the decorating hexagon motifs are marked in red.

Because these hexagonal patterns all share the same structure and construction, they can be characterized by a single number ρ: the ratio of the length of a side of each of the regular hexagons that appears in the pattern, to the length of a side of a hexagon in the underlying grid. We illustrate a few variations to this theme in Fig. 1. The simplest patterns place red hexagons motifs either at the center of each hexagonal cell in the black honeycomb grid (Fig. 1a), or at the vertices of the honeycomb (Fig. 1b). Other patterns have red hexagon motifs of uniform size and orientation at both the centers and vertices of the black hexagonal honeycomb grid, as illustrated in Figs. 1c-g. Finally, there are patterns where the red hexagons at the center of the honeycomb are rotated by 90° relative to those of the same size at the vertices of the honeycomb, as shown in Fig. 1h. The ratios ρ for all of the examples are provided in the caption.

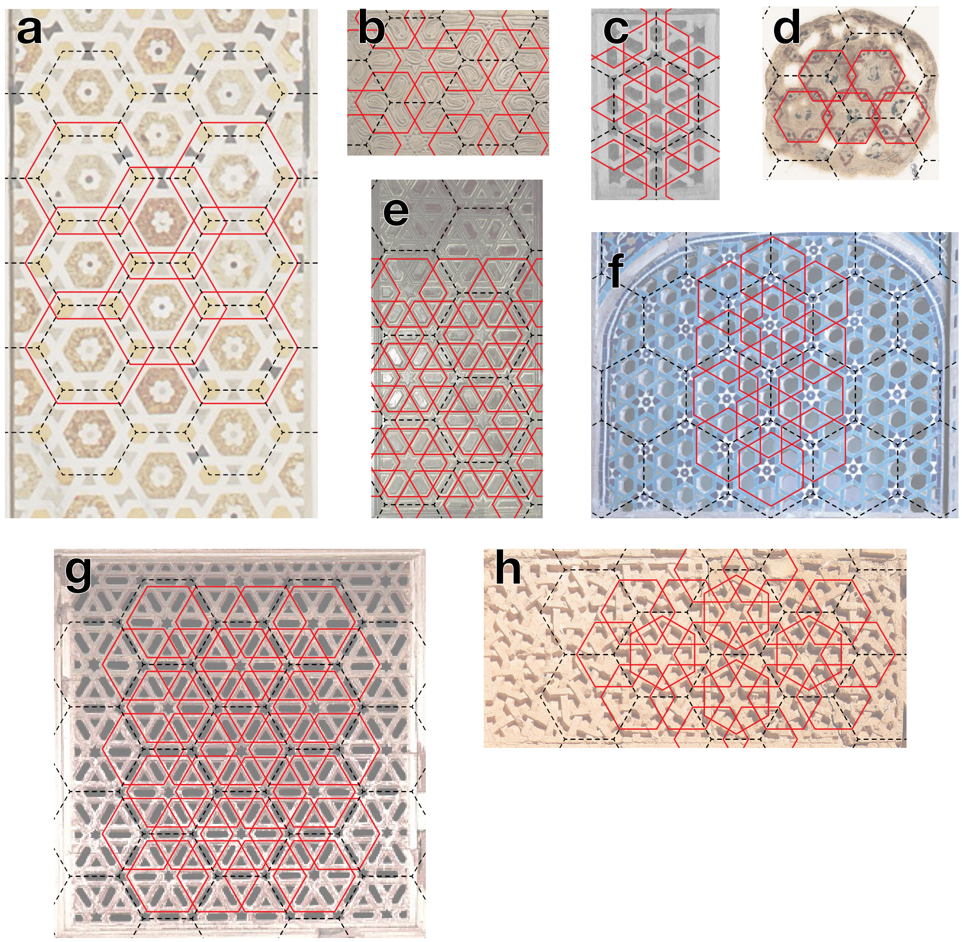

Figure 1: *Hexagonal tiling examples, scaled so that the underlying hexagonal grids (dashed black lines) are the same size for all patterns. Decorating hexagon motifs are shown with solid red line. (a) Itimad Al-Daula mausoleum, Agra, India (1622);* $\rho = 1.32$. *(b) Dado panel, Nishapur, Iran (10th cent.) in the Metropolitan Museum of Art, New York [40.170.442];* $\rho = 0.72$. *(c) The Great Mosque of Cordoba, Spain (987);* $\rho = 0.77$. *(d) Roundel textile fragment in the Ashmolean Museum, Oxford (10th-15th cent.);* $\rho = 0.59$ *(e) Eski Mosque, Edirne, Turkey (1414);* $\rho = 0.80$. *(f) Friday Mosque of Yazd, Iran (14th cent.);* $\rho = 0.76$. *(g) Alai Darwaza, Qutb complex, Delhi, India (1311);* $\rho = 0.84$. *(h) Agzikarahan, near Aksaray, Turkey (1231);* $\rho = 0.76$.

These patterns span a geographical range beyond Turkey, Iran, and India, and range from the 10th to the 17th century. They also represent a range of artistic and architectural traditions: one has been made on a piece of cloth, another on a door frame and yet another features as a window grille. What unites them is their construction method, based on the premise that by drawing lines and circles, many patterns can be created. Sixfold geometric design holds a special place because on a circle can be placed six circles of the same radius, spaced equally; the intersections created by this first step generate the foundation for a pattern: dividing the circle into six equal sections [1]. While the drafting of all of the hexagonal patterns we present here starts in this way, different design choices lead to the generation of the disparate hexagonal patterns, as we illustrate the step-by-step drawings in Fig. 2. Nevertheless, the patterns must be tessellated on a grid of hexagons for these patterns to make visual sense, and for their underlying structure to be easily perceived.

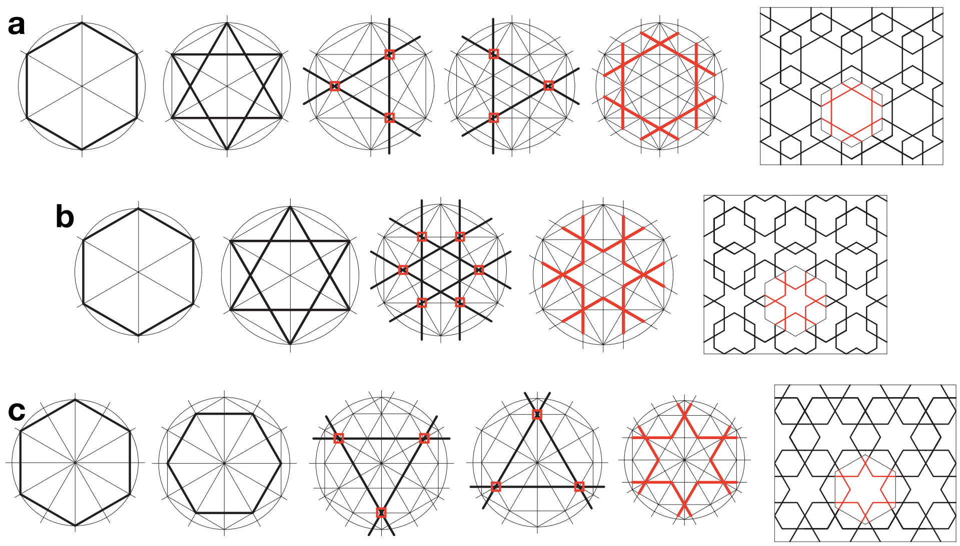

Figure 2: *Hexagonal tiling examples, with step-by-step drawings of the intermediate stages of construction, for the three classes of patterns described: (a) Itimad Al-Daula mausoleum, Agra, India (1622), shown in Fig. 1a. (b) Dado panel, Nishapur, Iran (10th cent.) in the Metropolitan Museum of Art, shown in Fig. 1b. (c) Friday Mosque of Yazd, Iran (14th cent.), shown in Fig. 1f.*

Our work suggests that a single number ρ can classify a large number of hexagonal patterns (within the three classes describing the placement and orientation of the secondary hexagons) in different media, and from different geographies and historical contexts, as shown in Fig. 1. This may help systematic description of historical tilings, and facilitate comparisons in a more quantitative way among all of the patterns that share the same underlying structure. Nevertheless, while these patterns share a common symmetry and hexagonal honeycomb grid, their construction requires different choices in design and drafting, as shown in Fig. 2. These patterns occur through a broad range of times and places throughout the Islamic world, suggesting that curious designers used simple tools to generate independently these related, but different patterns.

With any pattern analysis, a natural question arises as to the utilty of a classification scheme, like that we present here. Our main motivation is to provide one simple way of describing a family of complex-looking patterns, that in fact differ only in the magnitude of a single variable, thereby demonstrating that the visual diversity of this group of patterns derives ultimately from simplicity within the context of an elegant design tradition. Of course, there are alternate ways in which the patterns we present can be conceived; some of the richness of patterns themselves originates with the variation of personal experience—different people experience different aspects of the patterns in different ways, informing how different individuals choose to "read," analyse, deconstruct and reconstruct these patterns.

Our work here offers another alternative, that may reduce some of the initial complexity, by focusing on a single variable; we believe our approach could merit further exploration. We emphasize that our scheme is not uniquely optimal, or exclusive of other ways of classifying these patterns. Instead, we aim to provide not only a way to look to the past in historical patterns, but also a way to generate new patterns going forward, that abide by some of the rules that we observe and describe in Islamic geometrical design.

References

[1] Eric Broug, *Islamic Geometric Design* (New York: Thames and Hudson, 2013)

[2] Peter J. Lu and Paul J. Steinhardt, "Decagonal and Quasi-crystalline Tilings in Medieval Islamic Architecture," *Science* **315**, pp. 1106-1110 (2007).

[3] Issam El-Said and Ayse Parman, *Geometric Concepts in Islamic Art*, (London: World of Islam Festival, 1976).