Repeating Fractal Patterns with 4-Fold Symmetry

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Abstract

Previously we described an algorithm that can fill a region with an infinite sequence of randomly placed and progressively smaller shapes, producing a fractal pattern. In this paper we extend this algorithm to fill a fundamental region for the "wallpaper" group $p4$, then we tile the plane with copies of that region. This produces artistic patterns which have a pleasing combination of local randomness and global symmetry.

Introduction

In the past we have created pleasing patterns with an algorithm [1] that can fill a planar region with a series of ever smaller randomly-placed motifs. In this paper we extend that algorithm to create wallpaper patterns with $p4$ symmetry (we previously treated $p4mm$ patterns [2]). Schattschneider [3] gives a nice overview of wallpaper groups. Figure 1 shows such a random pattern of circles with $p4$ symmetry. To create our

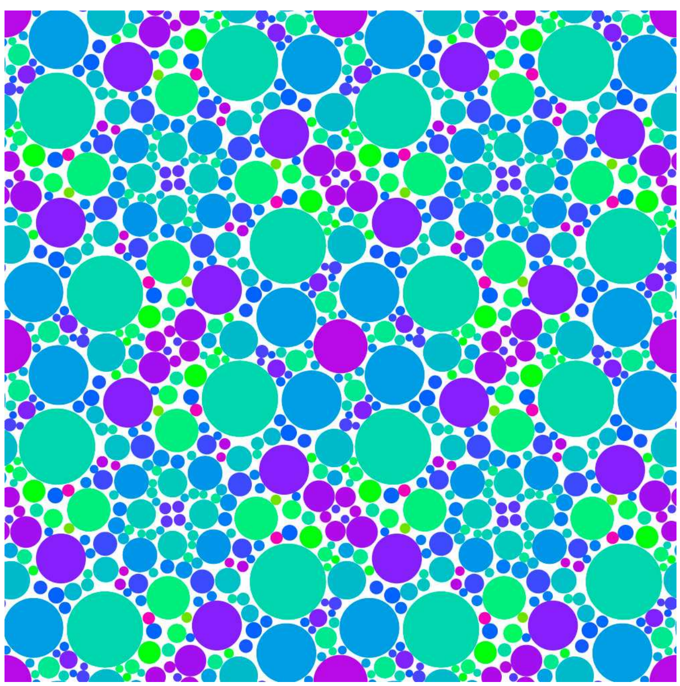

Figure 1: *A locally random circle fractal with global $p4$ symmetry.*

patterns, we fill a fundamental region for $p4$ with randomly placed, progressively smaller copies of a motif, possibly with different colors, such as the circles of Figure 1. This randomness generates a fractal pattern. Then copies of the filled fundamental region are used to tile the plane, yielding a locally fractal, but globally symmetric pattern. Here math provides the algorithm and art provides the coloring. In the next section we explain how the $p4$ algorithm works. Finally, we indicate directions of future work.

The Algorithm

The idea of the algorithm (below) is to randomly place progressively smaller motifs m_i within a region R so that they do not overlap any previously placed motif [1]. Here we show the original algorithm in normal type face with the modifications in bold that are needed to produce a $p4$ pattern:

For each $i = 0, 1, 2, \ldots$
  Repeat:
    Randomly choose a point within **fundamental** region R for $\boldsymbol{p4}$ to place m_i
    If $\boldsymbol{m_i}$ has 4-fold symmetry and overlaps a 4-fold rotation point
      Move $\boldsymbol{m_i}$ to be centered on that 4-fold rotation point
    If $\boldsymbol{m_i}$ has at least 2-fold symmetry and overlaps a 2-fold rotation point
      Move $\boldsymbol{m_i}$ to be centered on that 2-fold rotation point
  Until (m_i doesn't intersect any of $m_0, m_1, ..., m_{i-1}$)
  Add m_i to the list of successful *placements*
Until some stopping condition is met, such as a maximum value of i or a minimum value of A_i.

It has been found experimentally by the second author that this algorithm is non-halting if the areas of the m_is obey an inverse power law [1] (we note that exponentially decreasing areas cause halting).

Summary and Future Work

We have presented a method for creating patterns that generate global wallpaper patterns with $p4$ symmetry but are locally fractal in nature. Our goal is to make pleasing patterns with this kind of global symmetry but to also maintain local randomness. The methods presented here and in [2] should also work for other wallpaper groups. It would also be interesting to create corresponding spherical or hyperbolic patterns that are locally random, but have global symmetries.

References

[1] Doug Dunham and John Shier, The Art of Random Fractals, in *Bridges Seoul,* (eds. Gary Greenfield, George Hart, and Reza Sarhangi), Seoul, Korea, 2014, pp. 79–86. Also online at: `http://archive.bridgesmathart.org/2014/bridges2014-79.html`

[2] Doug Dunham and John Shier, Fractal Wallpaper Patterns, in *Bridges 2015,* Baltimore, Maryland, 2015, pp. 183–190. Also online at: `http://archive.bridgesmathart.org/2015/bridges2015-183.html`

[3] Doris Schattschneider, The Plane Symmetry Groups: Their Recognition and Notation, *American Mathematical Monthly*, 85, 6, 439-450, July, 1978. Wikipedia site for wallpaper groups: `http://en.wikipedia.org/wiki/Wallpaper_group` (accessed Jan. 24, 2016)