Two Non-Octave Tunings by Heinz Bohlen: A Practical Proposal

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Abstract

Although non-octave music is not a well-known subject among composers or musicologists, "the avoidance of octave doubling" was deemed especially important by the Modernist composer Arnold Schoenberg. This article introduces composers and performers to two non-octave tunings invented by a German engineer named Heinz Bohlen plus two octave-based tunings that can be used to approximate Bohlen's tunings. The principal aim is to encourage further practical experiments, not to indulge in phenomenological speculation. Relevant texts by William Brouncker, Walter O'Connell and Kees van Prooijen are briefly discussed.

Introduction

Heinz Bohlen was a German engineer that specialised in microwave electronics and communications. He was born in 1935 and died in 2016. In his spare time, Bohlen invented multiple non-octave tunings. Musicians, I propose, can experiment with two of Bohlen's non-octave tunings using fretted string instruments that are also able to play conventional music. This is fortunate: practical coincidences like this are anything but common.

Although non-octave music is not a well-known subject among composers or musicologists, "the avoidance of octave doubling" was deemed especially important by the Modernist composer Arnold Schoenberg [17]. Through his attempts to create something truly new with respect to Renaissance, Baroque, Classical and Romantic music, Schoenberg realised that the octave's familiarity posed a major problem. In Schoenberg's opinion, the octave creates a sense of emphasis, which is in turn linked to expectations of a conventional "root" or "tonic". Non-octave tunings, post-Schoenberg, offer a dramatic solution to the problem, without abolishing a mathematical theory of harmony.

Non-octave music should not be confused with quarter-tone music or the pseudo-sophisticated microtonal music that Plato's brother Glaucon made fun of in *The Republic* [14]. "They make complete fools of themselves with their 'close' intervals," he said, "applying their ears to the instrument as if they were eavesdropping on their neighbours". Schoenberg, likewise, derided the simplistic equivalence of small musical intervals with "a higher level of development" [18]. Non-octave music is something different, and it is arguably less pretentious. It simply involves a tuning system or a scale that repeats at an interval other than 2:1. Alternatively, one can make non-octave music with a set of pitches that do not repeat at any interval; but I cannot discuss this further in the present short paper.

Tuning theorists often refer to the interval of repetition as *the period*. Tuning systems' and scales' steps are commonly measured in cents. As the name implies, a cent is one-hundredth of a conventional semitone. The latter, to be precise, is defined as $\sqrt[12]{2}$, which means that a cent is $\sqrt[1200]{2}$.

A Period other than 2:1

To the best of my knowledge, the "Animadversions upon the Musick-Compendium of Renat. Des-Cartes" [1], published in 1653, was the first Anglophone text to establish non-octave periods. The anonymous author is believed to be William Brouncker, who, in 1662, became the first president of the Royal Society of London. See [20]. According to Susan Wollenberg [21], Brouncker was also the first English mathematician to use logarithms (invented circa 1614) for musical tunings. Why is Brouncker's work significant? Without logarithms, one would not have a modern theory of equal temperament; and without tunings that use a period other than 2:1, there would be no non-octave music.

In 1972, Bohlen invented a tuning system that consists of thirteen divisions of 3:1. (In conventional, diatonic theory, 3:1 is the perfect twelfth.) See [6]. There is a rational version of the system as well as a tempered version. The former consists of the frequency-ratios 1:1, 27:25, 25:21, 9:7, 7:5, 75:49, 5:3, 9:5, 49:25, 15:7, 7:3, 63:25, 25:9 and 3:1. The latter has an equal step, $\sqrt[13]{3}$, measured as 146.30 cents. On 1 September 1976, Bohlen submitted an article about his thirteen-note system to *Acustica*. The article was published in 1978 [2]. An English translation followed in 2001 [5].

The familiar, twelve-tone equal temperament only approximates one frequency-ratio that involves the prime number 7, namely the tritone (7:5). Bohlen's system, by contrast, makes use of multiple frequency-ratios that involve the prime number 7; and it excludes all frequency-ratios that involve the prime number 2, such as the octave (2:1), the perfect fifth (3:2), the perfect fourth (4:3), the major third (5:4) and the minor third (6:5). One should not, therefore, expect pieces of music to translate well from one system to the other. The harmonies engendered by Bohlen's system are truly exotic: this is both a gift and a curse. "An essential difficulty in confronting the new sounds," Bohlen noted, "is the inertia of established hearing habits, which try to force the listener again and again to perceive the novelty as a flawed reproduction of the well-known" [5].

In February 1978, Kees van Prooijen [15] submitted an article to *Interface*, which featured a section on equal temperaments that have "a higher harmonic than the second as the basis" – that is, a period other than 2:1. Unaware of Bohlen's work, Van Prooijen included the $\sqrt[13]{3}$ temperament among his own inventions – an innocent mistake. To date, composers and musicologists have hardly explored any of the other non-octave systems from Van Prooijen's article.

In "13 Tonstufen in der Duodezime", Bohlen specified two nine-note modes. Van Prooijen later established a seven-note, "lovely asymmetrical" mode. See [16]. For further examples of modes derived from Bohlen's thirteen-note system (both the rational version and the tempered version), see Todd Harrop [10], Ron Sword [19], and the section of [9] by Georg Hadju. For notation methods and examples of custom-made instruments, see the sections of [9] written by Nora-Louise Müller.

Φ:1 as the Period

In 1999, Bohlen invented another non-octave oddity, which he named *the 833 cents scale* [4, 7]. Bohlen did not set out to construct a scale with Φ:1 as the period, but this is exactly what resulted from his experiment. What does Φ:1 have to do with 833 cents? Φ:1 is approximately 1.618034, which in turn equates to 833.09 cents. The exact formula: $1200 \times \log_2(1.618034) = 833.09$ cents. Since Φ:1 is sharper than a minor sixth (8:5) but flatter than a neutral sixth (18:11), conventional terms are best left aside.

Between 1960 and 1977, theorists such as John Chowning, Walter O'Connell and Lorne Temes each devised systems and/or scales with Φ:1 as the period. See [8]. O'Connell [13] invented two Φ-based systems with equal steps. Each step from the first system, $\sqrt[25]{\Phi}$, measures 33.32 cents; each step from the second system, $\sqrt[18]{\Phi}$, measures 46.28 cents. Bohlen's 833 cents scale is incidentally very close to seven particular steps generated by the $\sqrt[25]{\Phi}$ system. The maximum difference is just 2.51 cents. See below.

Bohlen's 833 cents scale:	99.27, 235.77, 366.91, 466.18, 597.32, 733.82 and 833.09 cents
Relevant steps from $\sqrt[25]{\Phi}$:	99.97, 233.27, 366.56, 466.53, 599.83, 733.12 and 833.09 cents
Relevant steps from $\sqrt[36]{2}$:	100.00, 233.33, 366.67, 466.67, 600.00, 733.33 and 833.33 cents

Bohlen was not aware of O'Connell's $\sqrt[25]{\Phi}$ system when he invented his 833 cents scale, but he later acknowledged that he "re-invented O'Connell's wheel" [3]. This relationship has not been explored by either composers or musicologists.

Practical Suggestions

It follows that the $\sqrt[36]{2}$ system (with its 33.33-cent step) can be used to approximate O'Connell's $\sqrt[25]{\Phi}$ system (with its 33.32-cent step) as well as Bohlen's 833 cents scale. Given that $\sqrt[36]{2}$ is an octave-based system, this is somewhat amusing. Further to this, $\sqrt[36]{2}$ produces every interval from the conventional $\sqrt[12]{2}$ system and can thereby satisfy conservative composers and performers. Since 36 is a composite number, it is also possible to invent symmetrical modes for $\sqrt[36]{2}$, just as Olivier Messiaen did for $\sqrt[12]{2}$. See [12]. In sum, if musicians use the $\sqrt[36]{2}$ system, they can play established $\sqrt[12]{2}$ repertoire (diatonic, jazz and/or dodecaphonic) as well as new symmetrical modes, and they can closely approximate non-octave music derived from O'Connell's $\sqrt[25]{\Phi}$ system or Bohlen's 833 cents scale. Guitars with $\sqrt[36]{2}$ fretboards are considered playable by musicians such as Neil Haverstick; and since $\sqrt[36]{2}$ fretboards preserve all the frets from standard $\sqrt[12]{2}$ fretboards, they are not difficult for luthiers to make.

Alternatively, musicians could follow Melle Weijters and use $\sqrt[41]{2}$ fretboards to play conventional diatonic scales as well as each note from Bohlen's $\sqrt[13]{3}$ temperament. See [10]. How does this work? Five 29.26-cent steps from the $\sqrt[41]{2}$ system are equal to one 146.30-cent step from Bohlen's temperament. The $\sqrt[41]{2}$ system can also be used to approximate the 87.75-cent steps generated by $\sqrt[8]{(3/2)}$ – a lesser-known non-octave system. See [11]. $\sqrt[41]{2}$ fretboards do not preserve standard, 100.00-cent frets and thus cannot be based on $\sqrt[12]{2}$ fretboards. They have to be entirely custom-made.

There is, of course, a problem with both $\sqrt[36]{2}$ and $\sqrt[41]{2}$ fretboards: in comparison with $\sqrt[12]{2}$ fretboards, the spaces between frets are much smaller, which means that it is more difficult to play fast sequences of chords. The choice, then, is clear: one can either experiment with a proper, non-octave fretboard that has no extraneous frets and is easy to play, such as a $\sqrt[13]{3}$ fretboard, or one can opt for a complicated $\sqrt[36]{2}$ or $\sqrt[41]{2}$ fretboard and occasionally play octave-based music in ensembles alongside conventional instruments.

For my own composition work, I use custom-made, fretted string instruments that cannot play any standard repertoire. I acknowledge that this approach is not for everyone. If the field of non-octave music is to attract broader interest, less drastic options are likely required for performers of acoustic instruments; hence the present effort to review practical compromises.

Conclusion

There is plenty of material here for musicians willing to explore the paths stumbled upon by Brouncker, Bohlen and others, with no guarantees or clear ends. One must experiment. Theorists, in turn, can examine the controversial, hypothetical link between combination tones and perceived consonances. The hypothesis is relevant to Bohlen's thirteen-note system, the 833 cents scale and O'Connell's Φ-based systems. Cognitive science is not my area of expertise; thus I leave this final subject to others.

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