Baton Rolling on a Series of Curved Surfaces

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Abstract

We call an instrument that generates geometrically and aesthetically appealing spatio-temporal patterns through actual manipulation a *visual instrument*. We present a novel visual instrument that uses batons to roll on a series of curved surfaces. We fabricated a prototype of the instrument and confirmed that the batons can be smoothly rolled and manipulated on the surfaces and that the instrument leads to new types of rolling-based play and performances.

1 Introduction

Rolling a geometric object on a certain curve or surface has been studied since around the 17th century in both theoretical and practical standpoints, mainly in mathematics, physics, and for applications in astronomy [1] and mechanical engineering [2]. Rolling objects has been also successfully used in play, sports, and performing arts. For example, hula hoops and hoop rolling use hoops to roll around a human body and on the ground. In baton twirling, there are tricks in which a baton is rolled around the body in various ways. As for visual performances, which is the main topic of this paper, for example performing artist Greg Kennedy enters a huge transparent conic in which he rolls up to seven balls on its inner surface. The authors have also explored new kinds of rolling-based performances using a large sphere, cylinder (vertical and horizontal), and truncated cylindrical surfaces [3-6]. In [6], they call an instrument that is played for creating aesthetically and geometrically appealing spatio-temporal patterns a *visual instrument*, just like an instrument designed to make musical expressions through skillful manipulation is called a *musical instrument*. They also presented a visual instrument using a series of truncated cylindrical surfaces on which to roll a set of batons, which is later reviewed in Section 2. However, since the surfaces are only arranged in a straight line, there remain possibilities of using other surfaces and other ways of arrangements.

In this paper, we explore rolling batons on a series of surfaces in a more visually expressive and technically challenging way. Namely, we first allow the surfaces be arranged not in a straight line but in a circular form as shown in Figure 1. Furthermore, the surfaces are changed to have a dented part as a hyperbolic paraboloid has. These changes give us a new task of deciding the appropriate shapes for the

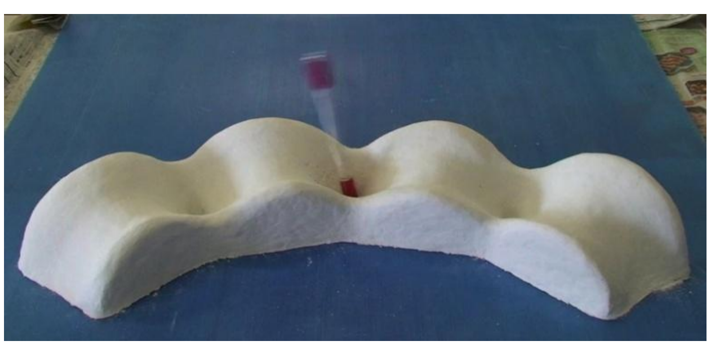
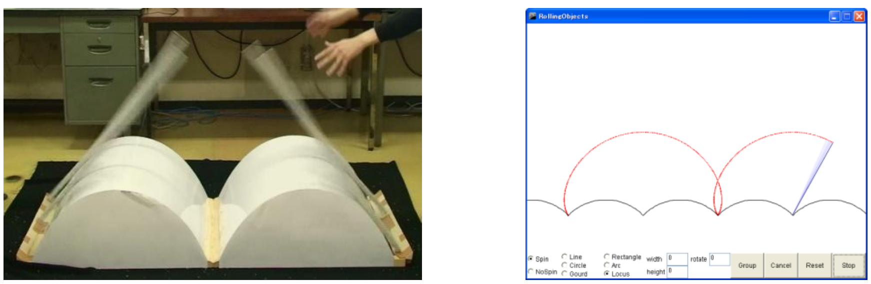


Figure 1: *Prototype of the new visual instrument with a series of curved surfaces and the rolling baton.*

surfaces and their arrangements to realize smooth rolling of batons. So far, we have made prototypes of the series of curved surfaces using clay and have confirmed that a baton can be rolled smoothly and continuously over the four surfaces in Figure 1.

2 Rolling Batons on Cylindrical Surfaces

We consider rolling a geometric object along the other mainly without slipping. The locus of a point on such a rolling object is called the *roulette*. We consider the case where one object is rolled on another fixed object. A pair of a straight line as a fixed curve and a circle as a rolling curve with the roulette being a cycloid is one of the most studied cases. The first author has developed several visual instruments where a geometric object such as a ball or a baton is rolled to create geometrically and aesthetically appealing visual patterns. The visual instrument that consists of a sequence of two or more truncated cylindrical surfaces with a baton to roll, presented in [6], is shown in Figure 2(a). We note that here rolling of a baton is considered not around its longitudinal symmetry axis but around a time-varying axis perpendicular to its longitudinal axis. Then the roulette of one of the end points of a baton is essentially planar as shown in Figure 2(b). So there remains the problem of making the baton move truly in 3D space by using other surfaces and other ways of arranging them.

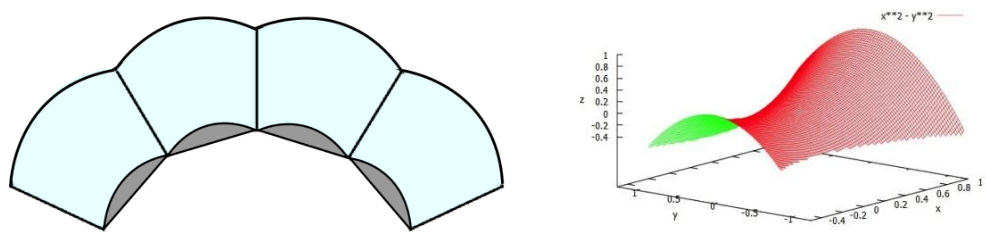


(a) (b)

Figure 2: (a) *Rolling batons on cylindrical surfaces*; (b) *a roulette of the rolling baton in 2D space*.

3 Beyond Planar Motion

Making the baton move roundly instead of straightly on a series of surfaces requires us a new technical challenge of actual fabrication and manipulation. We note that such a formation also has an advantage of allowing a player to easily catch the released baton at the opposite end. For this purpose, we first “curve” the series of surfaces from the straight line in Figure 2 to a circular form in Figure 3(a). The cylindrical surfaces are in effect changed to conical ones. Similarly to the case of cylindrical surfaces, the length of the baton is almost the same with that of the path on which the baton rolls at each of the conical surfaces. Then geometrically, it is easy to figure out how the baton should move in the formation. But to make this happen in physical setting requires a player to release of the baton quite accurately at the first surface with a right angle and velocity to continuously roll it over several surfaces in a row. Because of all this, we further deform the surfaces in a vertical direction, which means that the conical surface is changed to a saddle surface such as a hyperbolic paraboloid or a hyperboloid of one sheet. The example of the hyperbolic paraboloid is shown in Figure 3(b), having the equation $z = x^2 - y^2$ with $-0.5 \leq x \leq 1.0$, $-1.25 \leq y \leq 1.25$, and $-0.5 \leq z \leq 1.0$. When a baton is rolled on such a surface, even if there are some deviations of the baton from the ideal path, the baton tends to move close to the ideal path set in the dented part of the surfaces.



(a) (b)

Figure 3: (a) *A series of truncated conics in a circular form*; (b) *a truncated hyperbolic paraboloid*.

Now, we have had a plan of rolling a baton on the series of curved surfaces, possibly hyperbolic paraboloidal surfaces. Still, there are mainly the following two approaches for finding the appropriate surfaces and their formation: (i) the physical simulation-based approach; and (ii) the handcraft-based approach. We have so far succeeded in making software for physically simulating baton rolling only on cylindrical surfaces, without slipping, and without rolling of a baton along the longitudinal symmetry axis [7]. Extending this software for our setting and determining the geometric and physical parameters of the instrument is promising, but this time we make the second approach of handcrafting the appropriate surfaces directly using clay.

4 Fabrication and Demonstration of the Instrument

We explain some details of the fabrication of the surfaces and their manipulation. We use white clay as material for the surfaces. The shapes of the surfaces and the parameters of the baton such as the length, thickness, texture, and the weights at both ends were refined in the approach of trial-and-error to realize smooth rolling of the baton. Figures 4(a) and (b) show the instrument consisting of hyperbolic paraboloidal surfaces with a decent quality for our purpose. Each surface fits into a cube, 25 cm each side. The interior angle of each surface is about 30 degrees, so the four surfaces have totally 120 degrees. The length of the central curve of the surface on which to roll a baton is 22 cm, which is almost the same with the length of the baton. The baton shown in Figure 4(c) is made of wood and its thickness (i.e., the diameter) is about 7 mm. Similarly to the case of baton twirling, the baton has some weights at both ends to increase the moment of inertia and make it roll continuously. We also note that the weights at both ends make the baton a little thicker with the red tapes as shown in the figure but the difference is gradual and subtle, so it does not interfere with the smooth rolling motion of the baton.

Rolling of the baton on the instrument is demonstrated in Figure 5. In the figure, the player releases the baton from the right-hand side with some inclination. Then it smoothly and continuously rolls on the surfaces and reaches the left-hand side. Rolling the baton over three consecutive surfaces is not too difficult to achieve; rolling it over four surfaces is much more difficult and requires much practice for finding the proper launching conditions. This preliminary experiment implies that we can make feasible construction of a set of surfaces by craft-based trial and error.

(a) (b) (c)

Figure 4: (a) and (b) *A prototype of a series of curved surface in two views*; (c) *the baton*.

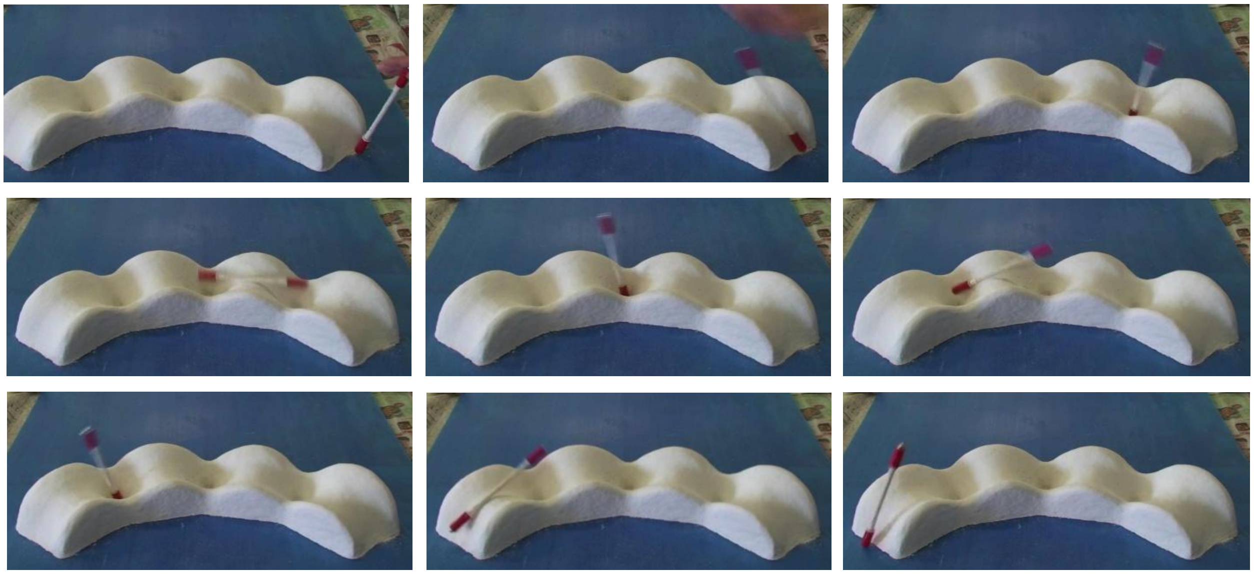


Figure 5: *Continuous rolling of a baton on the series of four hyperbolic paraboloidal surfaces*.

5 Concluding Remarks

We presented a new type of rolling-based visual instrument using a series of curved surfaces. There still remain some future work. Namely, we will develop a physical simulator for rolling batons on general surfaces in 3D space and find the appropriate geometric and physical parameters of them. To make full-size instruments and to create a performance piece with them should be also important tasks. At Bridges 2016, we will bring the instrument and demonstrate some manipulation.

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