

L-System Nomographs: Æsthetics to Calculation

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Abstract

We generate a nomograph that has branch-like ticks using the Lindenmayer System to simulate structures similar to those in plants. In so doing, we combine math and the art of nature.



Figure 1: *Tamarix africana* Poiret [1]. Image reproduced from the Digital Library of the Royal Botanical Garden of Madrid (CSIC).

Introduction

Beauty is bought by judgement of the eye [2]. Today we would say that beauty is in the eye of the beholder, and that æsthetics is a matter of opinion. Our opinion is that the structures that are found in nature are frequently pleasing to the eye. The repeated use of decorative floral patterns over millennia supports this theory. In the following, we try to mimic the decorative spirit seen a century or more ago by combining computer generated floral patterns with nomographs. The goal is to design a practical calculator that can also be seen as a piece of art.

Nomographs or nomograms are graphical calculators in which a straight line crossing the scales solves an algebraic equation. Nomographs were invented at the end of 19th century to satisfy a demand for fast and accurate calculations in the field [3, 4]. Today nomography is more or less a lost art due to the success of electronic calculators. However, nomographs are still practical in many situations where rapid and simple calculations are needed, for example in medicine [5, 6].

The Lindenmayer system, or L-system for short, is a set of grammar rules to generate structures similar to those found in plants [7]. The key idea is to present the structure as a series of symbols. For example, some symbols are branches, while others make the element turn through a given angle. The plant is built by using an initiator string and then modifying it with the given rules, where, in each generation or recursion step, symbols are replaced by other symbols.

In the following we combine nomographs and L-system graphs.

Example Nomograph

We first constructed a nomograph that has features taken from nature, but at the same time presents an exact equation. In other words, we try to generate something that is somewhere between math and the art of nature. Because the topic is related to plants, we chose the example of the Plastrochron Index nomograph that is used to calculate the fractional correction to the age of a plant [8].

The Plastrochron Index (*P.I.*) gives the developmental age of a plant by using sequential index number n of an organ, that is for example number of leaves over given reference length R . In order to calculate fractional *P.I.* one uses length of the leaf L_n that is longer than R and next leaf length L_{n+1} that is shorter than R [9]. The formula for calculating fractional *P.I.* according to [10] is

$$P.I. = n + (\log L_n - \log R) / (\log L_n - \log L_{n+1}). \tag{1}$$

We plot in Fig. 2a a typical contour plot relating the variables according to equation (1). This is the usual way to present equations graphically today. It is easy to read, but not very accurate, and has a neutral appearance.

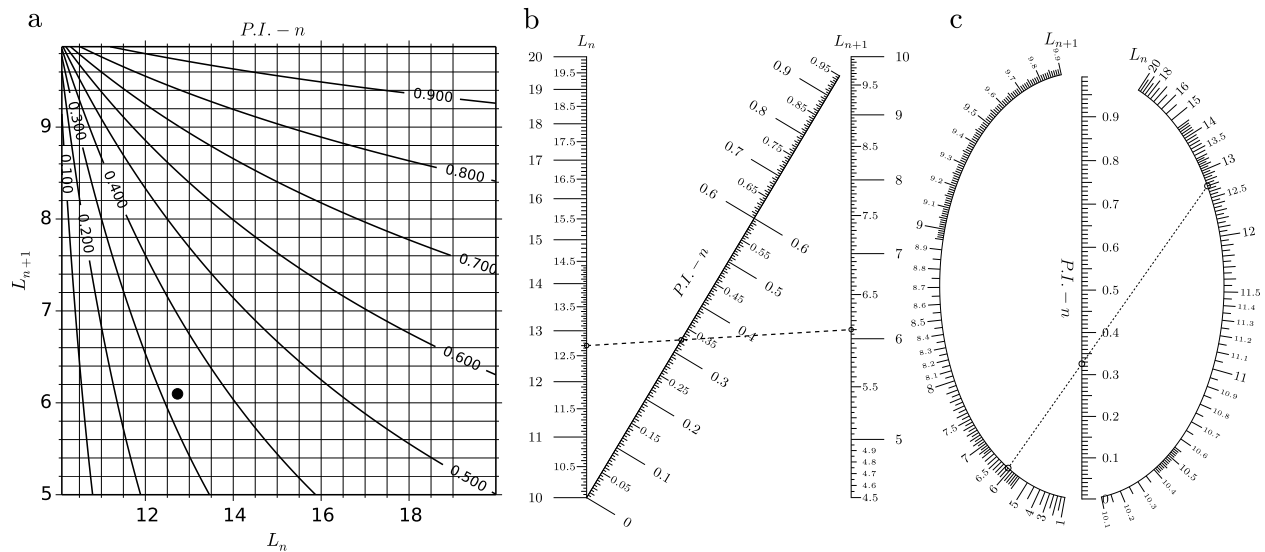


Figure 2: Comparison of graphs with $R = 10$. a) Contour plot b) N-type nomograph c) Circular nomograph. All graphs have marked a single solution of $L_n = 12.7$, $L_{n+1} = 6.1$ and $P.I. - n = 0.326$.

In order to present equation (1) as an N-nomograph as found in [8], the equation has to be transformed into form of $F_1(P.I. - n) = F_2(L_n)/F_3(L_{n+1})$, where $F_m(x)$ are functions. The Saint-Robert criterion shows that the transformation is possible, and a recipe for it can be found in [4, p.21]. The transformed form of equation (1) is

$$\frac{P.I. - n}{1 - (P.I. - n)} = \frac{\log L_n - 1}{1 - \log L_{n+1}} \tag{2}$$

and the resulting nomograph is presented in Fig. 2b.

Once we have made the transition from the contour plot of Fig. 2a to the nomograph of Fig. 2b, many new possibilities become available. In order to make the nomograph visually more appealing, we used a standard transformation to curvilinear scales (see e.g. Ref [11, p. 255]). The result is illustrated in Fig. 2c. Note that from a calculation point of view, Fig. 2b and Fig. 2c yield identical answers.

L-System

In order to learn and use the L-System, we employed [12]. The idea was to make the ticks on the scales look like branches. The following algorithm was used. The side of a branch is selected randomly. The initial seed is **F**, and it was replaced randomly in successive generations with **F[+F]F[-F][F+F]** or **F[+F]F[F-F]**. (**F**) means to “draw line forward”, (+) means “make a positive turn” and (-) means “make a negative turn.” The opening bracket (**[**) stores the state (point and heading) that is restored with closing bracket (**]**). The number of generations in ticks is adjusted in in order to enhance readability. Minor ticks are drawn as simple lines.

The result illustrated in Fig. 3 is the nomograph of Fig. 2c with modified ticks and texts. All graphs in this article were produced by using the Python language. The nomographs were generated with the help of the pyNomo Python library [13].

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References

- [1] *Flora forestal española [...] [2nd Ed.] Atlas de la Segunda parte*. 1890.
- [2] W. Shakespeare. *Love’s Labour’s Lost*.
- [3] M. Ocagne. *Traité de nomographie: Théorie des abaques.- Applications pratiques*. Gauthier-Villars, 1899.
- [4] H.A. Evesham. *The History and Development of Nomography*. Docent Press, 2010.
- [5] Harold Thimbleby and David Williams. Using nomograms to reduce harm from clinical calculations. In *Proceedings IEEE International Conference on Healthcare Informatics — ICHI 2013*, pages 461–470. IEEE, 2013.
- [6] A. La Marca, E. Papaleo, V. Grisendi, C. Argento, S. Giulini, and A. Volpe. Development of a nomogram based on markers of ovarian reserve for the individualisation of the follicle-stimulating hormone starting dose in in vitro fertilisation cycles. *BJOG*, 119, 2012.
- [7] A. Lindenmayer. Mathematical models for cellular interaction in development. part I and II. *Journal of Theoretical Biology*, pages 18:280–315, 1968.
- [8] Ralph O. Erickson. Nomogram for the plastochron index. *American Journal of Botany*, pages 350–351, 1960.
- [9] R. D. Meicenheimer. The plastochron index: still useful after nearly six decades. *American Journal of Botany*, pages 1821–35, 2014.
- [10] Ralph O. Erickson and Francis J. Michelini. The plastochron index. *American Journal of Botany*, pages 297–305, 1957.
- [11] E. Otto. *Nomography*. International series of monographs in pure and applied mathematics. Pergamon Press [distributed by] Macmillan, New York, 1963.
- [12] Przemyslaw Prusinkiewicz and Aristid Lindenmayer. *The Algorithmic Beauty of Plants*. Springer-Verlag New York, Inc., New York, NY, USA, 1996.
- [13] L. Roschier. <http://www.pynomo.org/> (as of Feb. 6, 2016).



Figure 3: The nomograph having L-System ticks and presenting equation (1).