Off the Wall: A Brief Report

Charlene Morrow
Faculty Emerita, Psychology & Education and Mathematics Departments
Mount Holyoke College
South Hadley, MA 01075
E-mail: cmorrow@mtholyoke.edu

Abstract

I describe the process I used to create my artwork entitled "Off the Wall", which will be displayed at the 2016 Bridges Mathematical Art Exhibition. Details about my initial inspiration from a painting by Frank Stella, mathematical concepts that guided my work, origami techniques used to create the work, and construction of the final artwork are discussed.

Inspiration

I was in the Whitney Museum in New York City, standing in the middle of a room in the retrospective exhibit of artworks by Frank Stella, feeling engulfed by the sheer size of the work as well as the volume of paint applied to canvas. I was waiting for my art viewing partner to put his coat in the coat check, and I said I would wait in the first gallery. Thus I was obligated to spend more time there than I might have otherwise. These were works that were perhaps eight feet tall, but seemingly simple in composition. Almost all were painted a dark color or black, from edge to edge. Many pieces had parallel lines in a slightly lighter color and in various configurations, sometimes turning 90 degree corners in unison. I wasn't bored. I found the simple geometry soothing and attractive, but I was reaching for a way to understand the paintings more deeply. As I stood in the middle of the room and tried to relax my mind, I suddenly began to see one of the works in 3D. The shading between the lines varied subtly from pair to pair of lines. This variation created the effect of light playing differently on surfaces oriented differently in relation to a light source. The effect on me was to bring the parallel lines more sharply to my attention. "Hmmm…" I thought. "Lines… parallel lines… folding lines… origami corrugations!" I definitely was onto a new way of looking at these paintings. My first question was to ask if I were to print the line design of the painting on a sheet of paper, could I fold the entire sheet without running into conflicts between folds in one section vs. folds in another adjacent section? Conflicts might happen, for instance, when a set of horizontal lines abutted a set of vertical lines.

Background

Paper Corrugation. This technique is a subfield of the body of origami techniques. Many, if not most, other origami techniques involve folds that serve to hide layers of paper within a model. These layers may only be for the purpose of shaping the model or they may serve to create a shadow effect if the model is backlit. Nevertheless, these hidden layers are no longer visible once the model is completed. On the contrary, in an origami corrugation, the entire paper surface remains visible. Folds are made in an alternating pattern of mountain folds (pointing "up") and valley folds (pointing "down") so that the width and/or depth of the footprint of the paper shrinks, but no paper is hidden. In the simplest corrugations, the folds are along parallel lines, as in a paper fan. But origami corrugations do not require that the fold lines be parallel or occur in 90º relation to each other. The fold lines can even be curved. In all cases the alternating mountain-valley pattern is present. Historically, examples of graceful and clever origami corrugations can be found in the work of David Huffman (see [1]) and Ron Resch (see [2]), and more recently in the work of the mathematician, Ekaterina Lukasheva [3].

A Mathematical Adventure in Stella Land

Frank Stella is well-known for his paintings on non-rectangular canvases (see [4], pp. 2-30 for relevant examples of his work). As I went beyond the first gallery in the retrospective at the Whitney Museum, I saw many more paintings with parallel lines throughout, and I began to see canvases that were L-shaped, star-shaped, V-shaped, and others with curved edges. The eight pointed star-shaped canvas (Figure 1) particularly caught my eye (also see [4] p. 26, *Plant City*), so I created a crease pattern in Adobe Illustrator, based on the lines that can be seen in the original painting, that I could print, cut, and fold (Figure 2). Figure 3 shows that actual folded object that was created. This object was even more interesting than I thought it would be, and I carried it around for weeks, showing it to friends and trying to think beyond just the piece in my hand.

Figure 1: "*Plant City" by Frank Stella, photographed by Char Morrow at the Whitney Museum. Shown here in black and white for emphasis of lines, painting is dark gold with brown lines.*

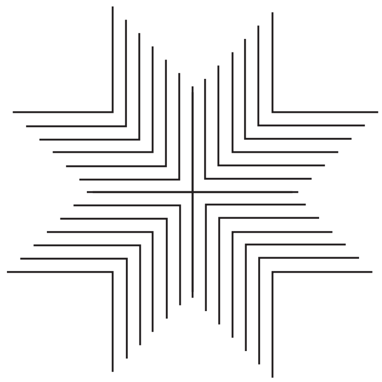


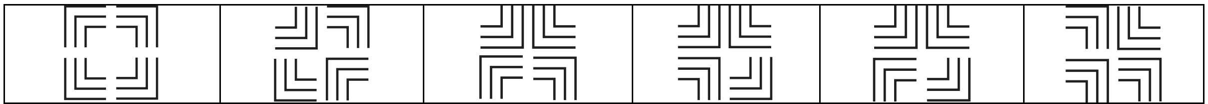
Figure 2: *Crease pattern created with Adobe Illustrator.*

Figure 3: *Star folded from one sheet of paper using crease pattern in Figure 2.*

Many people indicated a fascination with the folded object, and after much demonstration and playing, the piece finally split apart. At this moment I realized that the star was made up of four Vs, all pointing inward. In fact, I realized that many of the Stella paintings were comprised of configurations of Vs in different relationships to each other. For example, his L paintings are simply Vs. Another well-known painting is a linear string of four Vs pointing in various directions. Below I briefly describe the sequence (not entirely linear in real time) of my mathematical thoughts and explorations:

a) What would happen if I created a 2x2 grid and placed a V in each quadrant? Would the design I created through this process be foldable? By foldable I mean that it would be possible to make the alternating valley-mountain folds for one V without making it impossible to achieve that for any other of the four Vs in the design. Also each design would be folded from one piece of paper, which would remain connected in some way.

b) Should I allow repeats of direction or should the four Vs be strictly limited to one in each of the four possible directions? (See Figure 4 for examples.) In the "without replacement" case, there are 4!, or 24, unique designs if no rotations or reflections are allowed. However when rotations and reflections are considered, there are multiple repeats of designs. If organized based on which designs are identical under rotation and/or reflection, there are eight categories (or orbits, in algebraic language). In the "with replacement" case, there are 4^4, or 256, unique designs, with some number of orbits, not yet determined. I decided that it would be most feasible to start with the "without replacement" case.



Examples of four Vs arranged "**without** replacement" (i.e., each V pointing in a different direction). For each of the three designs above, directions are listed counterclockwise, starting at the upper left corner*: Left design: NE, SW, SE, and NE Middle design: SE, SW, NW, and NE Right design: SE, NW, NE, and SW *compass directions used for easy reference	Examples of four Vs arranged "**with** replacement" (i.e., possible repetitions of direction*): Left design: **SE**, NE, **SE**, and SW Middle design: **SE**, NW, **SE**, and SW Right design: **NE**, **NE**, **NE**, and SW *same references as for "without replacement;" bold indicates repeated direction

Figure 4: *Examples of four-V designs made "without replacement" and "with replacement."*

Conclusions and Observations. I folded all eight designs representing each of the eight orbits (see Figure 5). Note that all designs within an orbit differ only by rotation or reflection, which does not affect foldability, so only one design in an orbit needed to be tested. Thus it can easily be seen that all 24 designs for the "without replacement" case are foldable in the sense described above.

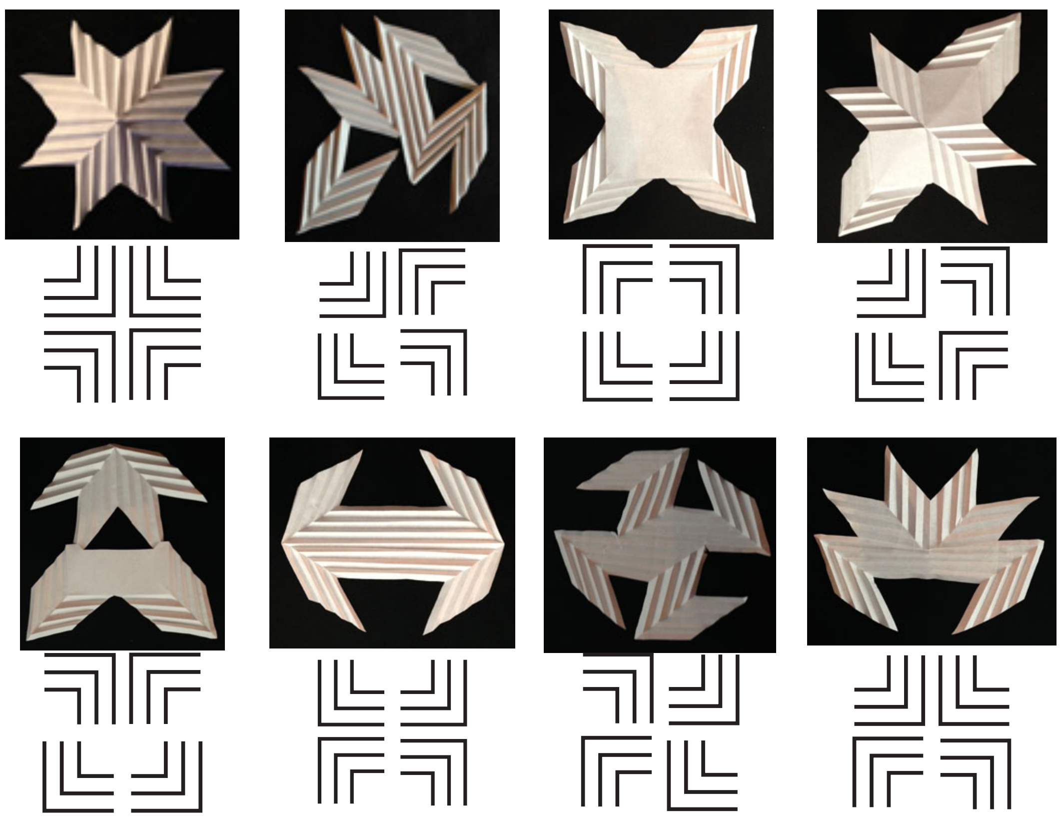


Figure 5: *Examples from all eight orbits of "without replacement" folded designs made with four Vs along with the schematic design for each design. From the schematic, placed below each design, it is easier to see the "without replacement" feature, i.e., each of the four Vs points in a different direction.*

When folding the printed designs, some decisions has to be made about what part of the paper to cut away and what to leave. One can see that three of the designs have flat, unfolded parts of the paper included with the design. If these unfolded parts of the paper were cut away, the design would no longer be connected. On the other hand, if I attempted to fold this part of the paper, the design would no longer be relatively flat and the Vs would be harder to see. In other designs (e.g., the 2nd, 3rd, and 4th in the second row of designs) this flat part of the paper, not technically part of the V could be folded without affecting the flatness or visibility of the Vs. In some ways my inability to apply a rule unambiguously was frustrating, but in other ways, it was interesting. I am still experiment with this set of designs as I think there is more to be learned.

I have not sorted the 256 "with replacement" cases into orbits, but with some preliminary observations, I have already seen that there are many more orbits than for the 24 "without replacement cases. (Note that this set of 24 cases is included in the 256.) I am very curious about whether these designs will also be foldable. This remains to be investigated.

One other observation I have made is that for all designs, the footprint of each V and the 2x2 box is the same size, but the space that each design takes up when folded is different. Thus some designs look smaller than other designs. One obvious reason is that unfolded areas of paper, found in some design, take up more space and keep the design spread out. However, there may be other interesting and more subtle aspects of collapse-ability that would be interesting to investigate.

The Artwork

My artwork, "Off the Wall," displayed in the 2016 Bridges Art Exhibition in Finland, (see Figure 6) is a "curiosity cabinet" of designs representing all eight orbits in the "without replacement" case. I take the term "curiosity cabinet" from a tradition in art where interesting objects are placed in relation to each other, often literally in a cabinet, for observation, consideration, and even surprise. Joseph Cornell (see [5]) was well-known for creating these works.

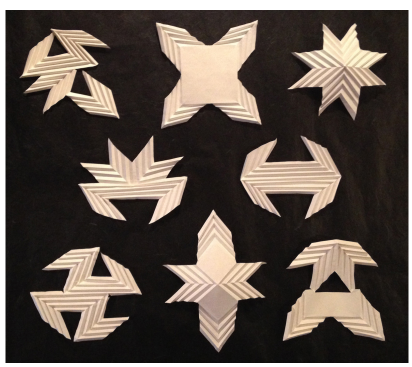


Figure 6: "Off the Wall," a *"curiosity cabinet" of folded designs, by Charlene Morrow.*

References

[1] E. D. Demaine, M. L. Demaine, and D. Koschitz, Reconstructing David Huffman's Legacy in Curved-Crease Folding. In *Origami*^{5} by P. Wang-Iverson, R. J. Lang and M. Yim. AK Peters, 2011.
[2] Ron Resch Gallery: < http://www.ronresch.org/ronresch/gallery/>
[3] E. Lukasheva's curved corrugation designs: <https://www.flickr.com/photos/kusudama-me/>
[4] M. Auping. *A Frank Stella Retrospective.* Yale University Press, 2015.
[5] WebMuseum,Paris: Joseph Cornell: < https://www.ibiblio.org/wm/paint/auth/cornell/>