A Recursion in Knitting

Susan Goldstine
Department of Mathematics and Computer Science
St. Mary's College of Maryland
47645 College Drive
St. Mary's City, MD 20686, USA
E-mail: sgoldstine@smcm.edu

Abstract

A knit shawl design with an unusual stitch pattern inspired an exploration of branched knitting, negative curvature, and Fibonacci recursion. The result is *Fibonacci Downpour*, a negatively curved knitted mesh that follows a simple recursive pattern. Here, we describe the inspiration for the artwork and the recursion that defines it.

Anne Lorenz-Panzer's Branched Knitting

My recent adventures in recursive knitting started with the *Not a Drop* shawl pattern from Arlene's World of Lace [1]. The unusual stitch pattern by designer Anne Lorenz-Panzer caught my attention; as shown in Figure 1, the shawl consists of vertical columns of stitches interrupted by teardrop shapes. The shawl's name is a play on the fact that the mesh in between the columns of stitches looks as though it was created by the more conventional technique of dropping stitches. In fact, there are no dropped stitches, and each drop shape is produced by a very clever combination of stitches that effectively splits the column of stitches into two identical columns at the pointed tip of the drop and then rejoins them at the rounded end.

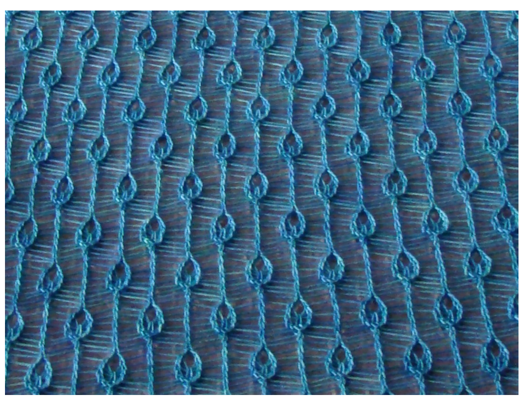

Figure 1: *Stitch pattern from the* Not a Drop *shawl designed by Anne Lorenz-Panzer. The columns of stitches are bifurcated and rejoined to form the teardrop shapes*.

As soon as I began knitting my own shawl (Figure 1), I became intrigued with the idea of uncoupling the branching of columns from forming drops, allowing for a tree-like design in knitting. The branching would increase the number of stitches per row, inducing the negative curvature familiar to anyone who has dabbled with hyperbolic crochet.

Daina Taimina's Hyperbolic Crochet

The key to hyperbolic crochet, as thoroughly discussed in Daina Taimina's landmark book [3], is that you can achieve constant negative curvature in crochet (and for that matter, in knitting) by distributing increases so that the number of stitches per row grows exponentially. More precisely, if you fix a natural number *n* and make an increase every *n* stitches throughout the crochet, you will produce an excellent approximation of a surface of constant negative curvature. This elegant pattern is what inspired me to learn crochet in the first place, and I returned to the idea in bead crochet for my 2014 artwork *Hyperbolic Constellation* (Figure 2). Here, every sixth bead is gold, and these gold beads mark the locations of the increases, demonstrating the unseen complexity in Taimina's hyperbolic technique.

Figure 2: Hyperbolic Constellation*, glass beads and crochet cotton thread, 2014.*

\

While my beaded and unbeaded hyperbolic crochet informed my thinking about branched knitting, it was clear to me that the process and result of a branched version of Lorenz-Panzer's shawl would be different in several key points. To faithfully represent constant negative curvature, hyperbolic crochet is typically very tight and rigid. By contrast, the *Not a Drop* stitch pattern is a loose mesh, and any curved surface it produced would be much more malleable. The focus of any such artwork, it seemed, should be more on an interesting choice of branching structure than on geometric precision.

A further complication arose from my desire to maintain the distinctive teardrop shapes that make *Not a Drop* such a pretty pattern in the first place. The tricky part is that, as seen in Figure 1, the drops take up enough space that need to be staggered; two drops formed in adjacent columns at the same time would crowd each other too much.

Recursive Roadblocks

As a mathematician searching for an interesting branching structure, it almost goes without saying that the Fibonacci numbers were on my mind. In fact, I had collaborated on a recursive Fibonacci tree-like structure in a much earlier artwork, a wire and paper Fibonacci mobile constructed with Alison Frane (Figure 3). Frane had observed that by viewing suspending two mobile segments from either end of a crossbar as a form of mobile addition, you can construct a mobile using the same recursion that produces the Fibonacci numbers [2].

On first glance, this seems like a process that would transfer easily into a branched knitting design, with the splitting columns replacing the wire crossbars and the drops replacing the suspended paper. However, there are a number of problems with this approach. First of all, the mobile's recursion works from the bottom up, which is exactly the opposite of the natural direction for the knitting. Instead of taking smaller mobile components and joining them together, the knitting takes individual columns and breaks them apart. Second of all, the mobile recursion does not provide an elegant solution to the problem of staggering the drops. Worse than that, it was not at all obvious where to place the drops to keep the knitting interesting; having large sections of knitting that were all branches and no drops seemed intolerably boring.

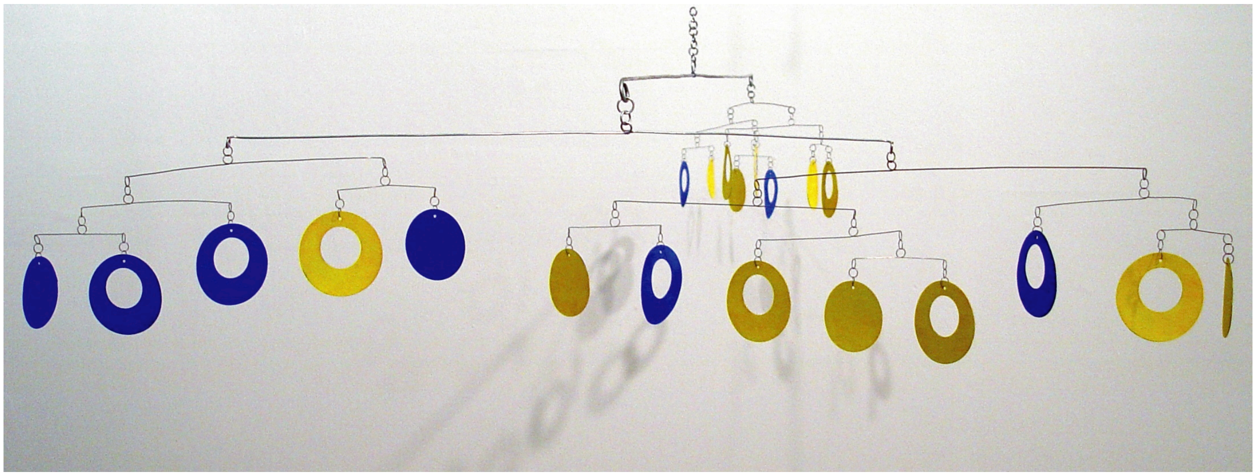

Figure 3: Fibonacci Mobile*, watercolor paper, watercolors, and steel wire, 2007, with Alison Frane.*

So, was there a different recursion that would interact well with the knitting process, produce an interesting distribution of drops, and ensure that two drops did not form right next to each other?

When It Rains, It Pours

Happily, the answer is yes, and the result is *Fibonacci Downpour* (Figure 4). The key is to incorporate both drops and branchings into the recursion, so that each level is a sequence of drops (1) and branchings (2). The piece starts with a single column of stitches in between two border columns. At the first level, this column (1) is branched (2). From there on, each drop (1) in the previous level is continued as a branching (2) in the next level, and each branching (2) in the previous level is continued as a drop followed by a branching (1 2) in the next level. The resulting numerical pattern is:

2
1 2
2 1 2
1 2 2 1 2
2 1 2 1 2 2 1 2
1 2 2 1 2 2 1 2 1 2 2 1 2
2 1 2 1 2 2 1 2 1 2 2 1 2 2 1 2 1 2 2 1 2
1 2 2 1 2 2 1 2 1 2 2 1 2 2 1 2 1 2 2 1 2 1 2 2 1 2 2 1 2 1 2 2 1 2

Notice that because of the recursive rules, there are never two consecutive 1's, which guarantees that drops will not appear in adjacent stitch columns. Furthermore, each level $n \geq 3$ is the concatenation of level $n - 2$ and $n - 1$, and the numbers of drops in level n, branchings in level n, columns of stitches entering level n and columns of stitches leaving level n are four consecutive Fibonacci numbers. (For instance, in the third level (2 1 2), there is one drop, two branchings, three columns coming into the level, and five columns coming out.)

Since the Fibonacci numbers grow asymptotically exponentially, in principle the fabric produced by this recursion is closer and closer to having constant negative curvature as more and more levels are added. In practice, the mesh is so loose that the form the knitting takes is just as defined by how the knitting is blocked and mounted as by the stitch pattern itself. With the bottom lashed to a circular hoop, the fountain of yarn takes on a pleasingly pseudospherical appearance, even if it is only a mirage.

Figure 4: Fibonacci Downpour*, merino yarn, cotton thread, and embroidery hoop, 2015.*

References

[1] Arlene's World of Lace, *Not a Drop,* downloadable pdf available through LoveKnitting website, http://www.loveknitting.com/us/independent-designers/?designer_name=36389.

[2] A. Frane and S. Goldstine, *Fibonacci Mobiles*, Math Horizons, November 2008, pp. 24—25
.
[3] D. Taimina, *Crocheting Adventures with Hyperbolic Planes*, A.K Peters/CRC Press, 2009.