Phylogenetic Analysis of the Ancient Greek Paeonic Rhythms

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Abstract

The ancient Greek paeonic (quintuple time) family of rhythms discussed by Aristotle and Aristoxenus provides a unique and much needed opportunity to test the evolutionary efficacy of the mathematical phylogenetic tools presently available. A paeonic rhythm has durational ratio 2:3, and can be notated succinctly using binary sequences of five symbols in length such as [x - x x -], where each symbol denotes a unit of time, the symbol 'x' denotes a sounded pulse (note onset), and the symbol '-' denotes a silent pulse (a rest). The rhythm [x - x x -], called the *cretic*, is the root of the paeonic genus. Seven variants of this prototype rhythm appeared gradually over a period ranging from the 7th Century BC to the 2nd Century AD. It is shown here that correlation and phylogenetic analyses of these rhythms support their documented historical evolution.

Introduction

In 1893, French archaeologists excavating the site of Delphi on the south-western promontory of Mount Parnassus on the mainland of Greece, were concentrating on a temple dedicated to the god Apollo, best known for being the site of an ancient oracle. Apollo was the god of music, poetry, truth, prophecy, healing, and sunlight, just to name a few of his laurels. The archaeologists made history when they discovered two hymns inscribed in stone fragments, in decipherable notation unambiguous in its rhythms [5, 9]. Delphi also contained a stadium where every four years the Pythian Games were held in honor of Apollo. The discovery of these ancient hymns at the site of the games inspired the French educator and historian Baron Pierre de Coubertin to attempt to revive the ancient Olympic games [3]. To this end he organized in 1894 a conference at the Sorbonne University in Paris, at which the ancient hymns were played to great effect, with the goal of creating an International Olympics Committee. The event was a resounding success, and two years later on April 6, 1896 the first modern olympic games were celebrated before 80,000 spectators in the Panathenaic Stadium in Athens [6].

The performance of the Delphic hymns at the conference in the Sorbonne must have sounded strange to many ears in the audience, since they were composed in the quintuple meter, which was rather unusual in the music that most listeners were accustomed to hearing at the time in Western Europe. The first ten measures of the first Delphic hymn are shown in box notation in Figure 1. Box notation uses two symbols to represent sounded and silent unit-time pulses (notes). In Figure 1 an empty box denotes a silent pulse and a filled box a sounded pulse. The rhythms of all the measures used in the Delphic hymns come in only four distinct varieties: [x - x x -], [x - x x x], [x x x x -], and [x x x x x]. However, in ancient Greece other quintuple rhythms are found both before and after the period at which the Delphic hymns were composed (2nd c. BC), and it is this family of rhythms that provides the focus for the study reported here.

There are very few documented examples in the history of the evolution of musical rhythm that yield evidence of the gradual development across centuries, of a prototype rhythm evolving to a set of its variants. The ancient Greek rhythmic paeonic (quintuple time) genus discussed by Aristotle and Aristoxenus is one such notable example, thus providing a unique and much needed opportunity to test the evolutionary efficacy of the mathematical phylogenetic tools available. A paeonic rhythm has durational ratio 2:3, and can be notated succinctly using the binary sequences described in the preceding. The rhythm [x - x x -] is the root

Pulse #	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
	●		●	●	●	●		●	●	●	●		●	●		●	●	●	●	●	●	●	●	●	●
Measure #			1					2					3					4					5		

Pulse #	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
	●	●	●	●		●		●	●		●	●	●	●	●	●	●	●	●		●	●	●	●	
Measure #			6					7					8					9					10		

Figure 1 : *The first ten measures of the first Delphic hymn.*

of the paeonic genus, and is called the *cretic* [11]. M. L. West documents seven variants of this prototype rhythm that appeared gradually over a period ranging from the 7th Century BC to the 2nd Century AD [10]. The eight rhythms listed in their chronological order of appearance in Table 5.1 of West's book, are reformatted in box notation in Figure 2. Here correlation and phylogenetic analyses using the tools available in the *SplitsTree-4* software package [1] are used to test whether mathematical evidence supports the evolution of the paeonic genus documented by M. L. West.

	1	2	3	4	5	7th c. BC	5th c. BC	3rd-2nd c. BC	2nd c. AD
1	●		●	●		✔	✔	✔	✔
2	●		●			✔	?		
3	●		●	●	●		✔	✔	✔
4	●	●	●	●			✔	✔	✔
5	●	●	●	●	●			✔	✔
6	●		●		●				✔
7	●	●	●		●				✔
8	●	●		●	●				✔

Figure 2 : *The increasing variety of quintuple rhythms from the 7th c. BC to the 2nd c. AD.*

Correlation Analysis

The first question posed in this study is the following. Can a mathematical measure of the distance between two sequences serve as a model for the chronological evolution (or historical appearance) of the ancient Greek paeonic rhythms? To answer this question the order of appearance of the eight paeonic rhythms in Figure 2 was tested for association with the *edit* distance of the root of the paeonic genus (rhythm No. 1) to all other seven rhythms. The edit distance used in this and the subsequent studies was motivated by the fact that it correlates well with human judgements of perception [4, 8]. The edit distance between two sequences (rhythms) is defined as the minimum number of mutations required to transform one sequence to the other. There are three types of mutations: (1) a symbol may be deleted, shortening the rhythm, (2) a symbol may be inserted, lengthening the rhythm, and (3) one symbol may be substituted by another [2]. A natural test to measure the association between two orderings is the Spearman rank correlation coefficient [7]. The edit distances from rhythm No. 1 to the eight rhythms are: 0, 1, 1, 1, 2, 2, 2, 2. The Spearman rank correlation between these distances and the chronological order is 0.91, and the two-tailed value of p is 0.00155.

Phylogenetic Analysis

The second question posed in this study asks whether a mathematical measure of the distance between two sequences can serve as a model for identifying the *central prototype* rhythm from which all the other ancient Greek paeonic rhythms are derived. To this end a phylogenetic analysis of the rhythms was carried out using the *BioNJ* algorithm in the *SplitsTree-4* software package [1]. The input to the *SplitsTree* program is in general any distance matrix that contains the distance (edit distance in the present study) between every pair of rhythms in the family. The *BioNJ* algorithm is a neighbor-joining algorithm that constructs a tree from the distance matrix using an agglomerative process consisting of iteratively picking a pair of rhythms, creating a new node which represents these rhythms, and updating the distance matrix by replacing both rhythms with the new node, while minimizing the the difference between the sum of distances in the matrix and the sum of geodesic distances in the tree.

Rhythm	x-xx-	x-x--	x-xxx	xxxx-	xxxxx	x-x-x	xxx-x	xx-xx
SumDist	**11**	**14**	**11**	**13**	**11**	**12**	**12**	**14**

Figure 3 : *The centrality measure* **SumDist** *of the eight rhythms documented by West [10].*

The edit distance matrix of the eight rhythms documented by West [10], yields the sum of the distances from each rhythm (a measure of centrality) to all the other seven rhythms, shown in Figure 3 (labelled **SumDist**). Thus the rhythm with the minimum total sum is considered to be the ancestral root, or central prototype, from which all the other rhythms may be generated with the minimum number of mutations. From Figure 3 it is observed that the oldest ancestral rhythm does have the minimum total edit distance of 11, and the most recent offspring has the largest distance of 14. However, the third and fifth rhythms are tied with the first, and the second rhythm is tied with the last, yielding a less than completely successful outcome.

No.	Rhythm	x-xx-	x-x--	x--x-	x-xxx	xxxx-	x-x-x	xx-x-	xxx-x	xx-xx
1	x-xx-	0	1	1	1	1	2	2	2	2
2	x-x--	1	0	2	2	2	1	2	2	3
3	x--x-	1	2	0	2	2	2	1	3	2
4	x-xxx	1	2	2	0	2	1	3	2	2
5	xxxx-	1	2	2	2	0	3	1	2	2
6	x-x-x	2	1	2	1	3	0	2	1	2
7	xx-x-	2	2	1	3	1	2	0	2	1
8	xxx-x	2	2	3	2	2	1	2	0	2
9	xx-xx	2	3	2	2	2	2	1	2	0
	TOTAL	**12**	**15**	**15**	**15**	**15**	**14**	**14**	**16**	**16**

Figure 4 : *The edit distance matrix for nine ancient Greek quintuple rhythms.*

However, the book by West [10] does not contain all the ancient Greek quintuple rhythms. In the 4th century BC, Aristoxenus, the Greek music theorist and student of Aristotle, wrote the *Elements of Rhythm*, which, according to C. F. Abdy Williams [11], contains two additional quintuple rhythms, namely [x - - x -] and [x x - x -]. Furthermore, the rhythm [x x x x x] was used rather infrequently in ancient Greece. To quote Abdy Williams, "a continuous use would not be made of such a rhythm; for its character is quite alien to the paeon. The reason Aristoxenus objects to giving five primary times to the paeon is that the ancients considered a succession of short notes mean and vulgar." Thus if the two rhythms found in the work of Aristoxenus are included, and the rhythm with five primary times (successive onsets) is deleted, one obtains the distance matrix in Figure 4, where the two additional rhythms are numbered 3 and 7 in the list. With these nine rhythms the results provide good agreement with the evolutionary data. The most ancient rhythm now is the unique rhythm with the smallest total sum of distances equal to 12. Figure 5 shows the *BioNJ* tree calculated from the matrix of Figure 4. The order in which the rhythms appear historically in West's table is indicated with numbers, and the two additional rhythms listed by Abdy Williams are circled. The most ancient rhythm is clearly situated in the center of the tree.

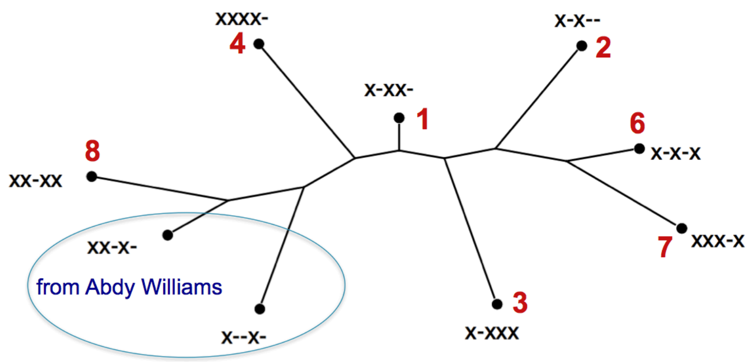

Figure 5 : *The BioNJ phylogenetic tree obtained for the nine ancient Greek quintuple rhythms.*

Conclusion

The correlation and phylogenetic analyses using the eight rhythms documented by West [10] provide moderate support of the chronological evolution of those rhythms. Furthermore, if the two rhythms listed by Abdy Williams are included, and the rhythm [x x x x x] is deleted from the group, then the phylogenetic analysis of the resulting nine rhythms provides strong support of the evolution of the ancient Greek quintuple rhythms.

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