Geometric Patterns as Material Things: The Making of Seljuk Patterns on Curved Surfaces

Begüm Hamzaoğlu*
Architectural Design Computing Program, Istanbul Technical University
Taşkışla, Taksim, Istanbul, 34437, Turkey
E-mail: hamzaoglu@itu.edu.tr

Mine Özkar
Department of Architecture, Istanbul Technical University
Taşkışla, Taksim, Istanbul, 34437, Turkey
Email: ozkar@itu.edu.tr

Abstract

There is very little information today on the historical techniques that medieval craftsmen used to apply the intricate geometric patterns on architectural surfaces. This paper focuses on the particular problem of applying geometric patterns on curved stone surfaces and explores possible technical application scenarios for a select group of 13th century Anatolian patterns. We illustrate hypothetical processes of how to apply three patterns on three different types of curved surfaces and discuss the relation between the surface geometry and the tools. Our main motivation is to shed light on how the geometric construction of the designs and the making of these patterns correlate.

Introduction

Geometric patterns consisting of polygons, stars and lines were used as ornaments on monumental building façades in medieval Anatolia, also known as the Seljuk period. The style was widely common throughout the larger region at the time. The question of how these intricate designs were constructed geometrically has drawn the attention of many interdisciplinary studies [1][2][3][4]. However, only a few of these studies focus on how abstract patterns were transformed into the material things that they are whether carved into stone, tiled in brick or ceramic, or arranged in wood using special crafting techniques [5]. Materials, tools and all other components of the making process such as the craftsman's hand movements are all factors in the formation of the shapes. The variation in the corpus is a result of this integrated process [6].

Only a few written original sources tell on the techniques from that period. One reports on regular meetings between craftsmen and geometers where the latter visually demonstrated geometric methods [7]. In addition to medieval treatises and enduring craft traditions, scholars mostly rely on clues from existing patterns to speculate on how they were made. Bakırer shows that the compass and circle traces found on a stone piece in Divriği Great Mosque in Sivas indicate that circular grids were used on site for either applying or constructing the geometric patterns on building surfaces [8]. This solidifies the link between the construction of a shape using circles and its application on the hard material surface. Figure 1 shows a partial and representative grid of interlacing circles drawn by a compass and an exemplary resulting pattern. After drawing the circles, a straightedge is used for drawing lines that cross intersection points. These lines divide the circles to equal parts and generate the polygons as the formal structure of the pattern. Polygons with different thickness and sizes can be constructed on a circular grid by drawing smaller circles at the intersection points on the grid and offsetting the inscribed polygon.

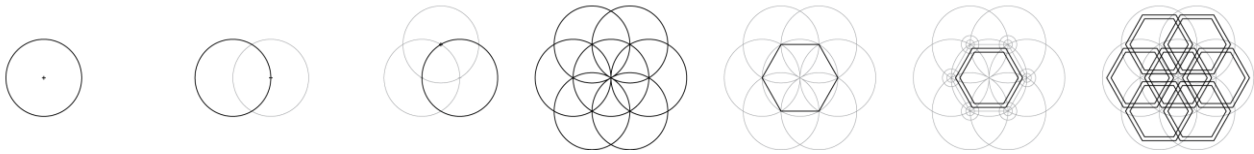

Figure 1: *A grid of interlacing circles drawn by a compass and an exemplary resulting pattern.*

One of the argued advantages of the compass-straightedge application is the possibility to transfer topologically consistent grids, and hence patterns, on uneven building surfaces and in different sizes [9]. In reality, patterns were also applied on different types of surfaces with positive, zero, or negative curvatures. For preserving the design symmetry when applied on such surfaces, these applications require either the knowledge of trigonometry or the know-how of simple drawing tools and techniques. This paper considers three Seljuk patterns and the physical application of their geometric designs on three differently curved stone surfaces. Considering their size, position, and material, the patterns seem to have been carved in situ. First, we show the circular grids that can be used for the construction of each pattern and model the surfaces they are on to understand their geometric features. Second, we demonstrate for each pattern two alternative methods that the craftsmen then could have followed to apply the circular grids on the corresponding stone surface using various tools such as paper, compass and rope. The investigation yields results for a discussion on which type of application works for different types of curved surfaces and the variables of this utility as well as on the impact of material application methods on local and global variations of the pattern design.

The Convex Cylindrical Surface

The first pattern uniformly wraps around the cylindrical surface of an engaged column at the entrance of the Tomb of Mama Hatun in Tercan near Erzincan, a yellow cut stone monument that dates back to the turn of the 13th century. The application of a two-dimensional pattern on the developable surface of cylinder is ideally similar to placing it on a flat surface. Still, there are two ways for doing it on a stone cylinder block.

The application of the design relies on the circular grid, which will serve as the underlying guide for generating the hexagonal pattern as shown in Figure 2. The lightly drawn circles are intermediary and are used to generate the next circles over. In the first alternative application, the circular grid is drawn on a sheet of paper or tissue that has a length equal to the circumference of the cylindrical surface. The radii of circles are equal and follow a linear and uniform repetition on the rectangular sheet. In this design, the ratio of the radius of a grid circle to the radius of the cylinder is $\pi/3$. The grid then can be carbon copied onto the surface of the cylinder (Figure 3) and embellished with actual tools and material. The paper wraps around the cylinder and acts as the template as its dark lines press onto the stone and mark it.

In a second alternative, the grid is applied directly onto the stone surface using a piece of rope, i.e. a primitive compass (Figure 4). As circles are drawn one by one, the center point of each is the reference for the next circle over. The rope follows the surface and produces circles the radii of which are the same as measured in the geodesic distance. On the developable surface, all these circles have the same radii as on the planar surface of the paper, and both applications deliver the same result. In this example, the hexagonal unit of the pattern is entirely visible from any angle albeit in a slender version as it wraps around the cylinder. If the ratio of the radii of the grid circle and the cylinder were larger than $\pi/3$, the design would look entirely different, perhaps not even uniform. The larger ratio could also negatively impact the second alternative where it would be difficult for the craftsmen to extend the rope on the surface as a compass. There is a limit to the proportional relation between the radii.

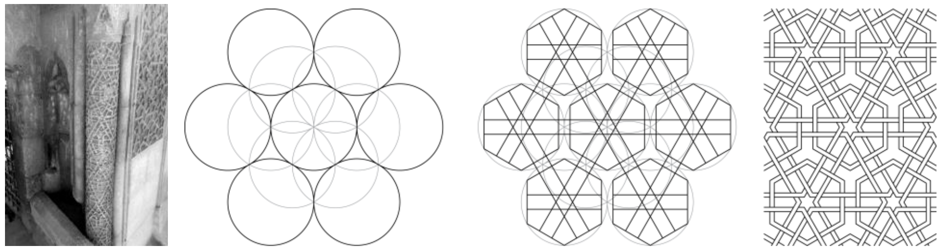

Figure 2: *Left - Photograph of the pattern on the engaged column at the entrance of Tomb of Mama Hatun (Photo credit: Mine Özkar). Right - A circular grid and the pattern constructed from it*

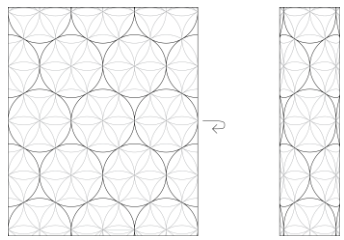

Figure 3: *Drawings that show how the paper wraps around the cylinder*

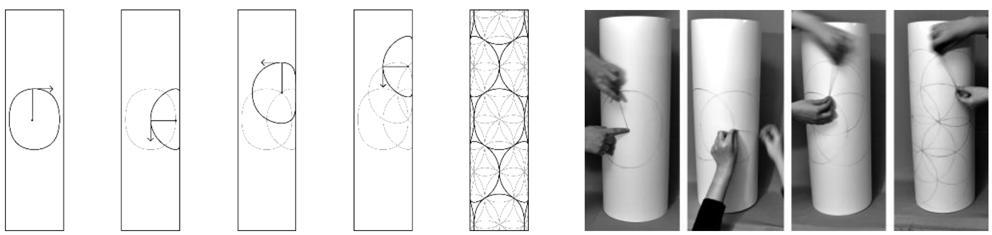

Figure 4: Left - *Drawings that show how each circle is drawn on the cylinder using a rope. Right – Photos showing the process*

The Curved Squinch

The second example is a pattern on a curved squinch from the same monument. The squinch is a typical muqarnas unit, a common feature in Islamic architecture used for covering vaults. The angle of the two vertical edges meeting at the top is usually 90°, 45° or 135° in muqarnas units [10]. Figure 6 illustrates the developable surface inside this particular squinch. Once again, a circular grid underlies the six-fold construction of the pattern (Figure 5) where the dodecagonal unit is offset from it as shown in Figure 1.

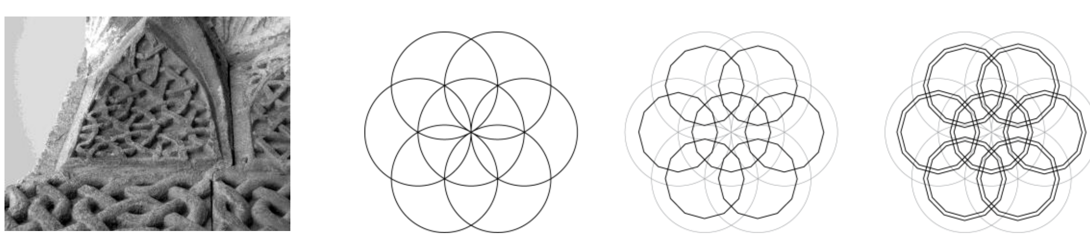

Figure 5: *Left - Photograph of the pattern on a curved squinch at the entrance of the Tomb of Mama Hatun (Photo Credit Mine Özkar). Right - A circular grid and the pattern constructed from it.*

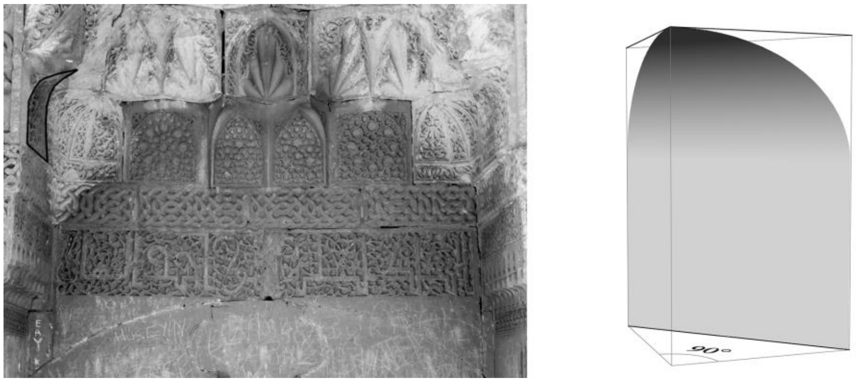

Figure 6: *Left – Photograph shows the location of the curved squinch (Photo Credit Mine Özkar). Right – The geometric model of the squinch that develops upwards then curves.*

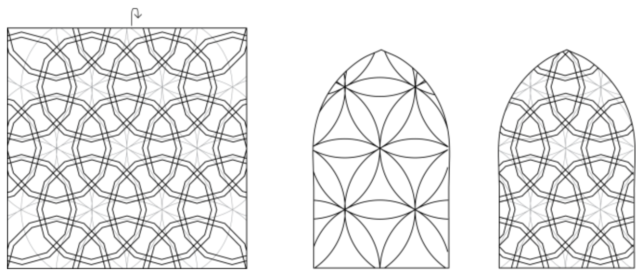

Figure 7: *Drawings show the drawing process using paper.*

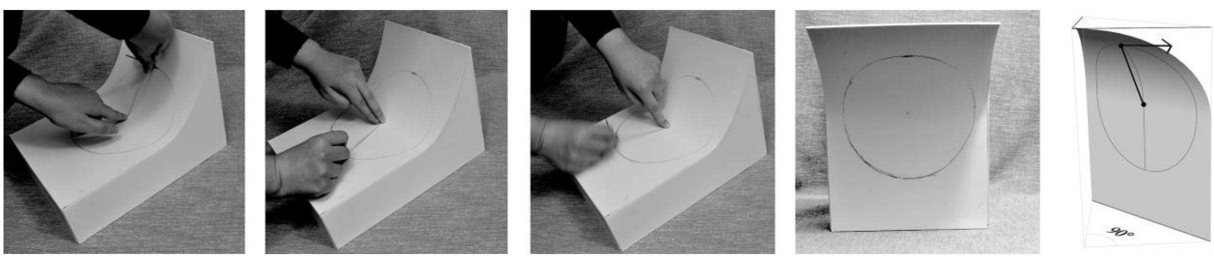

Figure 8: Left – *Photographs show the drawing process using rope. Far right – Drawing illustrates the stretched rope and the geodesic distance between the shape and the center.*

We pursue the same two alternatives from above for applying the pattern on the surface. In the first, the circles are drawn on the paper, which is then snug on the interior surface of the squinch for carbon copying (Figure 7). This is as straightforward as it was for the first example. Nevertheless, if the bottom edge of the squinch were curved, as it sometimes is in muqarnases, the surface then would not be developable and the paper template application would require extra steps such as modeling it as an interrupted surface.

In the second alternative, a rope compass is used for drawing circles directly onto the surface. Figure 8 illustrates the drawing process, in which the rope is stretched and turned 360 degrees around the center as the pen at its endpoint touches the curved surface. As a result of the concavity where the surface is bent, the geodesic distances vary and the resulting shape is not a perfect circle. This could be the case for this particular pattern as the dodecagonal motif is mostly on the flat surface and curves only at the top. The distortion has minimal effect on its perception from below, if not a positive one making it visible.

The Convex Spherical Surface

The last example is a pattern on a hemisphere-shaped stone surface from the entrance of the late 13th century Buruciye Madrasah in Sivas. The has interlocking components of various sizes. At the center, there is one central pentagon that is divided into five rhombuses, five triangles, and a pentagon as emphasized in the bottom-right image of Figure 9. The triangles and a pentagon constitute a star. These five triangles may also be perceived as five trapezoids because their lines are thickened up. Then there are ten other pentagons of the same size around the central polygon. Then, there is a single decagon that goes through the centers of these pentagons. Finally, towards the outer rims, there are ten additional smaller pentagons around the decagon that are interlocked with the bigger pentagons. The base that underlies this pattern includes circles of three different sizes. Figure 9 shows this base and the inscribed polygons both as wrapped on the hemisphere and as stretched on a plane. The five-fold symmetry prevails in both.

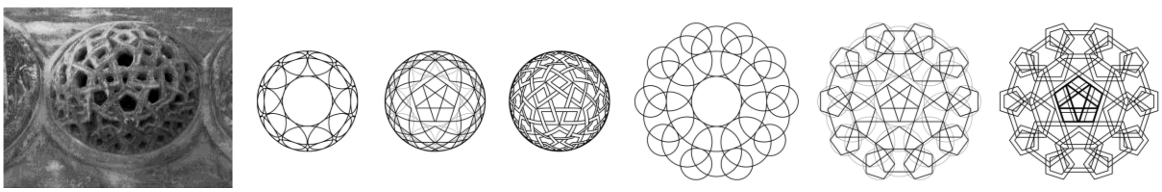

Figure 9: *Left - Photograph of the pattern (Photo credit Ezgi Baştuğ). Right - The circular base and the polygons for constructing the pattern.*

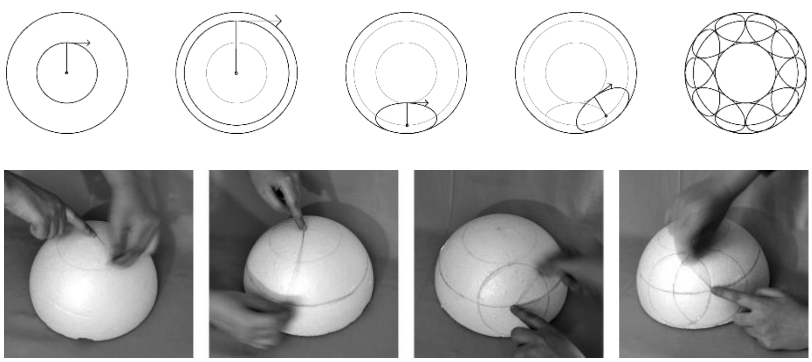

Figure 10: *Each circle is drawn on the hemisphere using a rope or a compass*

The first approach to apply the paper template on the hemisphere fails, as its surface is not developable. In the second alternative, the pattern is drawn on the hemispherical surface using a piece of rope or a regular compass (Figure 10). The stretched rope follows the surface as one of its ends is rotated around a central point. The radius of the circle is then equal to the geodesic distance between the circle and its center point on the hemisphere. A regular compass would yield the same resulting circle, and with greater accuracy as it would be free from any slips possible with the rope. The difference of this example from the previous two is that the design can only be constructed on a hemisphere and not on paper beforehand. The process of application starts with drawing two concentric circles with varying radii at the top of the hemisphere. The smaller one of these, with a diameter ½ the radius of the hemisphere, is for inscribing the pentagon and the bigger one is for the decagon. Then another circle that has the same radius equal to the small one is drawn around one random point on the big circle. The process continues with drawing nine more circles in a way that each time the next circle intersects at the center of the latest drawn circle. Finally, the smallest circles on the grid can be drawn around the intersection points of the ten existing circles around the center. In this way, different sized polygons can be adjusted on a spherical surface using simple tools.

Discussion and Conclusion

With the motivation to shed light on practical applications of Seljuk geometric design on actual material, we looked at two different ways that three patterns can be applied on curved stone surfaces. The first step is always the creation of the corresponding circular guidelines for generating the patterns. These "grids" of interlocking circles serve as guidelines for preserving the symmetry and can easily be drawn on developable surfaces using simple tools. So in some applications, it is possible that the guides, or even the full designs, were directly etched on the stone surface. However, when the surfaces are not developable, applications must have varied. We demonstrated that paper templates that wrap around developable curved surfaces are useful for applying patterns. The issue of scale, i.e. the ratio of sizes between elements of the pattern and the surface, is nontrivial. In the developable surfaces of the first two examples, where the circles of the guiding grid were well within the perceived boundaries of the surface from one viewpoint, it was also possible to apply the pattern using a rope compass. The exception in the second -concave- one was a real distortion due to varying geodesic distance. The application with the compass functioned also for the hemisphere with a convex surface and radial symmetry. The value of this simple investigation is firstly the demonstration of which type of hands-on application works for different types of curved surfaces and pattern-surface size proportions. Secondly, it is the point that in the application with the paper, the overall pattern is designed and calculated as a whole, whereas in the application with the compass, the pattern is created piecemeal and allows for local noise and variation. Immediate future work is twofold: one, more applications of these methods with variables of radius-surface size ratio and symmetry group of the pattern; two, defining algebraic representations of the application methods and integrating them into this framework for formalizing the making of geometric patterns on stone surfaces to be extended later to brick, ceramic, and wood.

Acknowledgements

This research is supported in part by TÜBİTAK under the project number 114K283.

References

[1] S. J. Abas and A. S. Salman, "Geometric and group-theoretic methods for computer graphics studies of Islamic symmetric patterns", *Computer Graphics Forum*, 11(1):43–53, 1992.
[2] C. S. Kaplan, "Computer Generated Islamic Star Patterns", *Bridges*, 105-112, 2000.
[3] C. S. Kaplan and D. Salesin, "Islamic Star Patterns in Absolute Geometry", *ACM Transactions on Graphics*, 23(2):97-119, 2004.
[4] P. Lu and P. J. Steinhardt, "Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture", *Science*, 315:1106–1110, 2007.
[5] Ö. Bakirer, *Selçuklu Öncesi ve Selçuklu Dönemi Anadolu Mimarisinde Tugla Kullanimi {The use of brick in Anatolian architecture in pre-Seljuk and Seljuk era*}, ODTÜ, 1981.
[6] M. Özkar and N. Lefford, "Modal relationships as stylistic features: Examples from Seljuk and Celtic patterns", *JASIST*, 57(11):1551-1560, 2006.
[7] A. Ozdural, "Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World", *Historia Mathematica*, 27:171–201, 2000.
[8] Ö. Bakırer, "The Story of the Three Graffiti", *Muqarnas*, 16:42-69, 1999.
[9] M. Özkar, "Repeating Circles Changing Stars, Learning from the Medieval Art of Visual Computation", In N. Lee (ed.), *Digital Da Vinci: Computers in the Arts and Sciences*. Springer, 49-64, 2014.
[10] Y. Dold-Samplonius, "Practical Arabic Mathematics: Measuring the Muqarnas by al-K¯ash¯I", *Centaurus*, 35:193-242, 1992/3.