Another Look at Pentagonal Persian Patterns

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Abstract

This approach of Pentagonal Persian Patterns is an alternative to the famous PIC theory. However, it turns out that the classification of pattern families and sets of tiles used is close to the one used by (at least some) masters of the art in Iran. A very simple transformation of one of these sets produce a new style of patterns. Then we dissect each tile of a specific set into rhombi, respecting the rules of the Binary tiling (not the Penrose tilling). Thus, applying the concept of X-Tiles to these rhombi, we get an equivalence relation between two families of traditional patterns.

(a)

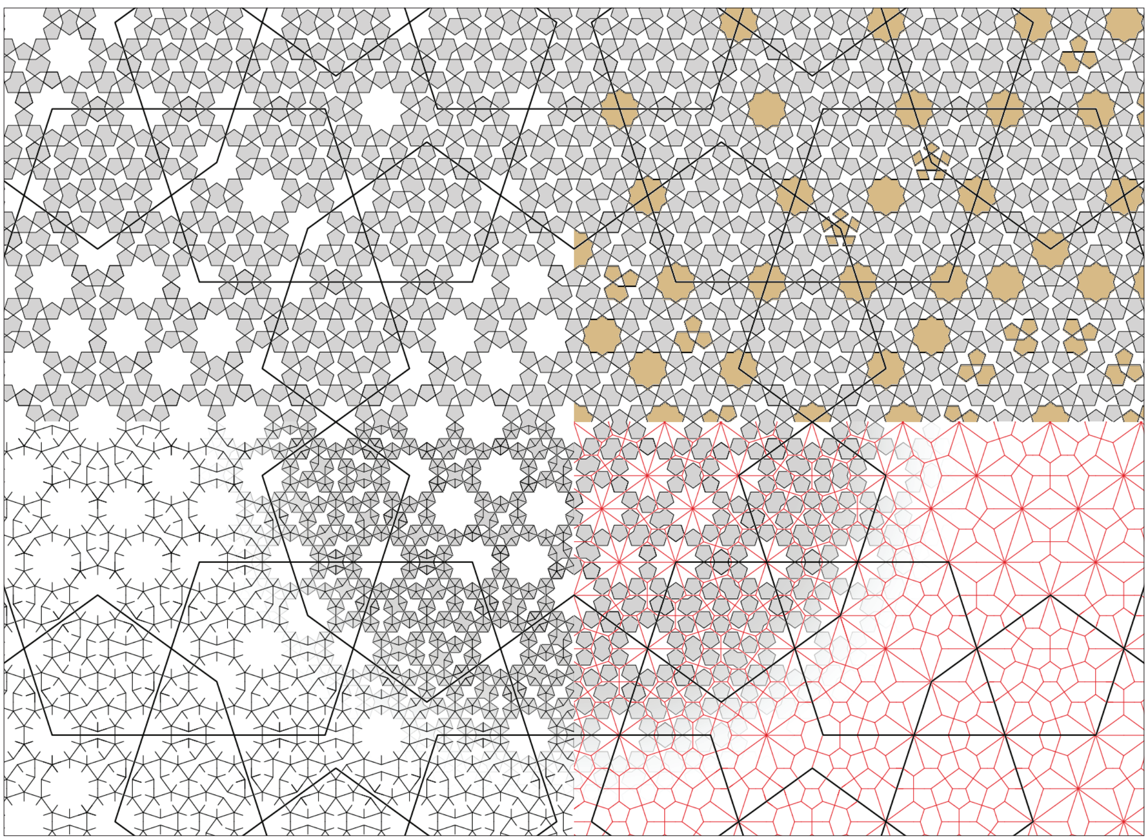

Figure 1 : *(a): Pattern of the Starry family, made from a process of self-similarity. (b): variations on the 10-Stars. (c): The "Positive Polygon Lines" automatically define the polygonal pattern of the PIC theory (the orphan lines have to be removed). (d): new pattern coming from the "Negative Polygon Lines".*

1. The “Starry Family”of Persian Patterns, *S*

The patterns of this family, which I refer to as the S-Patterns, are made from the set S (Fig. 2). Each tile can be drawn from the tile N1, the [10/3] star, which can be considered the mother of the tiles.
Why precisely that set of tiles? It comes from a search of self-similar systems for that kind of pattern [4]. Note that the tiles P4 and P5 are not that common.
S1 is a sub-set with remarkable properties. The variation $N1_2$ (Fig. 2), gives the geometric relationship between all the tiles of S1.

The “S-Tiles” (the tiles of the set S) are put together in such a way that each vertex is the crossing of only two lines. That is why any S-Pattern can be colored in two colors, like a chessboard. An amazing property is that in such a black and white coloration, each shape of tile is always the same color. That is why I define two classes of tiles, which I arbitrary name ‘Positive” and “Negative”.

As a result, any S-Pattern could be made with only the Positive tiles (connected by vertices, respecting the continuity of the lines), the holes in between being the place for the Negative tiles. And this is the easy way to create tilings when designing with computer. It is better to work with the Positive Tiles, as there are are fewer of them (in S and in S1 as well) than the Negative.

In Fig. 2 we have added some white lines on the positive tiles. Upon assembling the tiles to make a pattern, those “Positive Polygon Lines” automatically draw, disregarding orphans, the famous polygon pattern of the PIC theory (Fig. 1-c). So, this “hidden pattern” from which, according to the PIC theory, the actual pattern is supposed to be made, is in fact a kind of dual to the actual tiling.
Similarly we show (Fig. 2, top: dotted lines) the “Negative Polygon Lines” [4] on the Negative tiles. When applied to any S-Pattern, they give rise to new patterns (Fig. 1-d).

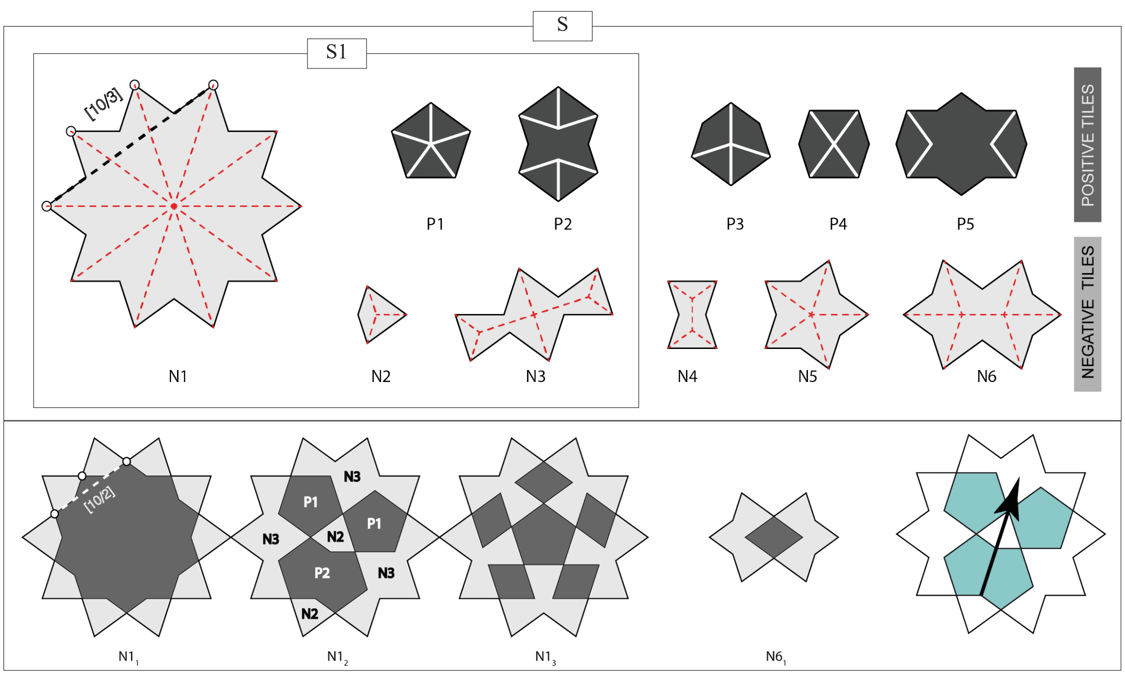

Figure 2 : *Top: The set* S *of Positive and Negative tiles used in the Starry family of patterns, S. Left, the sub-set* S1. *Bottom: Variations on N1 and N6. Left, orientation of* $N1_2$.

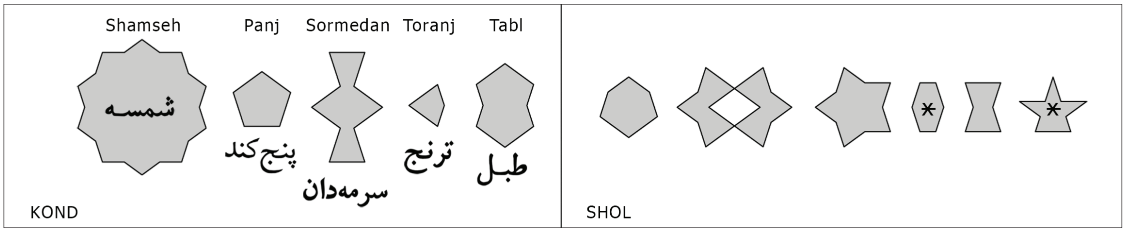

Figure 3 : *The traditional "Kond" and "Shol" Persian tiles. The tiles with a ✗ do not belong to S.*

It's not such a great surprise to recognize the similarity with the "*Kond*" and "*Shol*" traditional Persian set of tiles. The *Kond* set corresponds to the S1 set when the tile N1 is replaced by its variation $N1_1$.
Note that in traditional patterns, the tile N1 is always replaced by one of its variations (Fig. 2, bottom).
The two *Shol* tiles that do not belong to the set S arise when inserting a standard rosette (which belongs to the Floral Family F) into a S-Pattern. (Fig. 4).

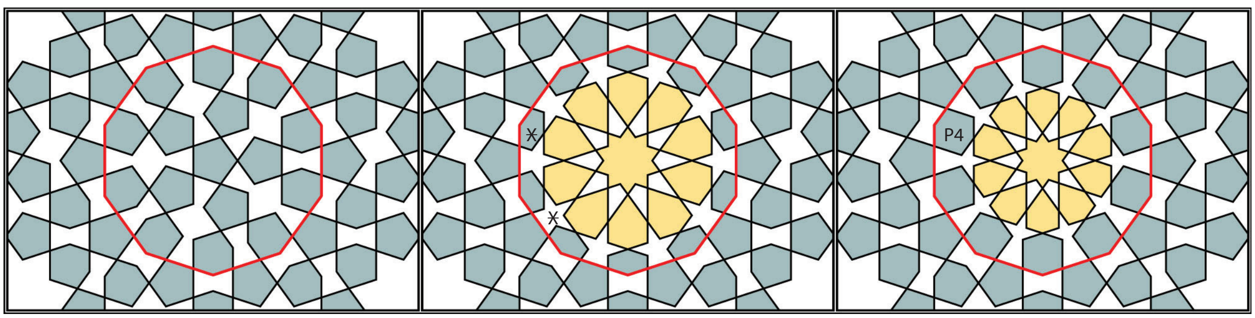

Figure 4 : *Left: A* S-*Pattern. Middle: The extra* Shol *tiles, coming from a floral variation. The petals of the rosette are as regular as possible (its four small edges are equal). Right: Similar but different: the hexagons are the regular P4 tiles, thus the scale of the rosette is smaller.*

Simplification is an effective process for analyzing a pattern: Replacing any variation of P1 by a simple P1 tile gives way to the root, the basis of this pattern. Thus some patterns that looks very different can be seen as different variations of the same simple basis. The only difficulty is to recognize those variations. An example is given in Fig. 5.

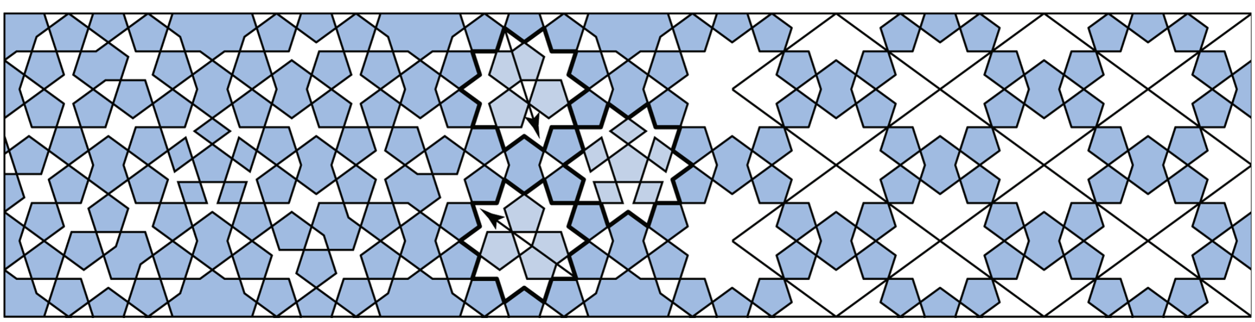

Figure 5 : *Left: This pattern that looks complex becomes simple when we can recognize the variations on the tiles N1. Right: All variations have been replaced by a simple star N1. We have added the structure of a rhombic lattice. This is the simplest S-Pattern.*
Note that the orientations of the $N1_2$ variations do not match the symmetry of the basis.

2. Two Systems of Rhombic Dissections of the Set S

We wonder if it is possible to find a dissection of each tile of the set S with "Penta-Rhombs" (the same as used in a Penrose tiling) in such a way that the tiles connect together make a pattern of those rhombs.

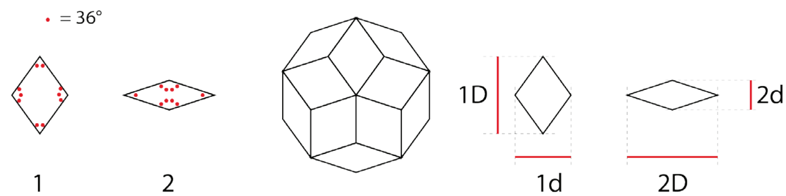

Figure 6 : *The two "Penta-Rhombs", angles and proportions. We refer as* 1 *the large rhomb, as* 2 *the thin one.* D *refers to a long diagonal,* d *to a short diagonal.*

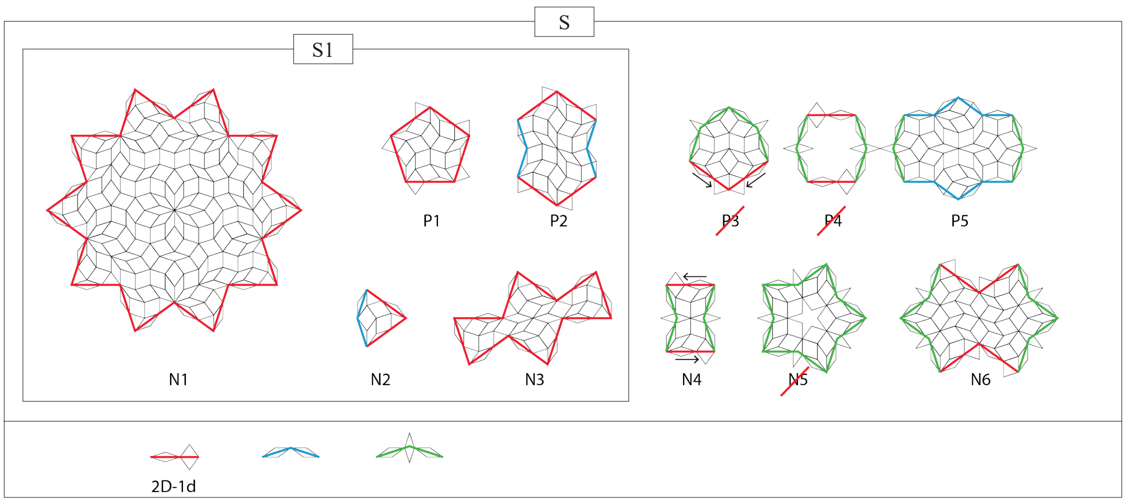

Figure 7 : *Dissected S-Tiles. This first solution doesn't works correctly for the tiles P3, P4, N4 and N5, but perfectly for the sub-set* S1*. Bottom, the mapping of the 3 different edges.*

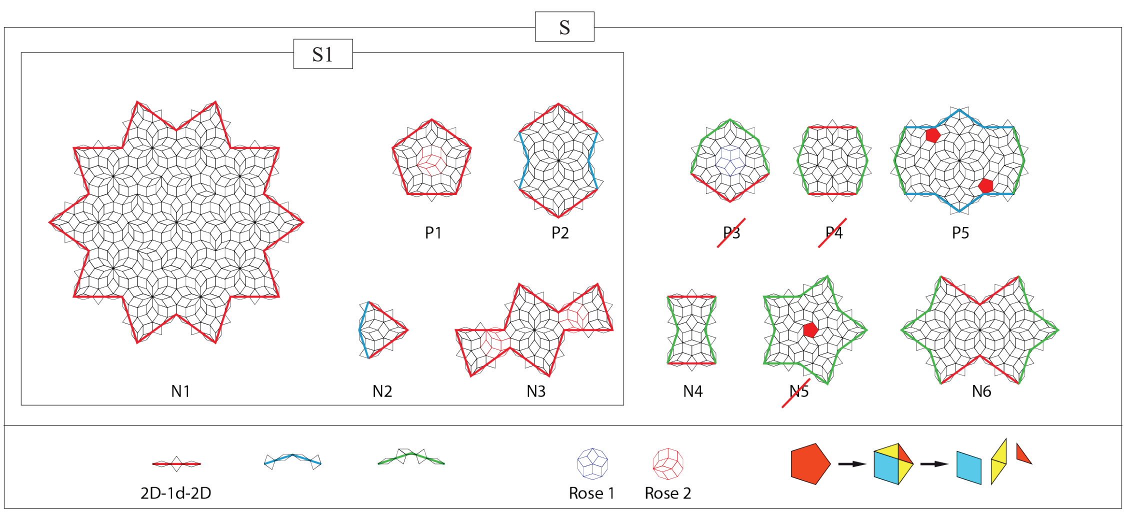

Figure 8 : *This solution works perfectly for the set* S1*. Bottom, the mapping of the 3 different edges, the two mapping of the "Rose", and the impossible dissection of the little pentagon (right) into rhombs.*

3. X-Tiles, Binary Tiling, and the Flower Family

Once having a dissection of the S-Tiles into Penta-Rhombs, it is natural to wonder if that dissection is compatible with a Penrose tiling. Yet, we can make a periodic pattern with the dissected S-Tiles, and since no such periodic tiling are possible with Penrose tiling, the answer is obviously negative. But... What's about the "X-Tiles"?

The X-Tiles comprise a system of two rhombi decorated with only two straight "X-lines" (in the shape of an X). They must be put together side by side, respecting the continuity of the X-Lines. Thus, the rules are not the same as for a Penrose pattern, but the same as for the "Binary Tiling". Once correctly assembled, if we remove the lines of the rhombus edges, the X-Lines will draw a pattern which belongs to the second Persian family of pentagonal patterns, which I refer to as the "Flower family" F. So we have a bridge between the two families:

Starry pattern => Dissection into Penta-Rhombs => X-Tiles replacement => Floral pattern

The shapes generated by the X-Lines belong to the *Tond* traditional Persian set of tiles (Fig. 11).

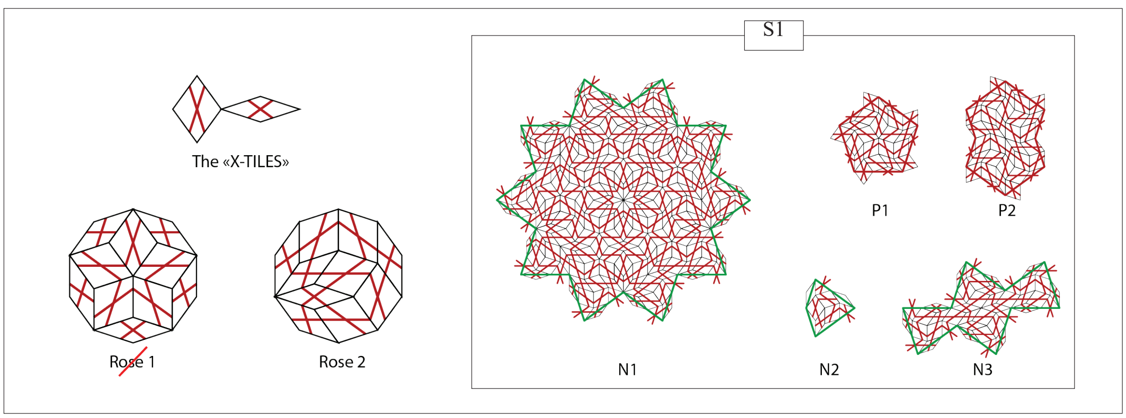

Figure 9 : *Left, X-Tiles and assembly rules. Right, X-Tiles apply to the first solution of dissection.*

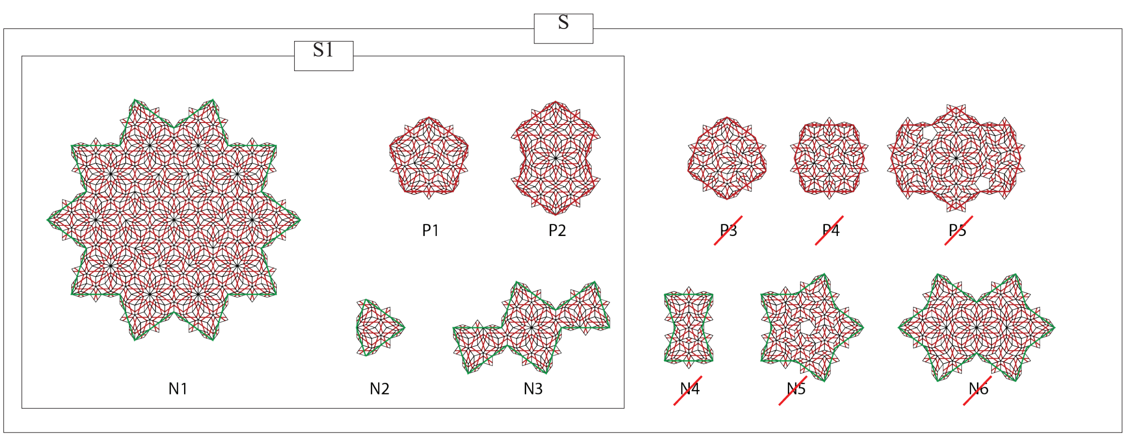

Figure 10 : *X-Tiles replacement from the second solution of dissection works only for the sub-set* S1. *Left, research of solutions for the other tiles: None works perfectly, because the continuity of the line cannot be preserved.*

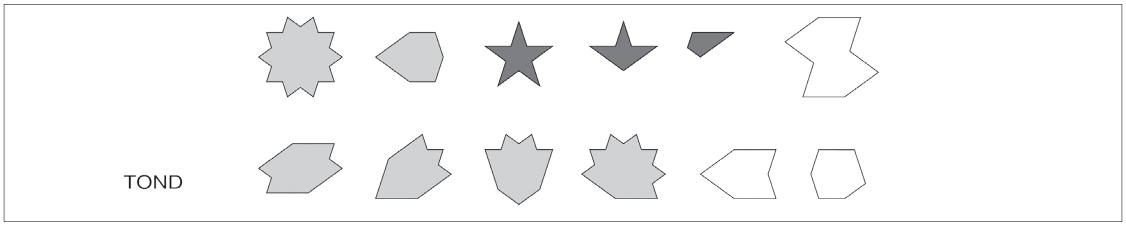

Figure 11 : *The traditional "Tond" set of tiles, from which are made the patterns of the "Flower family"* F*. In grey the 9 tiles that can produce the X-Tiles: 3 "Positive", in dark, and 6 "Negative" in light gray.*

Notes

An unexpected result of this research is the new patterns coming from the "Polygon Lines N" (Fig. 1).
The idea of a dissection of the X-tiles into the two Penta-Rhombs arises because I did that kind of thing previously with the octagonal family in Moroccan/Andalusian style. In that case, the tiles can be dissected into squares and diamonds.
To learn more about X-Tiles, see [3] and [4].
For more information about the PIC theory, see [1], [2], [5], [6] [7] [8].
Reference [9] is a very interesting Iranian book dedicated to a master in the art of patterns. The Persian tile names in Fig. 3, and of the pattern styles *Kond*, *Shol* and *Tond*, come from this book.

References

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