Dancing Math

Teaching and Learning in the Interplay between Aesthetic and Mathematical Literacy

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Abstract

This paper presents a teaching and learning program in mathematics drawing on the aesthetic expression of dance as the medium of seeking knowledge and meaning. Originally conceived as a joyful, effective and threshold lowering activity for learners apprehensive about the curriculum subject, including pre-service teachers, the program has been further developed to visualizing, in dancing, the learners' previous subject knowledge as a base for further acquisition of proficiencies in the discipline. The program's overall object is to create an effectual interactive space of learning with immediate interaction between teacher, peer learners and the math subject. Aesthetic literacy through dance and math numeracy are enhanced and interplay gainfully. Participants with varying cultural and language backgrounds, exchanging experiences and references, embark on joyful journeys through the field of knowledge. Data gathered from observations of lessons designed according to the program and dealing with a number of math curriculum items are presented and analysed. Conclusions are drawn for further investigation of the intersection of the proficiencies and literacies of aesthetics and mathematics.

Introduction

In this article a teaching and learning approach is presented, developed by the author, where aesthetic literacy interacts with and enhances math numeracy. The *Dancing Math* program draws on dancing in math class, rendering the learners' integrated and embodied knowledge visible and prompting interaction on a range of levels crucial in learning. Experiences by learners participating in the program are shared in lesson observation data. The framework for the series of observations consists of the Swedish curricula for pre-school (age 1-5), pre-school class and compulsory school (6-year olds and grades 1-6) [1]. Years of teaching *Dancing Math* in pre-school, compulsory and secondary school as well as in teacher education programs provide numerous testimonies of broken codes in learning mathematics and the discovery of math as an exciting subject matter existing all around us, illuminated by aesthetics.

Key Concepts – Interaction, the Space of Learning and the Dance Modality

The pedagogical concept of the *Dancing Math* teaching and learning mode is similar to the programs *Dancing through the Alphabet*, *Dancing Natural Sciences* and *Dancing is Being* [2], dealing with language, science and philosophy. The program is conceived on the notion *space of learning* [3], the interactive space where learning is made possible to occur.

Interaction and communication are pivotal in teaching and learning. The interplay between teacher and learner is uncontested. Peer to peer learning is another vital part of classroom interaction. Thirdly but not least, nothing is learned without the learner interacting with what is to be learned. In the classroom all three levels of interaction unfold interwoven.

A distinction is to be made between overall social aspects of teaching and learning, and its specific pedagogical nature. Hargreaves and Fullan [4] determine the teachers' human and social capital, their dexterity in interplaying with the pupils, as decisive for successful learning. Marton and Tsui [5], as well as Runesson [6] emphasize the significance of gathering around what is to be learned. The optimal *space of learning*, the space where learning occurs, evolves in the interplay between the teacher's and the learners' awareness of what is to be learned, and in the possibilities for communication [7]. Crucial then to the optimization of the teacher-learner interplay, Lo, Pong and Chik [8] point out, is the teacher's capability to imagine each and one of the learners' previous comprehensions and conceptions about the subject. Peer learners' sharing of pre-understanding, expressed and visualised in talk or other communication modes [9] including gesture and dance, is an indispensable activity in the *space of learning*.

The relevance of dance in teaching and learning may be discerned by examining its potential as mode of communication and interaction, creative expression, sensorimotor activity, embodiment of previous and new knowledge, its potential as artefact, metaphor and conceptualisation of subject matter. Additionally, dancing as part of socio-cultural learning entails attention to the learner's agency, to ethical aspects and to inherent aesthetic dimensions of a subject such as mathematics. Hands-on subject matter teaching and learning is combined with setting factual math proficiencies in a broader context of critical and liberal education and the challenges of democracy and coexistence in an accelerating changing world.

A Designed Space of Learning

A Designed Space. The class comes into the room decorated with items relatable to mathematics. Coated figures, Roman, Arabic and Indian, in different colours cover the floor. Hula-hoops in various tones lean to a wall, skipping ropes and small flags hang down from an other wall. There are hand drums, tambourines and castanets, an abacus, carved geometric forms in halves, quarters and eighths, eleven paper plates with three mandarins each. A large blue fabric lies by the window. The whiteboard is draped with a twelve-segments parachute in red, green, yellow and blue.

The immediate effect is spontaneous interaction, the learners chatting about figures, shapes and colours. Digits on the floor are pointed at and commented, cited in mother tongues and explained between peers. Fragments of previous subject-related knowledge, anchored in everyday life, are blended and shared. Before the teacher formally has started the lesson, the interplay with the math subject is incited, associatively, activating sight and tactility, triggering speech.

An Ethno-Mathematic Introduction – Cultural and Linguistic Aspects of Math. As the talk about figures, forms and tools fills the room, the teacher asks questions, encouraging further dialogue. Starting from the learners' existing subject knowledge, keys are given to counting and numeric thinking. How the numbers on old clocks go from I to XII. How it connects to language, pupils with Romance mother tongues being asked to share how 100 and 1000 sound, and C for per cent, M for millennium become clear. How much easer writing 2016 or 1916 is compared to with Roman figures. How 2 and 3 come from the Arabs' ٢ and ٣, which they in turn borrowed from India. How *cipher* and *zero* come from Arabic **صفر** [sifr] for *nought, void* or *space-in-between*, translated from the Sanskrit *sunya* for *void* or *empty*, the Indians' flash of genius to define *nil*, symbolized by a dot ٠.

Featuring linguistic and cultural aspects of mathematics learning has proven to be advantageous [10]. Gathering and talking around numbers and numeral systems, ways of counting in different languages are purposeful steps in establishing the *space of learning*. Math is unveiled as the sum of contributions by many cultures through the ages, leaving traces in language. The body of peer learners display considerable amassed knowledge in mathematics, spurring mathematical-logical literacy – numeracy.

Mixed-language math teaching for new-arrived young immigrants is increasingly advocated as a means to overcome language thresholds and achieve faster integration in school work [11, 12, 13]. One math teacher instructs new-arrived Syrian teenagers in Swedish, assisted by an Arab-speaking colleague. Building on the learners' existing math skills, sentence-structures and word orders in the new language are discussed [14]. Dancing in a setting with multi-linguistic learners offers a relief from battling the language barrier, engaging in joint speechless dance expression, yet enhancing general literacy as well as specific subject vocabulary while discussing the aesthetic activities.

Dancing Math – A Three-Step Process

The structure of a *Dancing Math* class, in each stage relating to a specific math subject matter, is threefold: dancing, wording, formalizing.

Dancing and Optimizing the Space of Learning. A gentle dance warm-up with turns and arm swings unfolds, with repeats at accelerating speed and intensity of movement. A math term is introduced in the directions – *double*! Following new instructions, configurations emerge on the floor, one by one, in pairs, quartets, octets. Spatial changes in the room are sensed, the dancing is concentrated and cooperative, bodies move and find each other speechless. When all have joined into one long waving row, the movement never stalling, a new term is inserted – *half*! Following concise directives, the groups backtrack from thirty-two or sixteen to eight, four, two, and back to one by one. Everyone on their own again, the music shifts, all engage in energetic disco dancing, imitating each other's improvisational moves.

No link to mathematics is explicitely mentioned in this stage of the lesson. Dance is the starting point and primary concern, mode of expression and resource. While each dance in the program is carefully designed to deal with the learning of a specific notion or axiom in math, it is initially dealt with and instructed as just dance. The vocabulary used is dance-adequate: walk around helter-skelter, arms down, straight backs, switch direction, sharp angles, accentuate, turn, stretch, on the tip of the toes, swing, move fast, slow down, relax, run, dodge, skip, gallop, etc. All dances and steps in the program derive from creative dance with children and youths in its own right [15]. No previous dance skills are required more than the ability to run, skip, gallop, turn, roll, sneak on the toes. Children, pupils and pre-service teachers with impairments participate vividly, adapting and interpreting the moves in own fashions.

The group's proficiency in math is visualized in the swift doubling and halving by the dancing bodies. In front of the peers and the teacher, proof is given of embodied pre-knowledge, not the least to the learner personally, as the dancing speaks back to the learner and reveals a mastering of a notion such as *double*.

While dancing, the participants constitute and build on to the space of learning on different levels. Most obvious is the engagement of bodies in speechless interaction and intense presence and concentration, joyfully and imaginatively. The specific design of the dances allows visualization of proficiencies and conceptual awareness in counting, adding and subtracting, tables of two or three, symmetrical and geometrical formations, triangular dramas, all anchored in earlier knowledge based on everyday observations, experiences and mathematical concepts.

Reasoning, Wording and Constituting Math Knowledge. In the next step, taking a breather, while spirits are high and concentration tight, the dance done is talked through from a mere mathematic angle. The teacher prompts for descriptions of the movements in the room, encouraging general and dance literacy development, scaffolding with appropriate wording for both dance moves and math thinking while explaining "double".

Two ways of explaining use to unfold. Some may say: Take the same amount and put them together, or: One added to one and then two added to two, and so forth. Others may say: The same amount once again, or: Two times the same, or: Twice as much. Thus a mathematical thinking, first embedded, embodied and perceived in the dance is now formulated in proper math terms. Math reasoning is visualized and becomes meaningful first through action, then speech. Learning mathematics is also learning a language. Socially contextualized, this is about learning a new discourse. Everyday speech meets subject terminology, speech as action is used to constitute new meaning in a specific setting, here making sense of math in math class.

This step, as well as the next, enfolds on a detail level adjusted to the participants' age. For the youngest, discussions and conclusions are kept at an elementary level, though not avoiding the introduction of notions such as addition and subtraction, multiplication by two, even and odd numbers, the circle, radius, circumference, area. Notions such as π, isosceles and congruent triangles, addition with disposition, polygons and fractions may be reserved for higher grades.

Formalizing Math Knowledge. In the third step, the math term, function, concept or axiom, first expressed in movement and then in verbal reasoning, is concluded in formalized language, formulas, equations and axioms, written or drawn on the whiteboard. As the explanations of double are being formulated, the thinking is physically visualized in the room with appropriate numbers of people. The reasoning about double in the line of taking an equal number and putting them together, adding 1 to 1, giving the sum 2, is written on the board, step by step, in formal math language: $1 + 1 = 2$, and properly worded as 1 added to 1 equals 2. The second line of explaining double, twice the number, leading to multiplying 1 by 2, giving the product 2, is written on the board as $1 \cdot 2 = 2$, in proper words: 1 multiplied by 2 equals 2.

The notion *half* is dealt with in the same manner, starting from thirty-two or sixteen. Some will say: Divide equally or: Divide into halves, some: Divide in two or Split in the middle. Everything is set up with pupils on the floor again, going from 32 or 16 to the quotes 8, 4, 2 and 1, written as $32 : 2 = 16$, $16 : 2 = 8$, $8 : 2 = 4$, $4 : 2 = 2$, $2 : 2 = 1$, properly articulated as 32 divided by 2 equals 16, etc.

Arrived back at 1, single pupils standing around, further halving is discussed, whether saying half of 1 makes sense. In many languages, e.g. English, this is easily worded, cutting an apple in halves – but a friend... For now, it is agreed upon to stick to whole numbers and whole friends. Setting up different numbers of peers, possible outcomes of halving 2, 3, 4, 5, etc. are checked.

Having established double and half, addition of the same number and multiplying by two, the next dances incorporate even and uneven numbers, the two-times table, the three-times table, multiplication by eleven, and by zero. Stomping, the two- and three-times tables get into the feet, and the other multiplication tables join in as the beat goes on, new rhythms going into feet and bodies. The learners have the tables at their feet, new idiomatic expressions are introduced, multiplication is no longer at sixes and sevens, math, dance and language are in full swing. Literacy development is boosted, everyone is on the up and up, in general as well as in the process of learning [16].

The tables, equations or axioms landing on the whiteboard, this phase concludes the process of learning, tying together knowledge expressed in dance, speech, writing and drawing. Conceptual change or readjustment is included in the dynamics, anchored in aesthetic expression, the embodied knowledge shared, the motor and speech dialogue, and the confirmation of accurately formulated mathematical axioms.

Multiple Semiotics in The Body's Dialogue of Knowledge and Meaning

A group of ten learners do the pals-of-ten dance, skipping back and forth, then verbally define the actions of addition or subtraction, and finally formalize the arithmetical operations on the whiteboard: 5 + 5 = 10, 3 + 7 = 10, 7 + 3 = 10, 10 + 0 = 10, 10 - 4 = 6, etc.

The interaction here discussed ensues on various levels: Between individual bodies within the group, through the dance expression, in relation to the physical space and the floor, in relation to the mathematical notions and operations in their abstractedness and how these are physically and dynamically concretized and then talked about, in the cognitive activity, in the interplay between teacher and learners, between learners, and between the teacher, the learners and the math subject matter learned.

Philosophically and epistemologically this touches Plato's and Aristotle's discussion whether math is a matter of ideas or reality. The frame of this paper does not allow for exhaustive comments on on-going discourses in the matter. One short epistemological answer to the antique question may be: Learning math is all happening within and between the learner's bodies. Plato sees math as *epistemic*, universal and absolute knowledge which we may reach through reasoning. Aristotle's responds in terms of all things being sentiently perceivable through concrete manifestations, representations, *mimesis*. If dancing in *Dancing Math* brings the learner closer to understanding the arithmetical operation, the question is how this unrolls.

Taking a huge leap forward in knowledge history, abandoning the idea of body and soul divided or reason separated from the senses, Merleau-Ponty [17] states we can only sense what can be observed. Ricœur [18] adds that we can only understand the self through signs, gestures, symbols and texts; how we interpret these signs is how we understand ourselves. Deleuze & Guattari [19], see the body as an interactive locus producing and displaying signs, meaning, desire and action, in constant adaption to contextual configurations. Goodwin [20] approaches human interaction taking account of the simultaneous use of multiple semiotic resources in speech, body action and gesture, sequential organization and graphic fields. He argues that the production and interpretation of social action evolves through a simultaneous deployment of a range of these different kinds of semiotic resources. Jointly engaging in the activities in the *Dancing Math* program, a multiplicity of semiotics is produced, the learner entering yet unknown patterns, being urged to restrain from spoken communication while dancing and find meaning through other modalities.

The dancing bodies become loci for production and projection of thought, sign and gesture in joint spatial and motoric activity, looking for meaning in interaction. This view on embodiment of semiotics falls into Stevens' [21] interactionist category, differing from the static conceptualist model, where phenomenological, unconscious primitives are seen as common internal concepts, developed in shared lived physical experience. Merleau-Ponty holds that the experiencing and handling subject is the own lived body short and sweet – we perceive the world in our bodies' dialogue with the experienced, amassing knowledge, in the body. The dynamic is explicitly dialogic, between the learner and that which is learned, through the body. The dancing and learning subject, bearer of life experience and previous subject knowledge, explores together with other bodies through action new knowledge and new perspectives. Merleau-Ponty sees the human as seeker of meaning, and the medium is the body. Body and mind are significances carrying meaning only in relation to a consciousness [22].

Kress [23] advocates semiotic diversity in learning, for the shaping of knowledge and in order to attend to and communicate about the world and our place in it. Semiotic resources are socially made, and no degree of power can act against the socially transformative force of interaction. The study of multimodality focuses on the bodilyness of those who make and remake signs in constant semiotic interaction, representing a move away from high abstraction to the specific, the material; from the mentalist to the bodily.

Dancing as mediating expression in mathematics learning offers a dialogical dynamic in the classroom between the familiar and experienced, through the body, undivided, one with reason, sensitivity and consciousness, interpreting and acquiring new subject knowledge, through symbols and gestures, increasing self-understanding and the awareness of an own place in the surrounding world.

Dancing Math Lesson Observation Data

Research in the intersection of art and science, dance and pedagogy, is also a search for an own ontology, epistemology and methodology. The theoretical background described above is one possible approach. Comprehensive studies need to be carried through and even a proper language needs to be developed, apt to capture the nature of the mental and cognitive, motor and sensuous processes evolving between individuals engaging in the aesthetic expression of dance, in educational as in other settings.

Concluding this presentation, empirical data are reviewed mapping curriculum content and the nature of classroom interactions in *Dancing Math* classes.

Observation Sheets. The data collected consist of over three hundred lesson observations. The focus of the observations was on the learners' activities and teacher performances during the lessons and on actual knowledge goals, articulated in the Swedish National curricula and mathematics syllabi. The sheets filled out are moulded on observation templates used by the Swedish School Inspection, adapted to *Dancing Math* classes and the *Space of learning* concept described above.

Data were gathered in a total of thirteen 90-minutes *Dancing Math* lessons, from a total body of 319 pre-service teachers participating in classes between 2014 and 2016. The participants study to be teachers in pre-school class through grade 3 (6-10 years) and 4-6 (11-13 years). The students attend a 3,5-year full-time education. Age varying between early 20's and late 50's, eager for and scrutinizing new teaching and learning practices. Proficiencies in mathematics vary from weak to sound.

Collected Observation Data.

Teaching patterns, ranging from learner-active whole-class teaching, learner-passive whole-class teaching, teacher-active or teacher-passive tutoring individually, two-by-two or in group.

Teaching media used, such as textbooks, aesthetic expression, visual art or other, audio-visual material, media designed by the teacher, artefacts and realia.

Time disposition of a 90-minutes' class, maintaining order, organisation, effective teaching time and comprehension follow-up.

Curriculum content covered, including focus on intellectual, practical, sentient and aesthetical aspects, the use of various modes of expressions of knowledge including rhythm, dance and creativity, incitement to critical thinking, preconditions to develop interest in mathematics and confidence in the ability to use mathematics in various contexts, opportunities to experience aesthetical values in mathematical patterns, forms and connections, preconditions to develop familiarity with basic mathematical concepts and modes of expression and their applicability.

Subject syllabus content dealt with, e.g. natural numbers and their properties, how numbers can be divided, how they can be used to specify quantities and order, how the positioning system is used to describe natural numbers, symbols for numbers and the historical development of symbols in cultures through history, parts of a whole and parts of a number, how parts are named and expressed as simple fractions, how simple fractions relate to natural numbers, properties of the four operations, the importance of the equals sign, basic geometric objects and their properties, including points, lines, quadrilaterals, triangles, circles, their relationships, various proportional relationships, including doubling and halving.

Teaching qualities, relating to degrees of secure, supportive and encouraging learning environment, degrees of clarity of goals and content of the lesson, degrees of individual adaptation, support and challenges and degrees of feedback, provided conditions for pupils' initiatives and peer interaction.

Free comments.

Findings. In teaching patterns, learner active whole class was observed as predominating, along with a high degree of teacher active and some degree of teacher passive tutoring in individual, pair and group work. None of the respondents observed learner-passive whole-class teaching.

A wide diversity of teaching media was utilised, no text material, primarily aesthetic (dance) expression, streamed and instrumental music, props such as coated figures, fabrics, garments, flags. Assessed time disposition averaged 78 min. for effective teaching time on a total of 90 min.'s lessons.

All of the curriculum and subject syllabus content listed in the observation sheets was marked to have been dealt with. Emphatically marked were the focus on intellectual, practical, sentient and aesthetical aspects, the prompting to critical thinking and the opportunities to develop math proficiencies in physical contexts and to discuss aesthetic expression with scientific content.

In teaching qualities, embracing and encouraging pupils' actions, initiatives and knowledge contributions was highly valued, as was the structure of the lesson and the logical flow of the dance activities, the discussions and the knowledge goals introduced.

Free comments stated the lessons as rewarding, fun, getting out a great deal, easier to understand difficult math concepts, learning with the whole body, offering lots of proficiencies to imply with the kids and pupils. New knowledge was stated to be likely to stick and last, the program was seen as a positive way of learning in stead of just books, aesthetics in learning was said to give great input, explaining abstract issues in a concrete way, the activities highly applicable in daily pedagogical work with children and pupils.

Conclusions. The observations indicate evidence of concentrated joint engagement in dance as aesthetic expression of subject matter, verbal interpretation and explanation of math concepts. Group dynamics are boosted in corporal interaction and overall intensive class activity while covering a comprehensive range of learning goals stated in the curriculum and syllabus. Mathematical codes are broken while dancing and discussing the dancing, involving neither textbooks or ex cathedra teaching.

Implications for Further Studies. The empirical data consist of observation data from fully three hundred lessons. However convincing the participants' response to the agency of dance as modality in teaching and learning math, the importance to be attached to the results can only be limited. Hopefully the concept may contribute to a deeper understanding of the essence and potential of the aesthetic expression of dance in teaching and learning subject matter.

The observations focused on the aesthetic and corporal activity, classroom interaction and the dealing with math subject matter. The framework of this article is to present a math teaching and learning mode starting from dance as aesthetic expression, knowledge embodiment and dialogue, and as agent in classroom interaction teacher-learner, peer to peer learner and the interplay with the subject matter learned.

Additional examination is needed of the nature of these interactions and on the essence and particularity of dance as aesthetic expression in math related matter. The issues may be examined in terms of creativity and imagination in learning, representation and perception, cognitive activity respectively socio-cultural learning, or aesthetic experiencing in knowledge shaping and meaning making.

In order to understand the full potential of the interplay of aesthetic expression and education generally and teaching and learning specifically, the perspective also needs to shift to studying dance as a field of knowledge and a literacy field in its own right. Further investigation of aesthetic aspects in

mathematics is another prompting area of inquiry – and thus of the potentials of the interplay of dance and math literacies.

Other issues for continued research are how aesthetically informed modes of knowledge shaping and meaning making interact holistically in curriculum subject matter learning, how social and cultural dimensions constitute literacies involved and how aesthetics and ethics inform mathematics learning.

In a bird's-eye view, further attention is needed on how academic and artistic research may cross-fertilise each other, and how practical artistic and pedagogical knowledge and theory can inform one another.

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