Shape-Changing Polyhedra

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Abstract

Shape-changing polyhedra are defined as three-dimensional polygonal and modular structures that interconnect and change shape and size. Applications include art, architecture, and engineering. Shape-changing polyhedra are composed of flexibly connected polygons. In their modular forms they can interconnect, one with another, such that when one shape-changes then all shape-change. Shape-changing polyhedra can be macro, micro, or nano sized. There may be thousands of interesting shape-changing polyhedra to discover.

Introduction

Historically the domain of three-dimensional polyhedra has mostly been explored with the concept of fixed connections, or with two dimensional fixed sets of fold lines (e.g. Origami), or with flexibly connected polyhedrons (e.g. where six might fold to compose a cube). This is true of such things as Platonic and Archimedean solids, Fig 1; Polygonal tessellations, Fig 2; of early Islamic muqarnas, Fig 3; the Geodesic domes of Buckminster Fuller, Fig 4; Origami, Fig 5; and Flexagons, Fig 6. This paper introduces a new and systematic means to explore the world of, "shape-changing," polyhedra starting with individual polygons that are flexibly connected according to various rule sets.

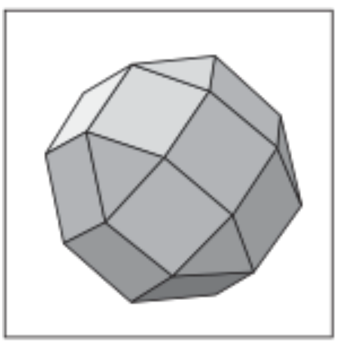

Figure 1: *Archimedean Solid*

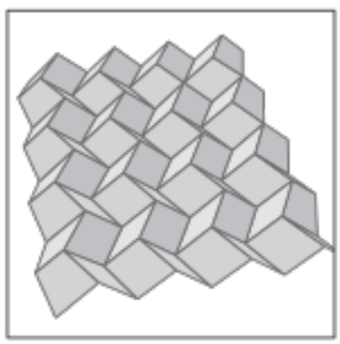

Figure 2: *2D/3D Tessellation*

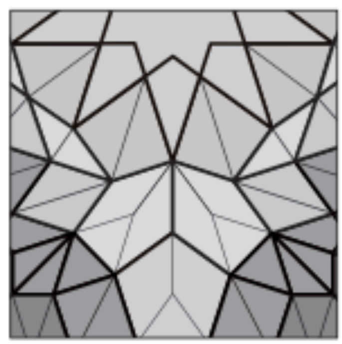

Figure 3: *Islamic Muqarnas*

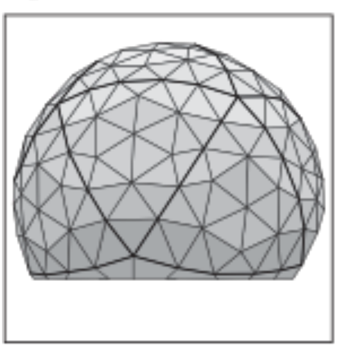

Figure 4: *Geodesic*

Figure 5: *Origami*

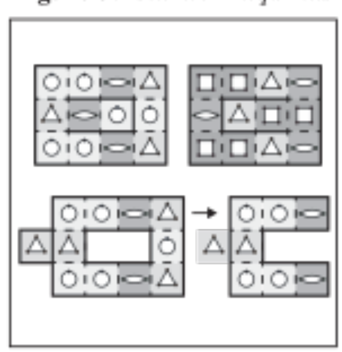

Figure 6: *Flexagon*

Current Initiatives

There are currently many initiatives that involve shape-changing geometries. For example, MIT's robots that transform from 2D to 3D, using Origami like polygonal folds, from two dimensions into three dimensions using electro-active polymers – where the robots walk away after assuming a 3D form, Fig 8. There's also the Barcelona Institute for Advanced Architecture of Catalonia, Spain, transforming 2D to 3D polygonal furniture that folds-up from a 2D tessellating hexagonal subdivision into various three dimensional structures Fig 9. The company Festo AG, in Germany, has developed flying and undersea craft driven by electro-actively connected ribs within helium and air filled polymer body envelopes, Fig 7 - and NASA is developing something similar using polymer-coated, electro-actively connected, and hydraulically-connected, wing ribs that will allow a wing to change-shape, twist, ripple, and even split, without any externally visible mechanical components. There are also initiatives with hydraulic powered buildings that will change shape to optimize their solar profiles and even a wind powered architectural concept where floor segments rotate to generate electricity.

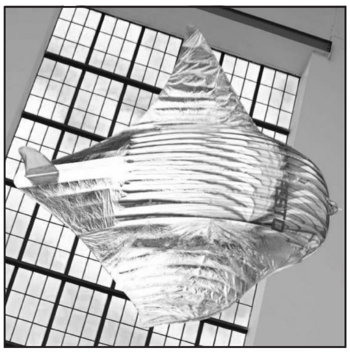

Figure 7: *Festo 'Air Ray' Photo © Festo AG & Co.*

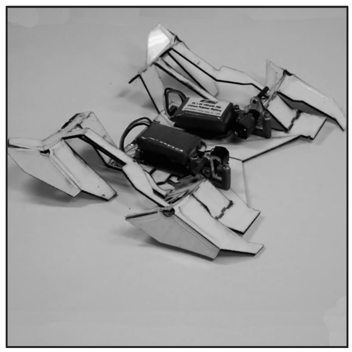

Figure 8: *MIT 2D/3D Robot Photo © WYSS Institute*

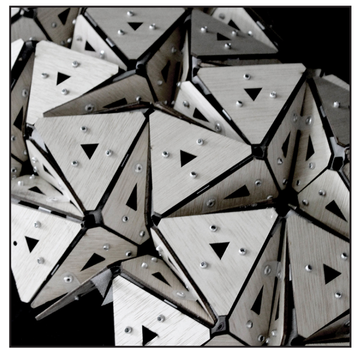

Figure 9: *BIAA Furniture Photo © BIAA*

Shape-Changing Polyhedra

Shape-changing polyhedra, as with polyhedral solids, are composed of polygons with the difference that polygonal edges are flexibly connected one with another. There are many ways to flexibly connect the edges of polygons. Examples are sticky tape, electroactive plastics, plastic binder clips connected with elastic bands, strips of silicone that can be bolted/riveted/bonded to edges, and two-joint hinges. Polygons can be made out of card, plastic, or metal. For experimentation thin (1mm) but fairly rigid plastic sheets can easily be taped and re-taped. Two-joint clip-on hinges, and polygons, can be 3D-printed and shaped to accommodate each other, or polygons can be 3D printed with clip-fit hinges on their edges. (Note A: Hinges need to be spaced to accommodate the thickness of the polygons).

Polygons can be combined in an indefinite number of ways making a logical combination system necessary. We therefore begin with a rule set that can evolve as new possibilities reveal themselves. 1. Start with a minimum number of polygons that will create a 3D tessellating module (see Note B). Call this minimum configuration a "Core." 2. Avoid triangulation where three polygons meet at a vertex (corner) to create a rigid geometry. 3. Establish and maintain symmetry. (Note B: where the term "tessellating module" is used in this paper as a 3D modular structure that will infinitely combine in a repeating pattern across one or more geometric planes).

A first example is that of a Core consisting of four 45º rhombus (R) connected edge to edge rotationally around a common 45º vertex. Call this "Core 1." Core 1 can be tessellated along the 'x' and 'y' axis, see Fig 10. (Note C: The term 'extension' is used when polygons are added within a core where the term 'combined' is used when polygons are added external to a core.)

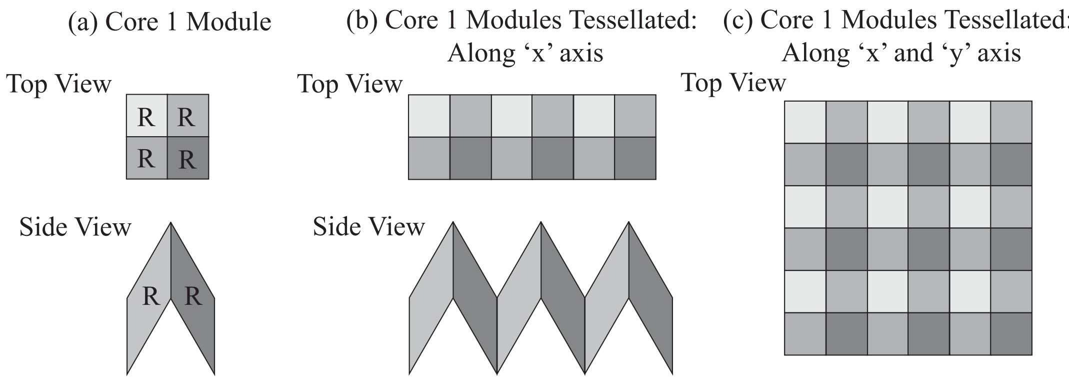

Figure 10: *Core 1 Module (a) and Tessellations (b) and (c)*

Core 1 can be developed further by combining it with squares (Sq.) on the open 'x' axis edges and then the combination can be tessellated along the 'x' axis, see Fig 11.

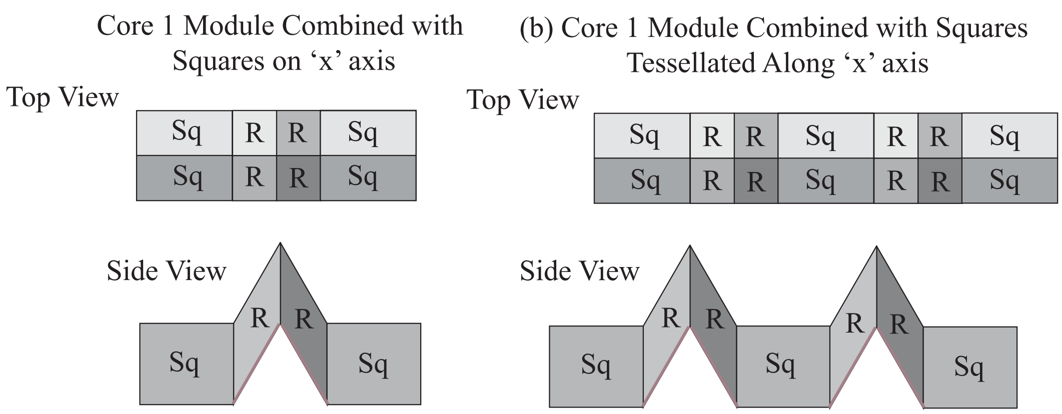

Figure 11: *Core 1 Combined with Squares (a) and Tessellated (b)*

Core 1 can be further developed by extending it with squares, Fig 12, and then combining it with squares along the 'x' and 'y' axis, Fig 13. The Core 1 development of Fig 13 can then be combined with its symmetrical opposite along the 'z' axis creating a closed-shell shape-changing polyhedra, see Fig 14.

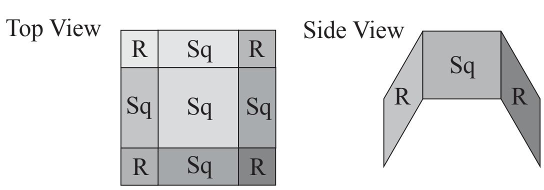

Figure 12: *Core 1 Module Extended with Squares*

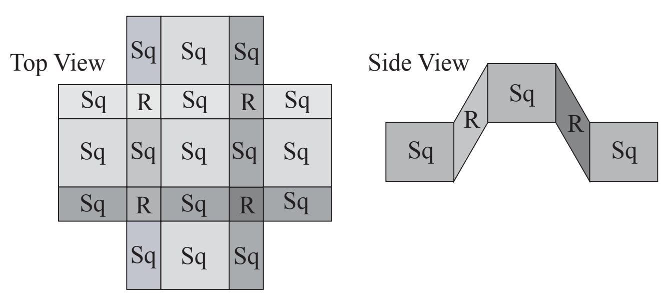

Figure 13: *Core 1 Module Extended and Combined with Squares*

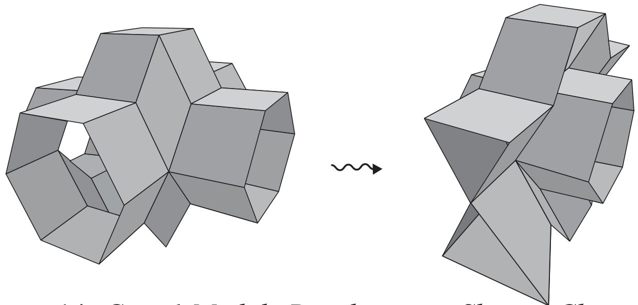

Figure 14: *Core 1 Module Development: Shape - Changing Module*

You will notice with Core 1 and its developments that the total angles of the polygons, meeting at a vertex, range from 180º to over 360º. (Note D: The wavy arrows in the drawings indicate shape-change).

Shape-Changing Polyhedra – Equilibrium Positions

The Core 1 development, Fig 14, as well as the other Core 1 developments shown, will, "shape-change." A common characteristic of many shape-changing polyhedra is that they shape-change from one "equilibrium" position to another. The Core 1 development shown in Fig 14 has two sets of six positions of equilibrium (6 along the 'x' axis plus 6 along the 'y' axis) where Fig 15 shows transitions from one position of stability to another. (Note E: The term "equilibrium" describes positions of stability).

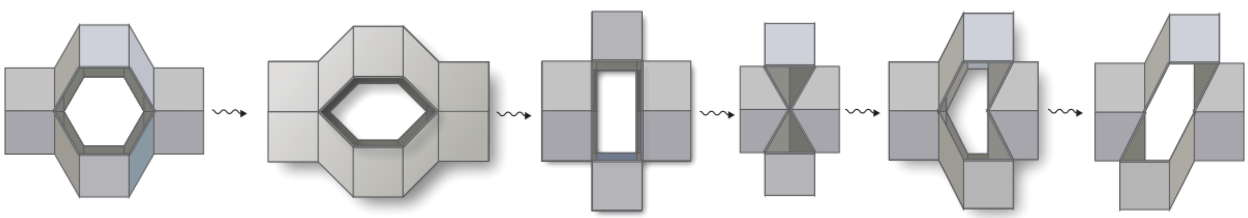

Figure 15: *Equilibrium Positions of the Fig 13 Core 1 Module Development*

Shape-Changing Polyhedra Combinations and Extensions

The Core modules of Shape-changing polyhedra can be extended, combined, and tessellated, in a multiplicity of ways. Tessellations can be stacked or nested in three-dimensional space. Tessellations will often retain shape-changing characteristics so that, theoretically, if one shape-changing module is shape-changed then it will immediately shape-change all connected modules – although in actuality stresses and strains might necessitate driving the shape-change at multiple points within a connected arrangement. Some modules will rotationally combine without creating a triangulation so that combinations will still shape-change. Some shape-changing Cores can be extended out from a centralized point or line of symmetry. Extended or combined modules will not always tessellate, for example, Cores composed of pentagons and 36º rhombi. Fig 16 shows tessellated and stacked combinations of the Fig 14 module where stacking and tessellations can follow multiple planes. Fig 17 shows how internal extensions can be added to the same module without altering the shape-changing characteristics of the combination (note the internal structure in the upper hexagonal cell). Fig 18 shows a tessellation of the Fig 14 module along both 'x' and, 'y' axis. Fig 19 shows how the Fig 14 module can be stacked, rotated, and combined. In the case of Fig 19 the combination of the 30º angled stack with the vertical stack creates an opposition to the direction of the shape-changing motion – so the overall arrangement becomes fixed.

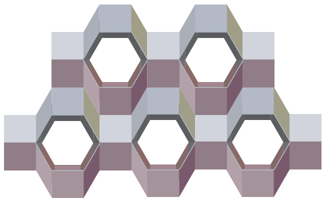

Figure 16: *Core 1 (Fig 13) Development Combinations*

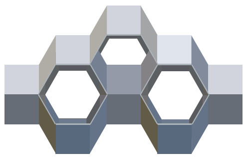

Figure 17: *Core 1 (Fig 13) Development Internal Extensions and Combinations*

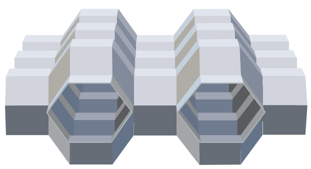

Figure 18: *Core 1 (Fig 13) Development Tessellating 'x' and 'y' axis*

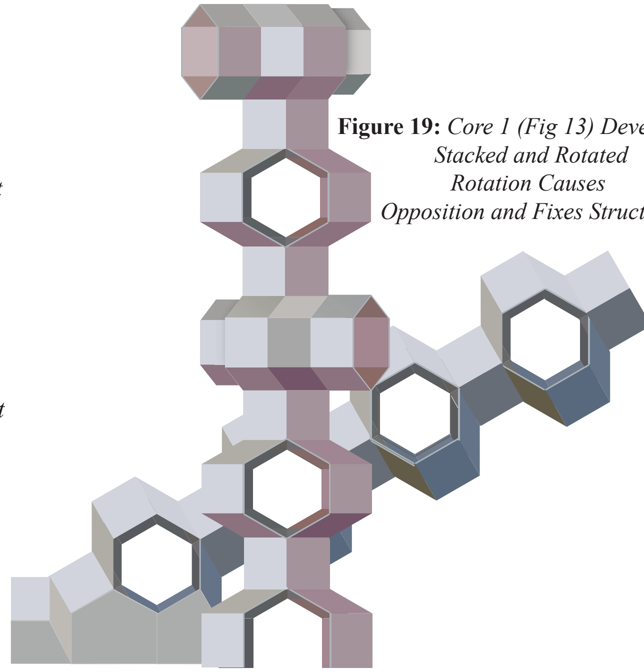

Figure 19: *Core 1 (Fig 13) Development Stacked and Rotated Rotation Causes Opposition and Fixes Structure*

Shape-Changing Polyhedra Core Development Logic

Establishing a "Core" basis for combining polygons is useful in that it allows a step-by-step means of exploring combinations in 3D where the number of potential combinations can seem overwhelming. Cores can be combined and extended in a multiplicity of ways. Many Cores can be connected and combined in 3D and in some cases one module can be rotated about another and still retain "shape-change," characteristics. A second Core example, Core 2, combines the same polygons as Core 1, Fig 20.

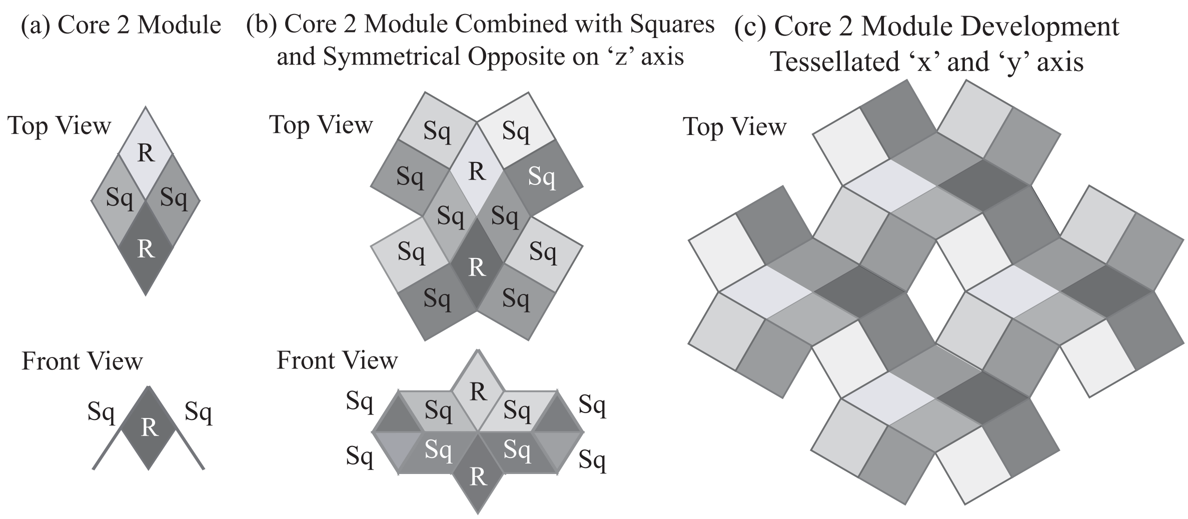

Figure 20: *Core 2 Module (a), Combined with Squares (b), Tessellated (c)*

Core 2 modules have just three "equilibrium" positions, Fig 21.

Figure 21: *Core 2 Equilibrium Positions - Top View*

Core 2 Modules can be combined in many ways including arrangements around a center point. Also, as with many other Cores and their developments, spaces between shape-changing polyhedra can be filled with other shape-changing or fixed polyhedra.

Examples of Other Cores

As a further example, a development of Core 3 combines the Core of four 60º rhombi (R) with an internal extension of eight 60º rhombi (R), Fig 22, creating a shape-changing module that can be tessellated. The extended Core 3 module has three equilibrium positions where two are flat (closed) and one is open, Fig 23. Core 3 can also be extended with squares as shown in Fig 23.

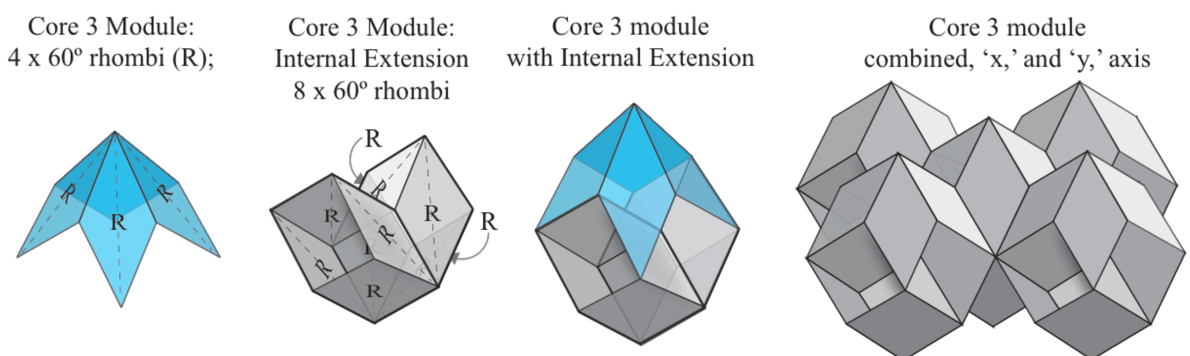

Figure 22: *Core 3 Module Developments*

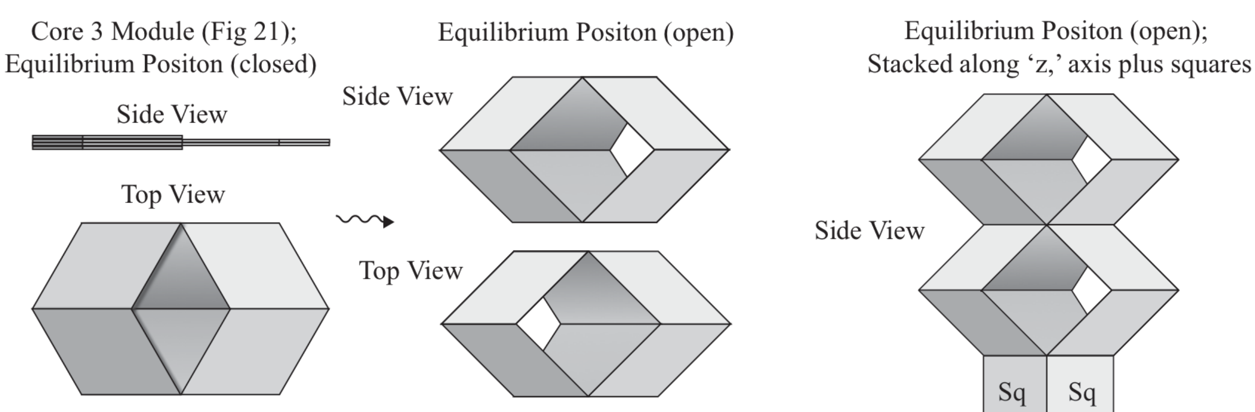

Figure 23: *Core 3 Equilibrium Positions and Extensions*

Core 4 has four equilateral triangles and five squares and the development has three equilibrium positions. Core 5 has six squares and the development has five equilibrium positions, Fig 25.

Core 4 Module (Eq. Triangle & Square); Extended and Combined, 'x,' 'y,' and 'z,'axis.

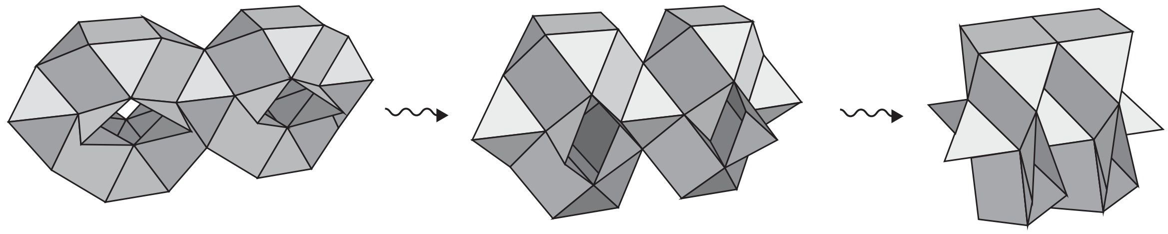

Figure 24: *Core 4 Development shows three equilibrium positions*

Core 5: Module (six squares): Development combines four cores (the dots define them).

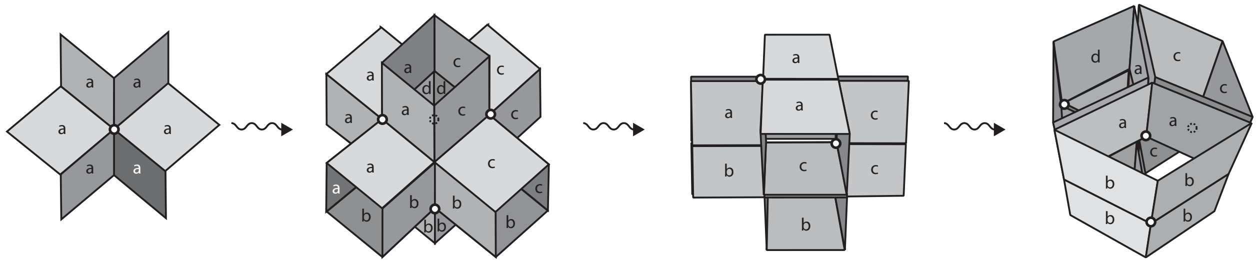

Figure 25: *Core 5 shows three of five equilibrium positions*

Examples of "Cores," in 2D and that fold-up into 3D configurations.

The logic of shape-changing polyhedra can be applied to 2D combinations of polygons that will shape-change into 3D even though this will overlap with more traditional, Origami like, methods of creating shape-changers. Core 1 2D consists of two 45º rhombi and two squares, Fig 26. Core 2 2D is the hexagonal tessellating core that was used to make the shape-changing furniture of the BIAA in Spain, Fig 27 and Fig 9. Core 3 2D is composed irregular polygons reflected along 1-axis of symmetry, Fig 28.

Core 1 2D: Module (45º rhombus + Square): Extension, 'x,' 'y,' axis; Folds-up from 2D into 3D

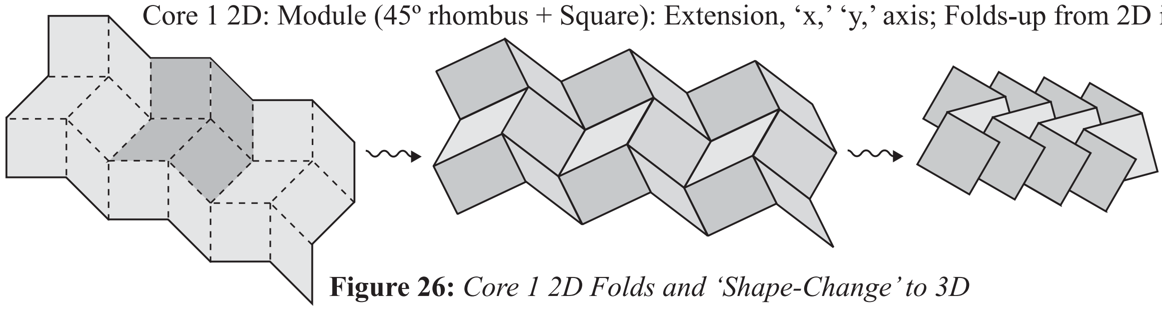

Figure 26: *Core 1 2D Folds and 'Shape-Change' to 3D*

Core 2 2D: Core folds up into 3D Core folds-up into 3D

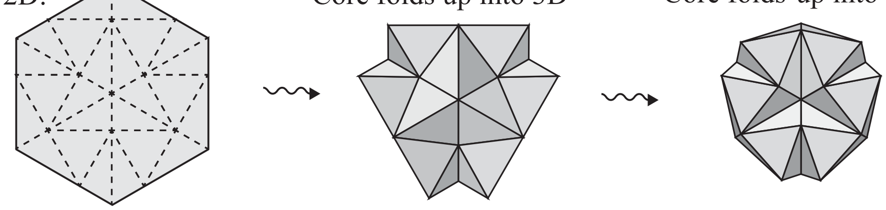

Figure 27: *Core 2 2D Fold Lines and 'Shape-Change' to 3D*

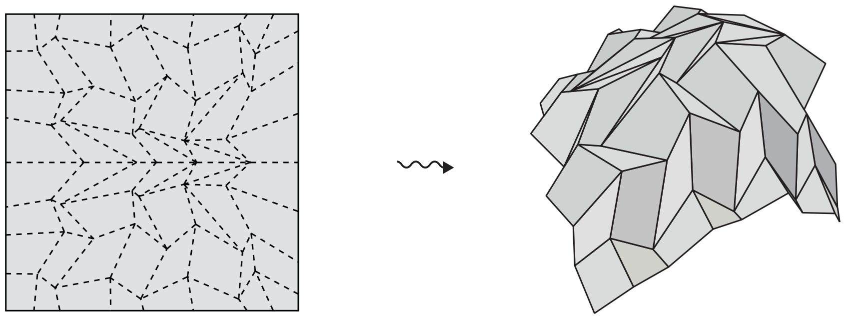

Figure 28: *Core 3 2D: 1-Axis of Symmetry - Irregular Polygons*

Core and Combination Starting Points

Any combination of polygons can be considered as starting points for Core development although using polygons that will tessellate in the more conventional manner on a two dimensional plane seems to be particularly fruitful as shown in Figs 10 to 27. For example one would expect that a Core with two hexagons, that are flexibly connected on one edge, with two 60º rhombi attached to two exposed edges on either side of the 'x' axis will create similar tessellating forms as the combined angles, when the Core is folded, will equal 180º on a flat plane.

Conclusion

This paper has been designed to introduce a means to explore the domain of shape-changing polyhedra. There may be thousands of interesting shape-changing polyhedra to discover – particularly when one considers that regular polygons, symmetrical polygons, and irregular polygons can be combined. Conceptually polygons can be macro, micro, or even nano-sized. Variations might include replacing polygons with pyramids, geodesic polygons, or any structure that has polygonal vertex points or outlines. I built my first shape-changers in 1972 as a result of wondering what 2D polygons would do in 3D space. Since then they have remained interesting but have mostly gathered dust until recent technological developments with for example, electro-formers, have made them seem ripe with potential applications.

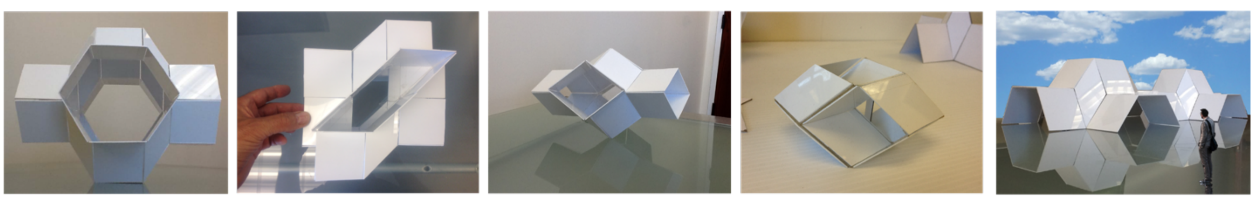

Figure 29: *Photos © Roger Burrows. Photos 1, 2,5 Core 1; Photo 3 Core 2; Photo 4 Core 3*

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Movie clips Cores 1, 2, 3 appear on www.rogerburrowsimages.com "Mathematics Meets Art Exhibition."

All line drawings in this paper are from "Think 3D," and © Roger Burrows, 2016.

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