Crystal Flowers in Halls of Mirrors: Mathematics Meets Art and Architecture

Kirsi Peltonen
Dept. of Mathematics and Systems Analysis, Aalto University
P.O.Box 11100, FI-00076 Aalto, Finland
kirsi.peltonen@aalto.fi

> To share in the delight and intellectual experience of mathematics—to fly where before we walked—that is the goal of mathematical education.
>
> *William Thurston*

Abstract

A transdisciplinary course, *Crystal Flowers in Halls of Mirrors: Mathematics meets Art and Architecture*, was established in 2013 at Aalto University to reveal contemporary mathematics to a diverse group of university students, from freshmen to graduate level. We especially wanted to address whether depth can be achieved at the same time as breadth in this context. After two successful implementations we have found new ways to increase interaction between mathematics and other fields.

Why Crystal Flowers?

There is both a need and an opportunity for students to engage with modern mathematics not contained in the current curriculum. This includes students in not only traditional schools of science and engineering, but also programs in arts and economics. To respond to these needs, we wanted to build a platform where not only students from diverse fields, but also teachers with different backgrounds, could share their ideas and views. Many recent achievements in mathematics have accessible layers, without heavy prerequisites, for which we wanted to find new space. As the skills of mathematics are of increasing importance to everybody, we wanted to create an open and encouraging learning environment for a broader audience than is addressed in conventional courses. Our goal was to share the beauty of mathematics that is obvious to researchers in the field by trying to find common language and places where it is possible to penetrate deeper. To show that mathematics is not only useful, but also fun, was one of our key motives.

First implementation 2013

The first implementation of Crystal Flowers saw daylight in spring 2013 after almost two years of planning and practical preparation. The crucial step came when a call for so-called Aalto courses was opened. Multidisciplinary courses open to all Aalto students were being sought. Martti Raevaara was a key person to provide us this opportunity. Funding for the course was provided in addition by Olavi Nevanlinna of the Aalto mathematics department. Together with Taneli Luotoniemi and Jouko Koskinen, we built up a semester-long 10-credit course comprising several components. Riikka Kangaslampi gave us valuable support and insight for implementing the learning environment. The contact meetings were held twice a week,

three hours each, in the inspiring atmosphere of the Aalto Design Factory [1]. Kalevi Ekman and his great staff and facilities provided important support.

The idea was to integrate traditional lectures and weekly exercise meetings into interactive gatherings that included group work. The assessments consisted of five parts: diary, weekly exercises, essay, project work, and group presentation. Weekly exercises had weight $1/3$, and the other parts each contributed $1/6$ to the final grading. The purpose of the diary was to include reflections about all meetings and events that took place during the course, so that the teachers had a possibility to get feedback and also to react in real time. Weekly exercises were designed so that teachers had the opportunity to give preparatory homework or tasks to be discussed after their lectures. Jouko Lehtomäki provided valuable mathematics support for exercises and practical matters. Essays were added to the program to give the students a possibility to deepen their skills in fields that were not handled during the lectures, but were somehow related. As peer-review was used to grade essays, this was also a potential channel to share knowledge between the students. Essays also provided a wonderful opportunity for teachers to learn from students coming from different fields. The last part of the course was reserved for group work, where student groups of four to six had the opportunity to implement an artwork reflecting the topics that were addressed in the course. This portion was also supported by designers Anssi Ahlgrén and Mikko Kauhanen from the Heureka science center. An exhibition of the project works was organized at the end of the course at the TUAS building in Otaniemi. During the building process, the workshop of the Aalto architecture department and its kind staff were utilized.

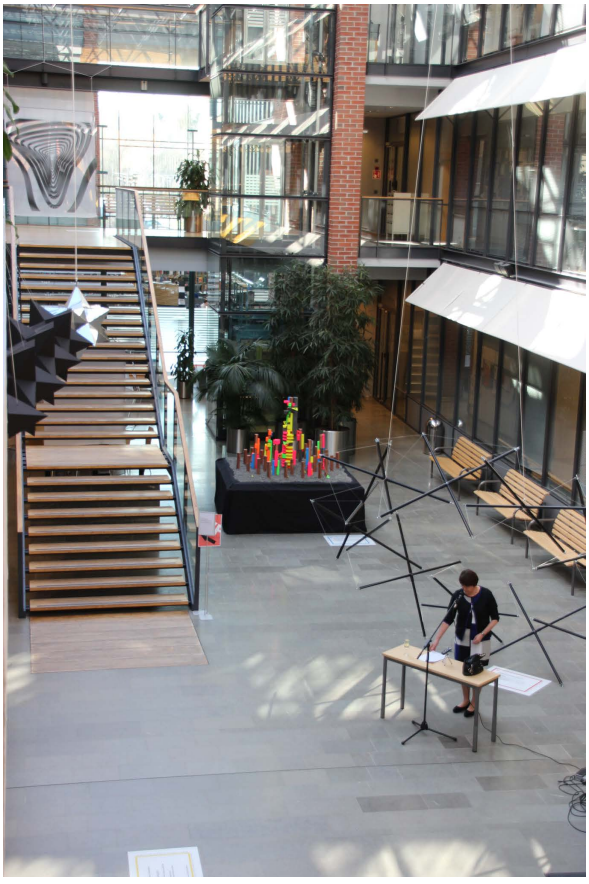

Figure 1: *President Tuula Teeri opening*

@Taina Hyppölä

Figure 2: *Exhibition poster: Taneli Luotoniemi*

@Eero Kaarlehto

The call for students was opened in fall 2012, and motivation letters were required of applicants. The original purpose was to take at most 24 participants, but as 36 students from all over Aalto sent convincing motivation letters, we were not able to select and took them all. In the end 31 students passed the course. We divided the students into diverse groups of six students at the very beginning to allow them to slowly get to know one another. Some small workshops were also organized along the way to make the treated material more concrete, and to help the groups find appropriate ways of working and learning together.

The topics of the course focused on low dimensional geometry and topology. Symmetries, tilings and their classifications provide a great source for mathematics that can be studied in many ways. I find Thurston-Conway orbifold notation to be a useful tool, applicable to broad audience. Analyzing tilings and trying to classify the symmetry motivates students to understand the need for proof. The pictures and explanations provided in [2] are a wonderful help for this. Through symmetries we then proceeded naturally to orbifolds and the classification of compact surfaces. Different models for geometry, polyhedra in low dimensions, and related structures provide a lot of interesting material where we used, for example, beautiful material from [10]. Fractals and chaos were also considered. There we used, for example, material from [8]. Outlines for the treated material were agreed upon by the teachers about half a year before the actual course started. Responsibilities of the individual contact meetings were also decided well in advance. In many places the same topics were treated a couple of times, but from different perspectives. This turned out to be a very fruitful technique for everybody, to learn new ways of thinking about even familiar objects. Although the program was essentially fixed beforehand, we wanted to be flexible with the students, and we did not hesitate to change our plans when needed.

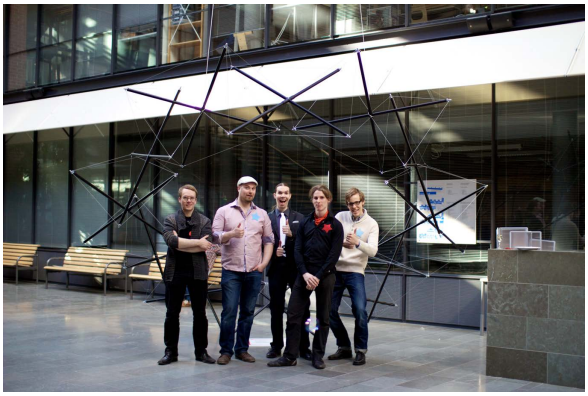

Figure 3: *Successful team and tensegrity*

@Taina Hyppölä

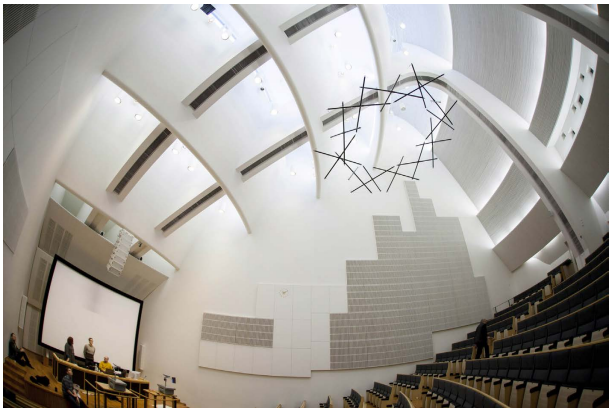

Figure 4: *B-hall Otakaari 1*

@Mikko Raskinen

Two visitors made the course implementation unforgettable. The first visitor was Jeff Weeks, who in addition to his beautiful lecture for a general audience, "The Shape of Space," [11] gave two wonderful hands-on learning sessions on orbifolds and the fourth dimension. His interactive tools like KaleidoPaint are incredibly helpful and fun in learning. For this and more recent applications see [12]. The second visitor was George Hart, who presented a talk titled, "From Mathematics to Sculpture" [4]. He also arranged a beautiful building session together with our students [5]. Figure 2 shows one of the sculptures just after the workshop. Both sculptures are still on display in the lobby of the computer science building in Otaniemi. The high quality of the student artworks surprised us all. Although we planned to have an exhibition lasting only one month, we can still find four of the students' artworks from 2013 on display in Otaniemi. A particularly beautiful example of this is the 'Innotorus' artwork of the student group Otso Helenius (ARTS), Jesper Jokilehto (ARTS), Eero Kaarlehto (ENG), Juha-Matti Lappalainen (ARTS), and Pekka Lehtelä (SCI). As they decided to come up with a sculpture of more than five meters diameter (Figure 3), it took us some time and effort to find a more permanent place for it. Art coordinator Tuula Isohanni described the hanging

process of this artwork as a small revolution. The ceiling of B-hall of the main building of Aalto University, designed by Alvar and Elissa Aalto, now serves as a perfect background for this piece (Figure 4).

Second implementation 2015

Aalto Science School support through Risto Nieminen and Juha Kinnunen made the second implementation of Crystal Flowers possible. After the first successful experiment, we wanted to keep the format and those elements that seemed to work best, but also bring new content and twist with elements existing in Aalto. Art pedagog Laura Isoniemi joined our team and brought new insight from the field of textile design. She organized interesting workshops on textile tying, dyeing and digital printing. These topics have natural connections to, for example, tilings and origami folding.

Figure 5: *Ice paper* @Robin Landsdorff

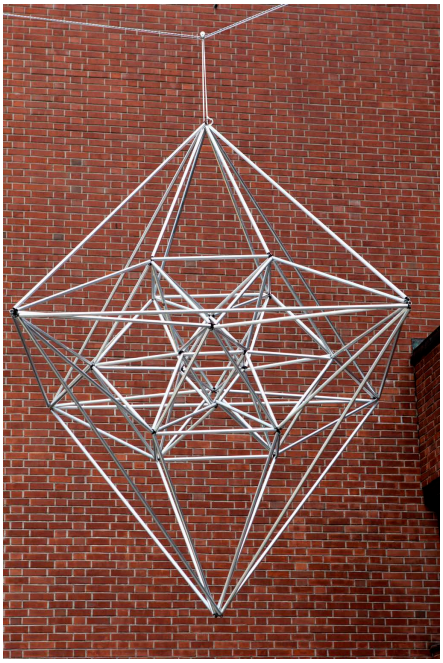

Figure 6: *Himmeli* @Adolfo Vera

Figure 7: *Paper Tessellation* @Adolfo Vera

As Kai Salmi from architecture and Marcelo Dias from physics came up with beautiful applications

related to origami, we decided to include more elements in this direction. Thomas Hull provides excellent material for exercises connecting mathematics and origami [6]. Katja Vuokko taught us basic principles in folding through her workshop, and Yves Klett gave a general audience talk "Technical Tessellations—Origami, Aerospace and Architecture" [7]. Yves also organized a workshop for students to introduce the engineering point of view to folding. Incredibly beautiful 'ice paper' was laser cut in Arabia workshop and folded by the students (Figure 5). Hannu Paajanen and Ville Arkonkoski provided us wonderful support for our experiments.

Urs B. Roth came up with his exceptional polyhedra and challenged our students to build them from plywood (Figure 10). His public lecture "Mathematics and Form: Recent Works of the Atelier für Konkrete Kunst" is available online [9]. Our second implementation attracted 50 students from all over Aalto and Helsinki University. Due to limited space we were forced to select 36 participants from as diverse fields as possible. Half of the students were from different areas and stages of art studies, and the other half from different engineering schools and the science school. Again the artworks produced by students amazed us all. Above, in Figure 6, we see a visualization of the 24-cell in dimension four. Sebastian Björkman (ARTS), Tristan Hamel (ARTS), Zsuzsanna Horvath (ARTS), Kaisa Kangas (Helsinki University Mathematics), and Saara Louhensalo (SCI) built this beautiful 'Himmeli' from aluminium tubes and rope. The artwork is visible in the courtyard of the Aalto University main building. In Figure 7 we see the 'Paper Tessellation' of Crystal Bennes (ARTS), Taina Hyppölä (CHEM), Lauri Laatu (SCI), Linda Lazarov (ARTS), Janne Rinta-Mänty (ARTS), and Mona Taponen (ARTS). It is a sculpture comprising various lengths of paper hand-folded in the 'waterbomb' tessellation attached to a steel scaffolding. Each section of pre-folded paper is between two to three meters in length.

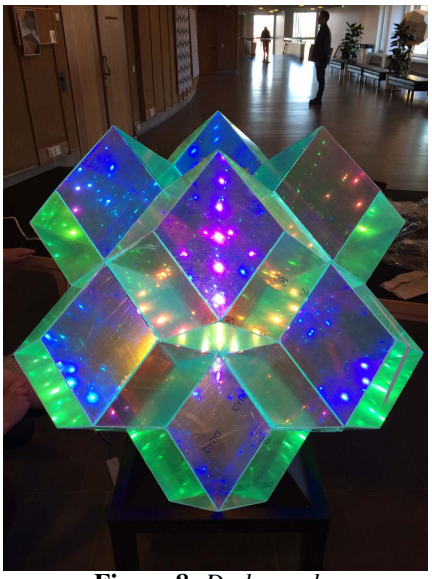

Figure 8: *Dodecacube*

@Meri Tuomela

Figure 9: *Clock*

@Adolfo Vera

The 'Rhombic Dodecacube' in Figure 8 was inspired by the structure of the space-filling rhombic dodecahedron. Beautiful acrylic and white LED lights were ingeniously combined to produce this most marvellous piece designed by Elias Axelsson (SCI), Hilla Fred (Helsinki University, Computer science), Henri Judin (ARTS), Viivi Livio (SCI), and Meri Tuomela (ARTS). In Figure 9 we see a 'Clock' designed by Liisi Huotari (ARTS), Satu Kruus (CHEM), Marloes Van Son (ARTS), and Siyan Zhuang (SCI). The artwork, which expresses time by using lights and shapes, was constructed from plywood, plexiglass, cardboard, LEDs, and Arduino. Figure 11 shows a beautiful ruled surface made of fishing line and metal rods

by Tomi Itäniemi(ARTS), Minna Kuusela (CHEM), Lauri Loiskekoski (SCI), Kai Salmi (ARTS), Johanna Strandman (ARTS), and Miikka Ullakko (ARTS) during the 2013 implementation. This artwork is still on display at the TUAS building in Otaniemi. Figure 12 shows an optical illusion created by the group Robin Landsdorff (ARTS), Rasmus Ruohola (SCI), Sara Saukkonen (ENG), and Riikka Schroderus (Helsinki University Mathematics) in 2015. From stainless steel and wood (aspen) they built a work consisting of two parts, which appear to form a continuous Möbius band when viewed from a certain viewpoint. This work is is visible in the courtyard of the main building of Aalto University. A video from the opening of the 2015 exhibition, and videos on student groups, can be found online [3].

Figure 10: *Some of the polyhedra from the workshop of Urs B. Roth.* @Adolfo Vera

Conclusion and Future Challenges

To successfully organize this type of a course requires extremely good teamwork among teachers. Not only must we agree on the topics to be treated, but we must ensure that the goals are truly common. Moreover, even the best plans do not guarantee great outcome unless students are properly taken into account. Collecting real-time feedback, discussions with teachers and again with students is important to clarify possible misunderstandings. Especially with students from diverse backgrounds, continuous support and reflection becomes ever more essential. Also grading becomes more challenging with heterogeneous groups. Clear and open principles that take the backgrounds of the students into account need to be agreed upon beforehand. The challenges of the group work are close to those in real-life working environments, where you cannot necessarily choose your team. This type of course provides a wonderful opportunity to learn to respect people coming from different fields. Mixed feelings, from fascination on different backgrounds to uncertainty on how to fit in, bring some extra flavor to difficulties in finding a common language and the challenge of how to agree on a joint project. From the plenitude of ideas and meandering views, it has been most interesting to see how mutual understanding can be attained to lead to joint intensive work with a beautiful outcome, and finally relief and even a hint of sadness when the work is over. To sustain this in the future

implementations, we need to direct our efforts to keep us and our students focused, find balance to support fruitful collaboration, and respond to individual interests. In the end we would like to find ourselves once again useless as teachers, with students activated to learn by themselves.

At the time of this writing, we are planing the next implementation for spring 2017. The plan is again to keep the parts that already work well, but bring some new insights from our past experience and contacts we have found along the way. As the next exhibition will be built for the Heureka science center, new levels are likely to be reached.

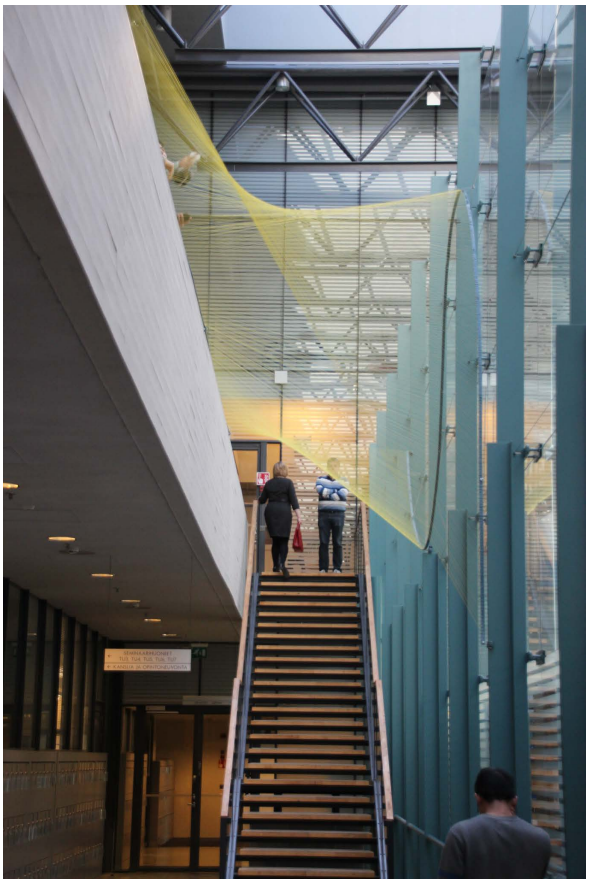

Figure 11: *Ventus Solaris*

@Eero Kaarlehto

Figure 12: *Möb&ius*

@Adolfo Vera

Acknowledgements

I am indebted to many people that have supported this project, but want especially thank Martti Raevaara for his support and continuous encouragement dating back to days when this project was just a dream. I am also grateful to Taneli Luotoniemi, Laura Isoniemi, Jouko Koskinen and our students for their shared experience and enthusiasm.

References

[1] Aalto Design Factory, http://designfactory.aalto.fi/.

[2] J. H. Conway, H. Burgiel and C. Goodman-Strauss, *Symmetries of Things*, Taylor & Francis, 2008.

[3] Crystal Flowers in Halls of Mirrors, http://sci.aalto.fi/en/about/scishowroom/crystalflowers/.

[4] G. Hart, "From Mathematics to Sculpture," https://www.youtube.com/watch?v=UA8oXGfNDEU.

[5] G. Hart, "Aalto," http://www.georgehart.com/Aalto/aalto.html.

[6] T. Hull, *Project Origami: Activities for Exploring Mathematics*, CRC Press, 2013.

[7] Yves Klett, "Technical Tessellations—Origami, Aerospace and Architecture," https://www.youtube.com/watch?v=xh6UNYjjjUA.

[8] D. Mumford, C. Series and D. Wright, *Indra's Pearls: The Vision of Felix Klein*, Cambridge University Press, 2002.

[9] Urs B. Roth, "Mathematics and Form: Recent Works of the Atelier für Konkrete Kunst," https://www.youtube.com/watch?v=XGnnC0IOzJ4.

[10] J. R. Weeks, *The Shape of Space*, Taylor & Francis, 2001.

[11] J. R. Weeks, "The Shape of Space," https://www.youtube.com/watch?v=j3BlLo1QfmU.

[12] J. R. Weeks, Topology and Geometry Software, http://geometrygames.org.