Highly Unlikely Triangles and Other Impossible Figures in Bead Weaving

Gwen L. Fisher
beAd Infinitum
Sunnyvale, CA, USA
E-mail: gwen@beadinfinitum.com

Abstract

The impossible triangle and other impossible figures are optical illusions that have inspired much artwork since their discovery by Oscar Reutersvärd in the 1930s. This paper shows how I use the impossible triangle and its variations to create a series of sculptures woven with seed beads and thread, using a bead weaving technique known as "cubic right angle weave" (CRAW). The flexibility of the beadwork eliminates the paradox of the optical illusion by substituting curves for the otherwise straight beams. The resulting "highly unlikely" beaded art objects generate surfaces that twist like Möbius bands around the objects, often making several distinct paths, which make interesting colorings possible. The many beaded examples include triangles, squares and frames. The beading techniques were then applied to tetrahedra and a dodecahedron that generate no corresponding optical illusion.

Impossible Triangles

The impossible triangle, also called the Penrose triangle, is a two-dimensional drawing that represents three straight beams with square cross sections, and the beams appear to meet at right angles. See Figure 1. It is the simplest example of an optical illusion called an *impossible figure* or a *paradox illusion* [13]. Seckel writes, "Impossible figures are two-dimensional line drawings which initially suggest the perception of a coherent three-dimensional object that is physically impossible to construct in a coherent way" [14]. Penrose and Penrose write, "Each individual part is acceptable as a representation of an object normally situated in three-dimensional space; and yet, owing to false connexions [sic] of the parts, acceptance of the whole figure on this basis leads to the illusory effect of an impossible structure" [12].

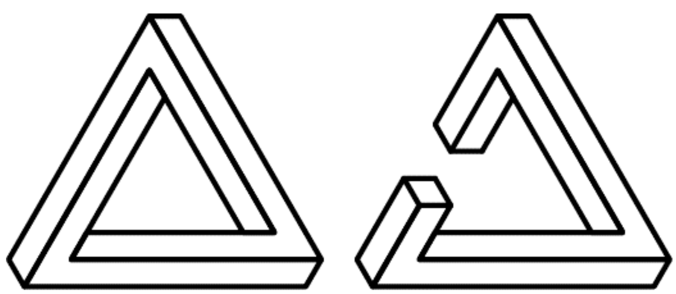

Figure 1: *The impossible triangle (left), a possible figure (right).*

The Swedish artist Oscar Reutersvärd drew the first impossible triangle in 1934. Since then, Reutersvärd has created literally hundreds of different impossible figures [13, 14]. The impossible triangle was independently devised and described by the mathematician Roger Penrose, who popularized it in the 1950s, most notably by showing it to the graphic designer, M.C. Escher, who, in turn, used it in his art. Escher drew impossible figures in 1954 [12], and his first major work exhibiting an impossible figure appears to be *Belvedere* in 1958 [13], followed by *Ascending and Descending* in 1960. Both of these lithographs incorporate impossible rectangles and cubes in the form of the impossible staircase, which was first drawn by Reutersvärd in 1937 and again by Roger Penrose's father, L. S. Penrose, in the 1950s

[12, 13]. It was not until 1961 that Escher employed the impossible triangle directly in his artwork when he created the lithograph *Waterfall* [3, 4, 5, 13]. Since then, many renowned artists have portrayed impossible figures in their art, including several sculptors. In particular, Mathieu Hamaekers built three-dimensional sculptures of impossible figures starting in 1985. From one specific viewing angle, Hamaekers' sculptures have the appearance of impossible figures made out of straight edges, but from any other angle, they reveal their true shape with curved beams. As early as 1997, Guido Moretti created cast bronze sculptures that transform from one line drawing of one impossible figure to a second as you move around the piece. See [13] for these and other examples.

In all of these incarnations, the impossible triangle appears to be a solid, three-dimensional object with straight sides (beams) and all right angles. (In the case of Hamaekers and Moretti, this view is accessible from just one or specific points of view.) An impossible triangle and other similar impossible figures are only impossible to construct in 3D if we assume the edges are straight and the connections are right angles. In contrast, there is nothing impossible about a two-dimensional drawing of an impossible triangle, and if you believe in Euclidean 3-manifolds, then you can make the topological equivalent of a "real" impossible triangle with them [8]. However, a true three-dimensional interpretation of an impossible object really is impossible. Hofstadter calls this "play[ing] with the level-distinction between *that which represents and that which is represented*. The 'artistic behavior' that results includes much self reference and many confusing and sometimes exhilarating paradoxical tangles" [11]. This paper shows a novel three-dimensional representation of impossible figures that eliminates this distinction and resolves the paradox in its own way. In particular, "that which represents" (i.e., a drawing) becomes a recipe for "that which is represented" (i.e., a beaded sculpture).

Beading the Highly Unlikely Triangle with Cubic Right Angle Weave

Figure 2 shows *highly unlikely triangles*, which are beaded versions of impossible triangles [6]. Because beadwork is flexible, a beaded version of an impossible triangle is quite possible, albeit, not very intuitive. Hence, I call it *highly unlikely*. To resolve the paradox of the impossible triangle, a highly unlikely triangle exhibits a quarter twist on each beam. The twist in the beadwork allows the impossible triangle to be constructed in 3D. The twist in the beaded version also destroys the optical illusion due to the curvature it introduces to the edges that appear straight in the 2D drawing.

Figure 2: *Highly unlikely triangles in three size.*

Constructing a quarter twist in each beam takes some thought. Figure 3 (left) shows my very first attempt at beading a highly unlikely triangle in 2006. Looking closely at the bottom beam, you will see that I accidentally omitted a quarter twist; the bottom beam is straight. The right photo in Figure 3 shows my second attempt, the first successful highly unlikely triangle. All three beads exhibit the desired quarter twist. I gave this piece to Roger Penrose at a math conference.

Figure 3: *Triangle with two quarter twists (left), highly unlikely triangle with three quarter twists (right).*

The conversion from an impossible triangle drawing to its beaded counterpart is straightforward. First imagine cutting the beams into rows of little cubes. In practice, one should use at least six cubes per beam to make enough length for the twist, but you can use as many as you want, and the beams can have different numbers of cubes from one another. The cubes identify short, black line segments that are all the same length (Figure 4, left). Place one bead on each line segment so that the hole of the bead is aligned with the line segment (Figure 4, right).

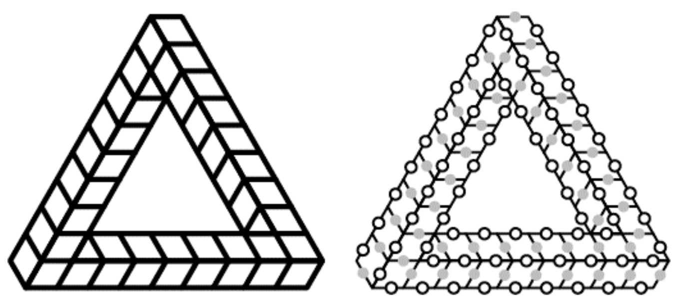

Figure 4: *Impossible triangle, line segments identified (left) and with beads (right).*

The beads are woven together in the arrangement shown in Figure 4 (right) with *beaded cubic right angle weave (CRAW)* (See Figure 5), a relatively new bead weaving stitch. David Chatt seems to be the first to have published instructions for a variant of CRAW in 2005 [9], but friends' family heirlooms suggest that CRAW predates Chatt's 2005 publication by at least a half century. Many tutorials exist for CRAW, both in print and on YouTube. For example, see [2] for how to weave a straight row of cubes (a beam); see [1] for how to turn corners; and see [7] for both of those, together with how to weave *embellishment* at the vertices. The embellishment consists of a single (smaller) bead added at each vertex after all of the beads on the line segments are woven together. All of the beadwork in the photos herein shows embellishment. For example, the small round beads in Figure 2 (center) are embellishment, and the longer tube beads are the beads on the line segments shown in Figure 4 (right). For complete instructions on how to weave a highly unlikely triangle, see [6].

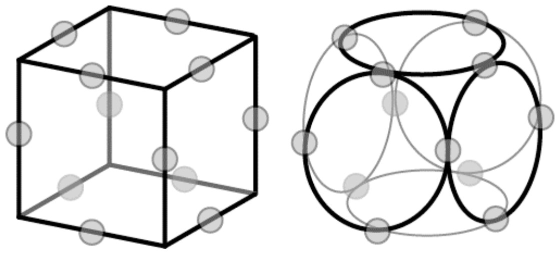

Figure 5: *A cube with one bead per line segment (left), one unit of beaded CRAW (right).*

To distinguish edges and faces of the impossible triangle depicted in Figure 4 (left), the corresponding beaded versions in Figure 4 (right) shows gray beads on the faces of the beads, and white beads along the edges (with the exception of the three gray beads at the extreme corners to keep the gray path continuous). The white path of the edges travels all the way around the triangle four times before coming back to where it started (See also, Figure 2, left). The gray path of the faces does the same. As a flat strip, the gray path forms a 3-loop Möbius strip [15].

Highly Unlikely Squares and Frames

A *highly unlikely polygon* is a beaded polygon whose edges are rectangular beams with a quarter twist. For a highly unlikely polygon with an odd number of beams (e.g., the triangle), there is a single path on the face that travels around the polygon four times. When the polygon is a square, the path on the faces separates into four, distinct paths that each travel all the way around the polygon only once. Similarly, there are four paths for the edges. These eight paths allow for interesting colorings like those shown in Figure 6.

Figure 6: *Highly unlikely squares (left and center), impossible square (right).*

The hexagon in Figure 7 has two paths on the faces that each travel around the hexagon twice.

Figure 7: *Highly unlikely hexagon (left), impossible hexagon (right).*

The *weird link* in Figure 8 has four beams, making it a quadrilateral, a kite, in particular. It has a quarter twist on the two short beams that twist in opposite directions.

Figure 8: *Weird link (left), associated impossible figure (right).*

The *weird square earrings* in Figure 9 (left, center) have twists on all four beams, but the twists do not all have the same directional rotation like they do on the highly unlikely polygons. The rotations go clockwise, clockwise, counter clockwise, counter clockwise. The coloring creates the effect of front, back, and "jam" in the middle, as in, jam is the layer between two slices of bread.

Figure 9: *Weird square Earrings (left, center), associated impossible figure.*

Two photos of the same *highly unlikely frame* are shown in Figure 10 (left and center). This object has half twists on the three long beams. By using half twists rather than quarter twists shown in Figure 10 (right), one can color the piece with a front, back, and jam.

Figure 10: *Highly unlikely frame with half twists (left, center), impossible frame with quarter twists (right).*

I was lured into beading the frame in Figure 11 by its complexity. This design is the compilation of six impossible quadrilaterals.

Figure 11: *Highly unlikely frame (left and center), the associated impossible figure (right).*

Unlikely Polyhedra

Armed with techniques for beading structures made with beams with twists, I applied these techniques to a couple of different polyhedra, described next. As far as I can tell, there are no optical illusion drawings (i.e., impossible figures) that correspond to these pieces.

Tetrahedra. One way to make an unlikely tetrahedron is to weave the frame of a tetrahedron with an unlikely triangle on each of its four faces. Before I built it the first one, shown in Figure 12, I had no idea if all of the twists would work together, and I was quite pleased to find that they did. All six twists have the same handedness. I used one color for the CRAW and one color for the embellishment, but it would be prettier (and trickier!) if I had colored the edge and face beads in different colors.

Figure 12: *Unlikely tetrahedron, one color.*

So the next one I beaded, shown in Figure 13, includes different colors for the edges and faces. To distinguish between the faces and edges, I needed to color the corner cubes differently from what I had done for the polygons. Fortunately, the faces still align to make obvious paths that weave around the sculpture. However, we lose the idea of simple edge path that continues around the sculpture because every inside (or outside) corner has three edges meeting there.

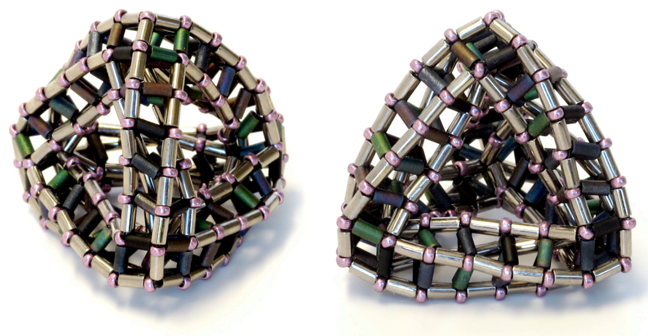

Figure 13: *Unlikely tetrahedron, edges and faces colored.*

The faces form three distinct paths that twist around the sculpture in unexpected ways. That led me to make the piece in Figure 14. Each beam includes all three colors of faces, with one color on two opposite faces. Like a Möbius band, as you follow a path around the piece, sometimes when you get back to a beam, you return to the opposite face. Thus, it feels like you have to travel around the sculpture twice just to get back to where you started. This is akin to what Hofstadter calls a *strange loop*. "The 'strange loop' phenomenon occurs whenever, by moving upwards (or downwards) through the levels of some hierarchical system, we unexpectedly find ourselves right back where we started" [10]. In the case of the tetrahedron, the faces of the beams correspond to the levels in the hierarchy.

Figure 14: *Highly unlikely tetrahedron, edges and three face-paths colored.*

Dodecahedron. The most recent piece this ongoing series is the unlikely dodecahedron with one unlikely pentagon on each face. Some day I hope to make one that shows the colors of the ten distinct paths.

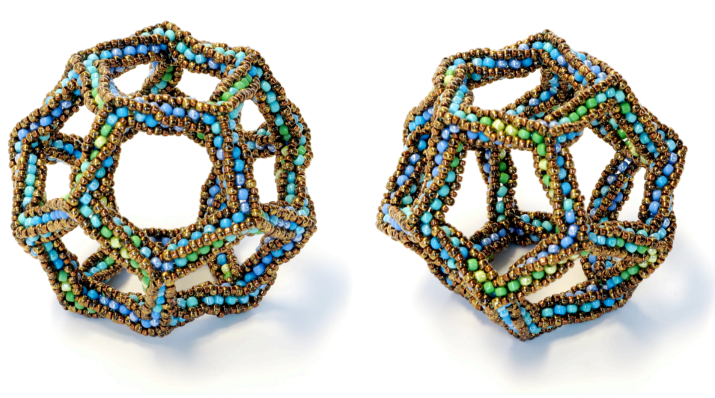

Figure 15: *Unlikely dodecahedron.*

Future Works

My series of beading unlikely objects is ongoing. Many more impossible figures are possible to create with bead weaving. In particular, nobody has ever beaded an impossible staircase. Also, other polyhedra in which every vertex has valence three (e.g., a cube) can be bead woven using the techniques described.

References

[1] H. Collin, *Cubic Right Angle Weave - how to create shapes*, YouTube video, https://www.youtube.com/watch?v=d9jE2dldiVo, Accessed April 22, 2015.

[2] H. Collin, *Cubic Right Angle Weave video Tutorial*, YouTube video, https://www.youtube.com/watch?v=P1Ib0qsQSy4, Accessed April 22, 2015.

[3] M. C. Escher, *Escher on Esher: Exploring the Infinite*, Harry N. Abrams, Inc., New York, pp. 78-79. 1986.

[4] M. C. Escher, *M. C. Escher: 29 Master Prints*, Harry N. Abrams, Inc., New York, p. 38. 1981.

[5] M. C. Escher & J.L.Locher, *The Infinite World of M.C. Escher*, Abradlae Press/Harry M. Abrams, Inc., New York, pp. 145. 1986.

[6] G. L. Fisher, *Highly Unlikely Triangles Beaded with Cubic Right Angle Weave*, Self Published on Etsy, https://www.etsy.com/listing/204753180/, pp. 2, 14. 2014.

[7] G. L. Fisher, *How to Weave Cubic Right Angle Weave with Beads*, YouTube video, https://www.youtube.com/watch?v=BSM9OiO4xjk, Accessed April 22, 2015.

[8] G. K. Francis, *A Topological Picture Book,* Springer-Verlag, New York, pp.68-75, 1987.

[9] V. Hector, *The Art of Beadwork*, Watson-Guptill Publications, pp 90-94, 2005.

[10] D. R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid,* Basic Books, Inc., New York, p. 10. 1979.

[11] D. R. Hofstadter, *Metamagical Themas: Questing for the Essence of Mind and Pattern,* Basic Books, Inc., New York, p. 18. 1985.

[12] L. S. Penrose and R. Penrose, "Impossible Objects: A Special Type of Visual Illusion," *British Journal of Psychology,* 49(1), pp. 31–33, February 1958.

[13] A. Seckel, *Masters of Deception*, Sterling Publishing Co., Ltd., New York, pp. 67-87, 90, 132-136, 179-184. 262. 2004.

[14] A. Seckel, *Optical Illusions: The Science of Visual Perception*, Firelfy Books Ltd., New York, pp. 309. 2006.

[15] Wikipedia, *Penrose Triangle*, http://en.wikipedia.org/wiki/Penrose_triangle, Accessed April 21, 2015.