The Concept of Elevation applied to Flat Tiling Patterns

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Abstract

In their book “La Divina Proportione”, Luca Pacioli and Leonardo da Vinci described and illustrated an operation which you can apply to a polyhedron, called Elevation. In this paper I want to show how you can make models of these elevations from simple elements, material that can be used for workshops. Luca Pacioli and Leonardo da Vinci applied the elevation operation only to polyhedra, but the concept can also be applied at 2D tiling patterns, resulting in interesting single and double weaving patterns.

1. Introduction

1.1. Elevation. In La Divina Proportione [1] by Luca Pacioli and Leonardo da Vinci, an interesting new concept that can be applied on polyhedra is introduced. The concept of Elevation that they introduced was also the subject of my paper for Bridges in 2014 [2], in which the difference between Elevation and Stellation was explained.

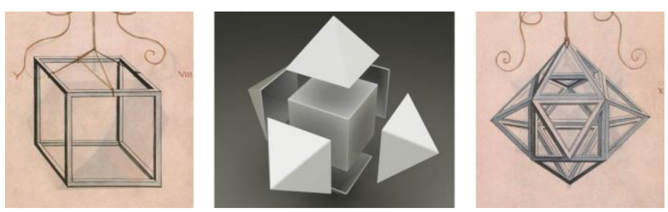
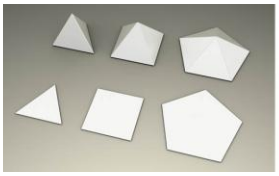


Figure 1: *Elevated cube*

Pacioli doesn’t give a real definition of “Elevation” but his descriptions are very clear: he describes the elevated version of the cube (Figure 1) as follows: “… it is enclosed by 24 triangular faces. This polyhedron is built out of 6 four-sided pyramids, together building the outside of the object as you can see it with your eyes. And there is also a cube inside, on which the pyramids are placed. But this cube can only be seen by imagination, because it is covered by the pyramids. The 6 square faces are the bottom faces of the 6 pyramids.” [3]. There is another way to describe the process: Elevation for polyhedra is the process of pulling each midpoint of all of the faces outwards until the triangles formed by those midpoints with two adjacent vertices of the original face form are equilateral. A generalization can be made by not demanding that the triangles must be equilateral.

1.2. Building models. The way Leonardo da Vinci made the drawings of the elevated polyhedra suggest that there were real hanging models. And indeed it is very inspiring to have real models of these constructions. Therefore I started to try to find an easy way to make paper models of the designs presented by Luca Pacioli and Leonardo da Vinci. The Elevation of a polyhedra is not just a collection of pyramids, as in Figure 2), but there is also something inside. It is in fact a double layer construction. I had to find a way to keep this visible. The solution for this was found in M.C. Escher's print "Gravity", a drawing of a stellated dodecahedron (Figure 3). So by opening up the pyramids I succeeded in developing simple basic elements (Figure 4) that could be made of paper, and still allowing a look at inside layer of the construction.



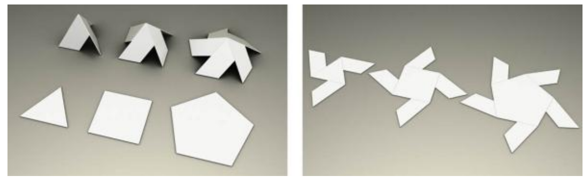


Figure 2: *Elevation of faces.* **Figure 3**: *M.C. Escher* **Figure 4**: *Developing the elements for the models.*

1.3. The Elevated Platonic Solids. To build the models of the Elevated Platonic Solids (Figure 5) we need only three different elements that have to be glued together. Because I was not happy with this solution, which especially at workshops caused a lot of trouble, I decided to develop the element in such a way that glue was no longer needed. This resulted in the element of Figure 6.

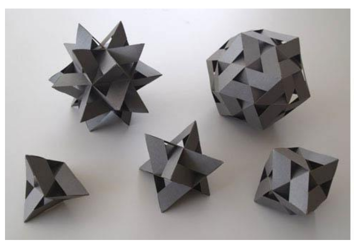



Figure 5: *Elevations of the Platonic solids.* **Figure 6**: *Element for construction.*

This new element, based on the equilateral triangle, can be used to build the models of da Vinci's tetrahedron, octahedron and icosahedron (Figure 7). Note that I took the liberty to "elevate" not exactly to the height that Pacioli described. But the original construction can be easily recognized (Figure 8).


Figure 7: *Elevations of tetrahedron, octahedron and icosahedron.* **Figure 8**: *Models of the elevations.*

1.4. Variation of the Shape of the Element. Because, with my way of making the models, the height of the elevation was no longer fixed, I decided to experiment a little more with this height as well as with the shape of the connection part of the elements. In Figure 9a a new shape is shown. To build the model of the elevated tetrahedron you still need four elements, but the elevation height is chosen so that he outer shape is a cube. So with four equal parts we have build a cube now (Figure 9b).

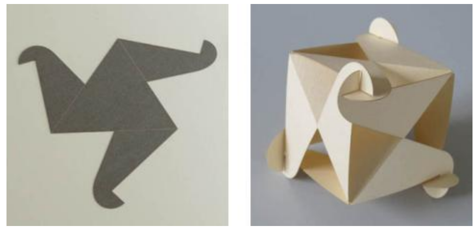


Figure 9a,b: *The new element and the new model of an elevated tetrahedron*

2. Flat Patterns

2.1. Triangular Tiling Pattern. Because Pacioli and da Vinci used only equilateral triangles for the pyramids of the elevations, they could not make elevations of polyhedra with faces other than triangles, squares or pentagons (Figure 10). We can however extend the original concept of elevation to the field of flat tilings with triangular and/or square tiles (Figure 11).

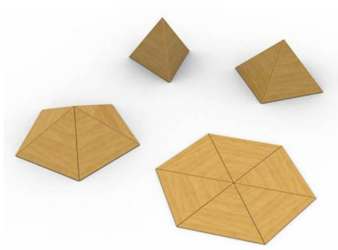


Figure 10: *Extension to flat patterns.*

Figure 11: *Elevated triangular pattern.*

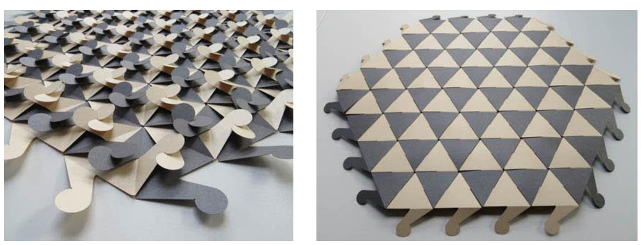


Figure 12: *Paper model of the elevated triangular pattern.*

Figure 13: *Demonstrating the strength.*

To make the flat pattern models we can use the same paper elements as we have used for the polyhedra (Figure 12). The resulting structure turned out to be very stable as is shown by one of the workshop participants (Figure 13). Another thing that was introduced in the workshops was the use of different color patterns (Figure 14).

Figure 14: *Introducing color patterns.*

2.2. Double Weave. In La Divina Proporione [1] the concept of elevation is introduced as an operation applied on polyhedra in one direction only: pyramids are placed on the faces at the outside of the polyhedron. When working on flat patterns we can apply this operation in both directions. When we make pairs of elements as is shown in Figure 15, we can indeed create a structure that can be seen as an elevation in two directions. In Figure 16 we can see the front and the back side of such a double woven elevated pattern.

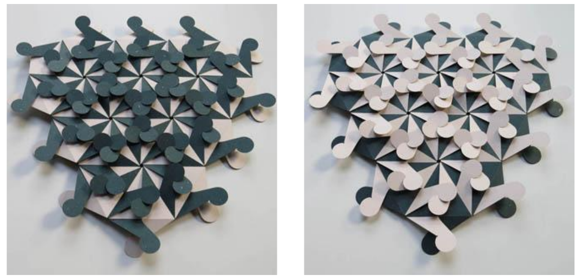
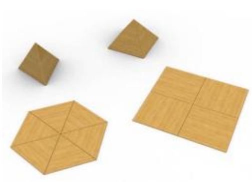
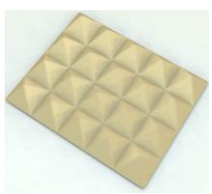


Figure 15: *Double element.* **Figure 16**: *Double woven elevated pattern.*

3. Archimedean Tiling Patterns

3.1. Weaving structures. The concept of elevation applied to flat patterns leads to interesting weaving. Elevation of squares, like in Da Vinci's elevated cube, leads to the pattern of Figure 18. But in the plane there are many tiling patterns that can be studied now.





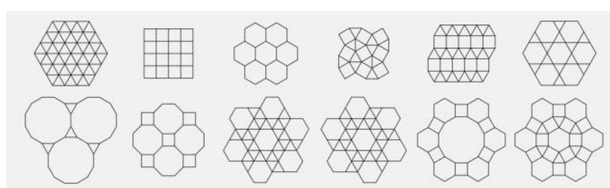
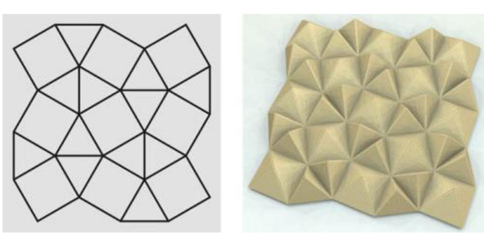


Figure 17: *Regular tiling patterns.* **Figure 18**: *Square pattern.* **Figure 19**: *Archimedean tiling patterns.*

We will start examining the possibilities of elevation of the Archimedean patterns and the duals of these patterns (Figure 19). As in the elevations of Luca Pacioli and Leonardo da Vinci, I will limit myself to tiles with 3, 4 or 5 edges.

3.2. Tiling 33434. There are two Archimedean patterns in which triangles and squares are combined. In Figure 20 we see the pattern 33434 with the elevation, which is then transformed to a weaving structure in Figure 21.



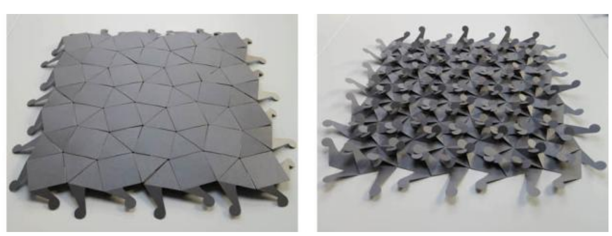


Figure 20: *Tiling pattern 33434.* **Figure 21**: *Model of the elevation of the pattern 33434.*

3.3. Dual Tiling Patterns. Besides the four Archimedean patterns we have discussed so far, there are no patterns with tiles with a maximum of five edges. But we can create more basic patterns by making the dual patterns of the Archimedean patterns, as is shown in Figure 22 for the pattern 33434.

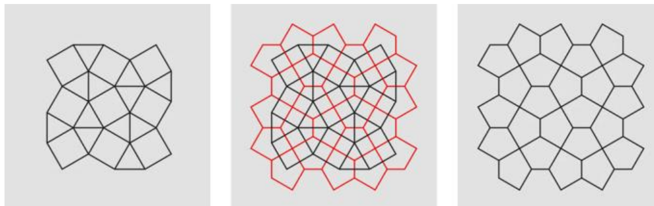


Figure 22: *Creating the dual pattern of tiling pattern 33434.*

And now this pattern with pentagonal tiles can be used to make the elevation (Figure23).

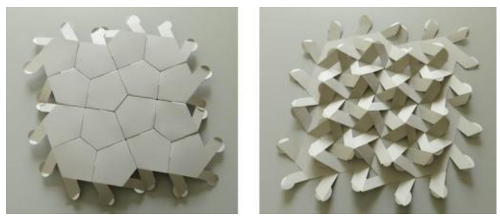


Figure 23: *Model of the elevation.*

All patterns dual to the Archimedean patterns, except 333333, have tiles with a maximum of 5 edges. In Figures 24 and 25 the process is shown starting with the Archimedean tilings 488 and 3636. In both cases the double woven elevation technique, as described in Section 2.2, is applied.

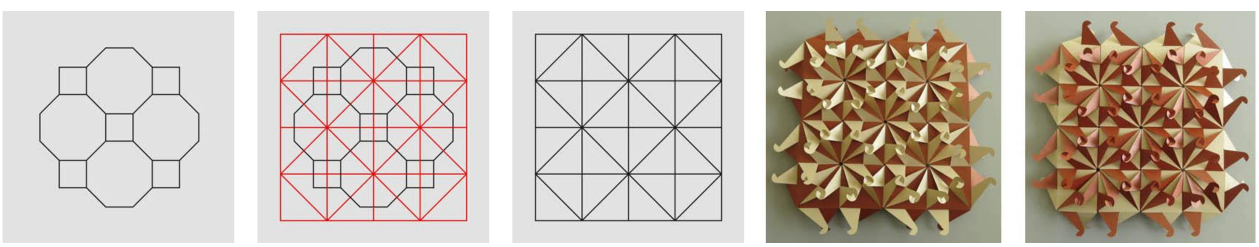


Figure 24: *Creating the dual pattern of tiling pattern 488 and the double woven elevation structure.*

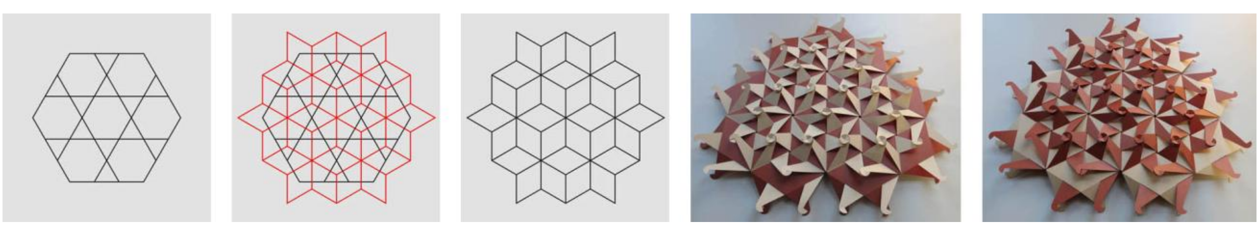


Figure 25: *Creating the dual pattern of tiling pattern 3636 and the double woven elevation structure.*

3.3. Color Patterns. Working with colors of the elements can be used to show sub patterns in the structure. A nice example is the dual pattern of the Archimedean tiling 3464 (Figure 26). When you want to use different colors for adjacent tiles you need at least three different colors.

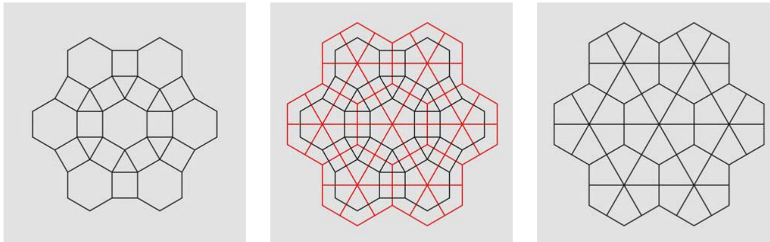


Figure 26: *Creating the dual pattern of tiling pattern 3464.*

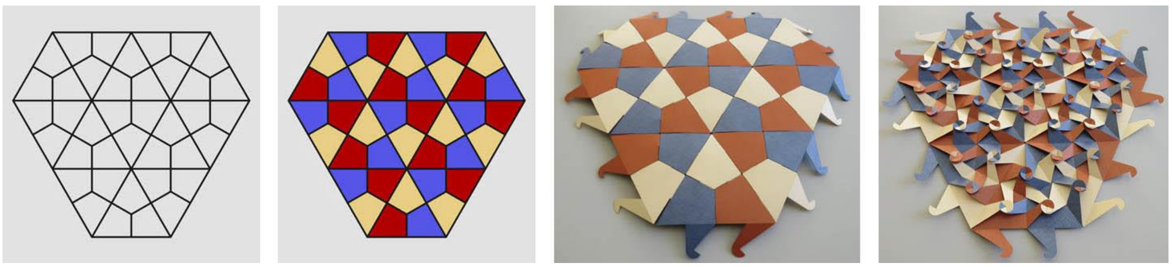


Figure 27: *Quadrangle tiling patterns (1) and first coloring.*

But we can see this tiling pattern as a combination of two other patterns: the triangular pattern, Archimedean pattern 333333 and the hexagonal pattern (Archimedean pattern 666). And by the use of colors we can focus on either of them (Figures 28 and 29).

Figure 28: *Second coloring.*

Figure 29: *Third coloring.*

4. Other Flat Patterns

4.1. Other Quadrangular Tiling Patterns. Now we have explored the basic techniques of applying the concept of elevation on flat tiling patterns we can look for other tiling patterns that we can use to create interesting paper structures. The first examples are both based on quadrangular tiling patterns.

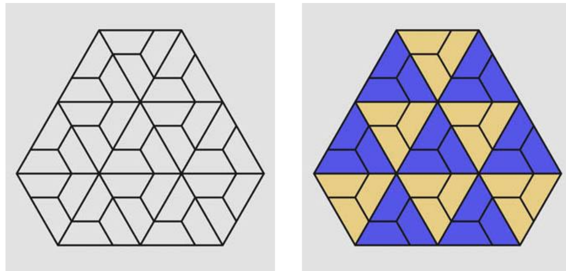


Figure 30: *Quadrangular tiling pattern (1)*

This pattern is chosen because it shows that in some cases the elevation leads to more than one element needed to build the structure (Figure 31).

Figure 31: *Building elements and model of the elevation, front and back.*

The pattern can be used both for single and double woven structures and also playing with different color schemes will give nice results (Figure 32 and 33).

Figure 32: *Double woven elevation.* **Figure 33**: *Second color scheme and model of the elevation, front and back.*

A second example shows the variety of the shape of the tiles that can be used (Figures 34, 35 and 36).

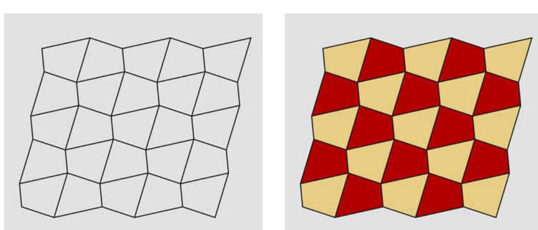


Figure 34: *Quadrangular tiling pattern (2).* **Figure 35**: *Paper elements for the elevation model.*

Figure 36: *Model of the elevation. Single weave front and back, double weave front and back.*

4.2. Spiraling Tiling Patterns. To show the many interesting possibilities of applying the concept of elevation to flat tiling patterns I want to add two more examples, both based on spiraling patterns. The first spiraling pattern is created from equilateral tiles and square tiles only (Figure 37). The second is based on a tiling pattern derived from an Archimedean spiral pattern.

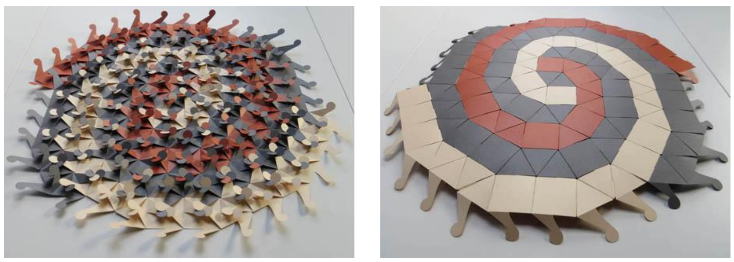


Figure 37: *Elevation of a spiraling tiling pattern*.

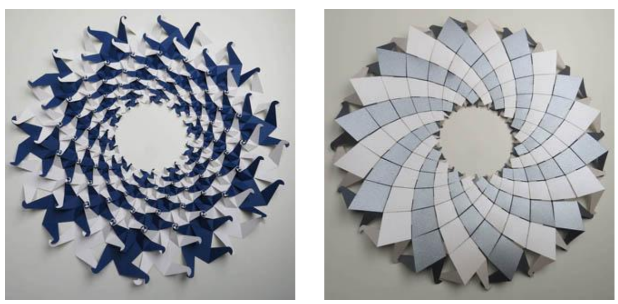


Figure 38: *Elevation of an Archimedean Spiral tiling pattern.*

References

[1] Luca Pacioli – Leonardo da Vinci, *La Divina Proportione*, 1509, Ed. Akal, S.A., Madrid, 1991
[2] Rinus Roelofs, *Elevations and Stellations*, Bridges Proceedings, Seoul, 2014.
[3] Luca Pacioli, *Divina Proportione: Die Lehre Vom Goldenen Schnitt*, 1509, Ed. Carl Graeser, Wien, 1896.
[4] B. Grünbaum – G.C. Shephard, *Tilings and Patterns*, W.H. Freeman and Company, New York, 1987.