

## Connecting with the Sierpinski Tetrahedron

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### Abstract

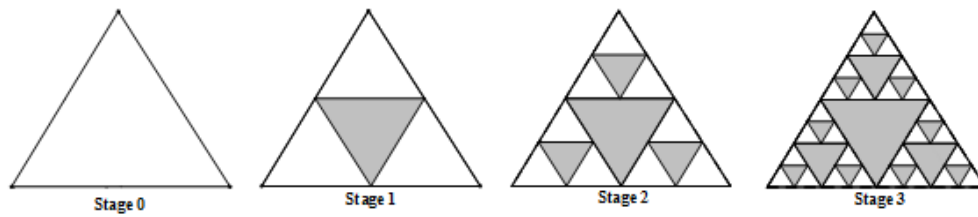
This workshop for Bridges participants will explore mathematical topics related to undergraduate geometry embodied in the Sierpinski tetrahedron. Come prepared to create a Stage 2 or 3 Sierpinski tetrahedron. Participants will be able to decide how best to structure and tailor the assembly and display process. There will be a variety of choices in terms of paper weight and designs. All materials provided.



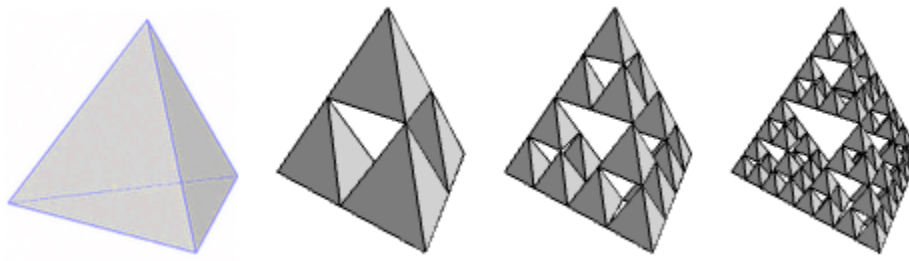
**Figure 1:** *Student-created Sierpinski tetrahedron sculpture.*

This workshop activity engages participants at various levels and can also be used for team-building with math department students and faculty. Some people find assembling the physical components while contemplating the mathematics involved a relaxing and interesting project. The mathematical content can be adapted to the various audiences: secondary students, undergraduates taking a liberal arts mathematics class or mathematics majors.

The Sierpinski tetrahedron fractal sculpture provides a creative link to the geometry mathematics curriculum that is accessible to students of varying levels. The workshop will introduce the Sierpinski triangle in 2D and then look at its 3D analogue, the Sierpinski tetrahedron.



**Figure 2** *Sierpinski triangle, Stages 0-3 (author created)*



**Figure 3** *Sierpinski tetrahedron, Stages 0-3 [1]*

Fractal geometry provides a powerful connection between mathematics and the visual arts [2,3]. The Sierpinski tetrahedron has inspired sculptures in various media and is recognized in the art community. In Minnesota, Breckenridge Senior High School students welded 4,096 equilateral metal triangles together to create a Stage-5 Sierpinski Tetrahedron. Artist Gwen Fisher has small versions of the Sierpinski tetrahedron using beadwork, and also a very large scale Stage 3 Sierpinski Tetrahedron, *Bat Country*. The *Bat Country* sculpture was created using baseball bats and softballs, and was selected as one of the 2013 Burning Man Honorary Art Projects.

Participants will be encouraged to adapt their experience building a Sierpinski tetrahedron sculpture to their own context as a classroom or informal learning group activity. The self-similarity is experienced in the construction process itself. Foundational to developing a robust understanding of the fractal process is familiarity with the idea of self-similarity and the scaling relationships with area and volume. The power, utility and beauty of mathematics take shape as participants learn that some very interesting things happen as the stage number approaches infinity. Students will be able to make connections with surface area and volume. Generalizing the calculations related to finding the sum of the edge lengths, surface area and volume at different iterations can motivate students to appreciate the power of algebra to represent their conclusions. More advanced students can calculate the fractal dimension of the Sierpinski triangle and the Sierpinski tetrahedron. Ideally this activity would either precede learning about fractals and fractal dimensions or serve as a culminating activity after these topics have been explored in more depth. A follow-up to this activity could be to demonstrate that the fractal dimension of the Sierpinski triangle is between 1 and 2, and the dimension of the Sierpinski tetrahedron is exactly 2 (Hausdorff-Besicovitch dimension). Explorations in the computer lab with Iterated Function Systems would be a technology-rich complement to this hands-on activity.



**Figure 4** Cutting the nets for the tetrahedrons.

This workshop provides an opportunity for participants to try this out and take the concept back to their classroom or campus. Template nets available online in various sizes and designs will be used to create the tetrahedrons for the construction of the Sierpinski tetrahedron [4,5]. The benefit of this type of workshop experience is that the final design and concept will have gone through at least a couple iterative stages of design process, resulting in a smoother adaptation to individual contexts. There are multiple choices to make in terms of color, paper weight, and maintaining structural integrity. Tetrahedrons will be constructed and then combined to form larger ones, culminating in a Stage 2 or 3 Sierpinski tetrahedron that participants can take with them. Once the sculpture moves beyond Stage 4, there are also considerations as to how to support the structure to keep it from collapsing.

Carpenter and Lehrer (1999) [6] identify five forms of mental activity from which mathematical understanding develops:

- a) constructing relationships
- b) extending and applying mathematical knowledge
- c) reflecting about experiences
- d) articulating what one knows
- e) making mathematical knowledge one's own

Participants will have an opportunity to reflect on the experience so that it can be adapted for specific classroom use.

## References

- [1] Eric W. Weisstein, "Tetrix." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com/Tetrix.html> (permission granted for non-commercial use, accessed April 20, 2015)
- [2] A. Crannell and M. Frantz, *Viewpoints: Mathematical Perspective and Fractal Geometry in Art*, Princeton University Press, 2011.
- [3] D. Jarvis and I. Naested, *Exploring the Math and Art Connection: teaching and learning between the lines*, Brush Education, Inc., Canada, 2012.

- [4] National Science Foundation Project Pattern Exploration: Integrating Mathematics and Science for the Middle Grades, 1999  
[http://math.fau.edu/Teacher/CATEs\\_PDF/3%20sierpinski/sierpinski%20unit%204.pdf](http://math.fau.edu/Teacher/CATEs_PDF/3%20sierpinski/sierpinski%20unit%204.pdf), (accessed 1/31/2015)
- [5] Matt Parker, Sierpinski Tetrahedron Fractal Worksheets, <http://www.think-maths.co.uk/> (accessed 1/31/2015)
- [6] T. P. Carpenter and R. Lehrer, Teaching and learning mathematics with understanding. In E. Fennema and T. Romberg (Eds.), *Mathematics Classrooms that Promote Understanding*, pp. 19-32, Mahwah, NJ: Lawrence Erlbaum Associates, 1999.

Link to workshop handouts:  
<http://tinyurl.com/mm5erkw>



**Figure 5** *Sierpinski Tetrahedron close-up.*