Mathematics through the Lens of a Kaleidoscope: A Student Centered Approach to Building Bridges between Mathematics and Art

Gail Kaplan*
Department of Mathematics
Towson University
Towson, Maryland 21252, USA
gkaplan@towson.edu

Rachael Gross
Department of Mathematics
Towson University
Towson, Maryland 21252, USA
rgross@towson.edu

Kim Krusen McComas
Department of Curriculum and Instruction
University of Arkansas
Fayetteville, Arkansas 72701
kmccomas@uark.edu

Abstract

This workshop paper explores the interdisciplinary connections between the visually appealing art of kaleidoscopes and fascinating mathematics associated with their construction, as well as the mathematics found in the images produced.

Introduction

Life is like an ever shifting kaleidoscope – a slight change, and all patterns alter.

Sharon Salzburg

Hold a kaleidoscope up to the light and peer through. A glorious vision of colors and shapes is transformed by the slightest twist. If you look beyond the magic, you will find angles, reflections, rotations, circumferences, surface areas, symmetries, and more.

Sir David Brewster invented the kaleidoscope in 1816. The patent application grants David Brewster the right to “make, use, exercise, and vend my new optical instrument called the kaleidoscope for exhibiting and treating beautiful forms and patterns or great use in all the ornamental arts with England, Wales, and the Town of Berwick-upon-Tweed . . .” Brewster further explains that "The kaleidoscope . . . is an instrument for creating and exhibiting an infinite variety of beautiful forms, and is constructed in such a manner as . . . to please the eye by an ever-varying succession of splendid tints and symmetrical forms . . .” Brewster named his invention “kaleidoscope” from the Greek “kalos eidos scopos,” literally “beautiful form-watcher”.

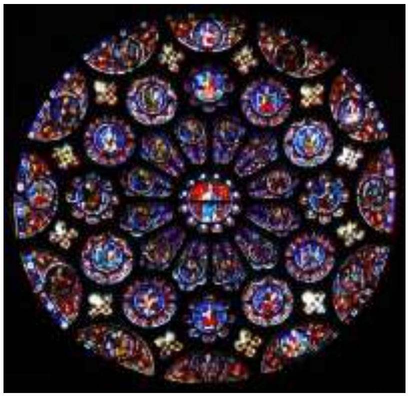


Figure 1: *The rose window of the Chartres Cathedral, built in the thirteenth century, has the same visual appeal as the view from the lens of a kaleidoscope.*

In the Victorian era experiential toys such as the kaleidoscope, called philosophical toys, inspired society to consider scientific questions about how things worked. Philosophical toys became very popular in the 1800's as scientists explored light and optics. These toys provided delightful entertainment. The type of kaleidoscope we will explore was patented by an American man named Charles Bush. In 1873, Bush received a patent for the sealed container used to make the designs. He used liquids of different densities to obtain glorious designs.

Overview of the Project

Kaleidoscopes are inherently interesting, providing an engaging, experiential opportunity to expand mathematical knowledge and understanding. We can improve motivation for learning mathematics by presenting meaningful projects. Students are more willing to persist at a task when motivated. When students see value in a task, such as appreciating the mathematical applications of a kaleidoscope, effort toward the task increases. When teachers provide opportunities for student autonomy, such as student-centered exploration of mathematical relationships, students have more control over their learning and motivation flourishes [1].

The kaleidoscope is an ideal instrument to inspire mathematics students. We explore an interdisciplinary discovery project in which students enrich their mathematical knowledge by building and exploring kaleidoscopes. Students work together to examine mathematics through hands-on investigations. Students make, test, and modify conjectures, enhancing mathematical reasoning and communication skills as they solve mathematical problems.

This project emphasizes connections between mathematics and the world around us, sparking excitement among students and encouraging discussion about geometry. Students begin to understand why we study rigid motions and symmetry. The kaleidoscope project not only teaches students about applications of geometry, but also demonstrates connections between mathematics and the arts and sciences. These interdisciplinary connections can be further explored by looking at the symmetries of artwork, studying the refraction of light, or discussing the importance of problem-solving skills to inventors and engineers. The project can lead to lessons on non-rigid motions, Frieze patterns, wallpaper patterns, tessellations, and introductory group theory. Our exploration helps students discover that Geometry exists everywhere.

The moment a sample kaleidoscope is passed around, it stimulates a high level of enthusiasm. Students cannot wait to build their own; they anticipate fun-filled classes and a fabulous objet d'art. Many are curious to learn how kaleidoscopes relate to math. One student commented, "When I came to class the day we were going to do this kaleidoscope project I thought it was going to be a blow-off project, but when I saw this packet, I knew it was going to be different." As students begin the exploratory activities of the kaleidoscope project, they discover the mathematical richness that lies within what they once considered a mere childhood toy.

The kaleidoscope journey begins with an opportunity for creating and testing conjectures about the different symmetries that result as we change the angle measurement between two mirrors, supporting the increased emphasis on reasoning and proof as an integral part of the daily classroom experience. Students place a hinged mirror at a 120° angle, place a bead between the mirrors, and count the number of beads they see (Figure 2). Students repeat the experiment with various angles and make conjectures connecting angle measure to the number of images. One student wrote, "I discovered that if I divide the angle measure into 360°, I will get the number of bead images." Students test their conjectures by comparing experimental results to their predictions. If the conjecture is incorrect, students revise the conjecture and test the revision. This is reasoning and problem solving at its best. Students are doing mathematics while playing with beads and mirrors!

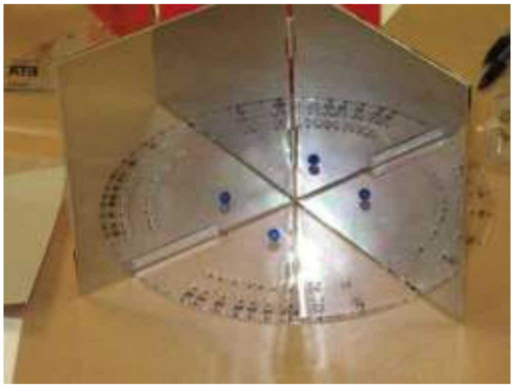


Figure 2: *Hinged Mirror with Bead.*

After discovering the "mystery of the mirrors," as one student puts it, we begin construction. To decorate the kaleidoscope, we create "wrapping paper" for the kaleidoscope. Using cardboard tubes from toilet paper rolls as models, students confirm that an unrolled cylinder is a rectangle. Students discover the relationship between the lateral area of a cylinder and the area of any parallelogram obtained by slicing the cylinder from top to bottom, gaining a deeper and practical understanding of the relationship between the area of a rectangle and of a parallelogram with equal height and width. Students cut along the seam of the cardboard around the tube, then unroll the tube to reveal a parallelogram. The area of the flattened tube does not depend on the way it is cut. Therefore, the areas of the two shapes must be the same.

After completing this exploration, students apply their findings by cutting rectangular wraps and testing the fit. This is an example of hands-on experience allowing for self-assessment: the wrap fits or it does not! One group quickly discovered that cutting the "correct" rectangle is not ideal; a wider rectangle allows for some overlap to ensure no cardboard shows. Students adjust the theoretical formula for practical application. The artistry of decorating the kaleidoscope begins!

With new kaleidoscopes in hand, students view the beautiful images and begin to analyze the symmetries within (Figure 3). Following a brief review of reflection and rotation, students sketch the images seen through the lenses of their kaleidoscopes and look for symmetries (Figure 4). This provides another opportunity to experience a practical application of theoretical ideas in geometry.

Figure 3: *Viewing the Images.*

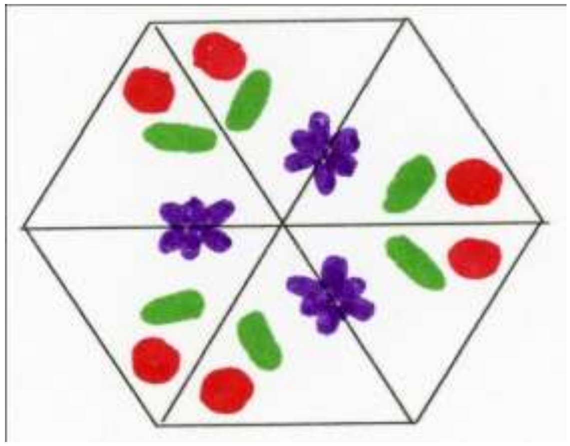


Figure 4: *Sample Sketch.*

After completing the project, the room is alive with delight and excitement. Students previously intimidated by geometry show new signs of life! One student wrote, “I have really enjoyed this project. It was so fun! So fun that for a straight week, Geometry was fun. I knew everything; I was not lost, confused, or overwhelmed.” Motivation theory tells us that *how a person feels about their ability* (self-efficacy) is more powerful than their *actual ability* [1]. Students need opportunities to believe in themselves as mathematicians; the kaleidoscope project is the catalyst. As elated students leave the classroom proudly carrying their kaleidoscopes, others question whether they will have the opportunity to build a kaleidoscope. The answer is simple: "Yes, if you take my class next year!"

As teachers, there are certain words from students that touch our hearts. Consider the response of one student who tapped the spirits of geometry’s ancient Greek philosophical roots when asked what he learned from making a kaleidoscope:

> I made a kaleidoscope. Or did I? Indeed I did, and from this journey into the bowels of geometry, I learned many things. I have discovered that the principles of geometry have withstood the test of time. From the humble kaleidoscope to rocket science, these century old principles have manifested themselves in all parts of the natural universe. In geometry over the past few days, we have caught but a mere glimpse of the very tip of the iceberg – a fleeting glance at the wider spectrum of geometric mathematics. A simple kaleidoscope assignment has given us the opportunity to open our minds and leave “the cave”, as described by Plato. We will continue to learn many fascinating new things on into the future having had the avenues of our existence unblocked.

What more could a teacher desire?

The Project
Part I: Making Conjectures

Angles

When a light ray hits a mirror, the angle between the incident ray and the mirror is congruent to the angle between the reflected ray and the mirror. Kaleidoscopes have at least two mirrors. The way the light reflects depends on the angle between the mirrors.

1. As you work through this problem, you will examine the different symmetries that result as we change the angle measurement between two mirrors.

    a. Place the hinged mirror on the protractor to form a 120° angle. Place a bead between the mirrors. How many beads do you see? (Don't forget to count the original bead!

    b. Repeat part (a) with a 90° angle. How many beads do you see?

    c. Repeat part (a) with a 45° angle. How many beads do you see?

    d. Make a conjecture about the relationship between the measure of the angle and the number of images.

    e. Using your conjecture, how many images would you expect to see if the mirrors form a 60° angle? A 30° angle?

    f. Use the mirrors to test your conjecture. If your conjecture is not correct, redo parts (d) and (e).

2. We will place 3 mirrors inside our Kaleidoscopes to form an equilateral triangle.

    a. What is the measure of each interior angle of an equilateral triangle?

    b. How many images of an object should we expect to see using this shape?

Area

The kaleidoscopes are constructed from two cylindrical tubes. We now consider how to determine the size of the paper we will need to cover the tubes. To begin, use the cylinders provided and make a single cut from the top to the bottom. Experiment and determine if it is possible to obtain the following when you unroll the cylinder. . If it is not possible, explain why.

  a. a rectangle

  b. a parallelogram that is not a rectangle

  c. a square

If you have an infinite supply of tubes, all identical in size, how many different parallelograms is it possible to create?

What do all of the parallelograms cut from the cylinders have in common?

Imagine that we cut the cylinder starting at the top and cutting straight down. What shape is obtained?

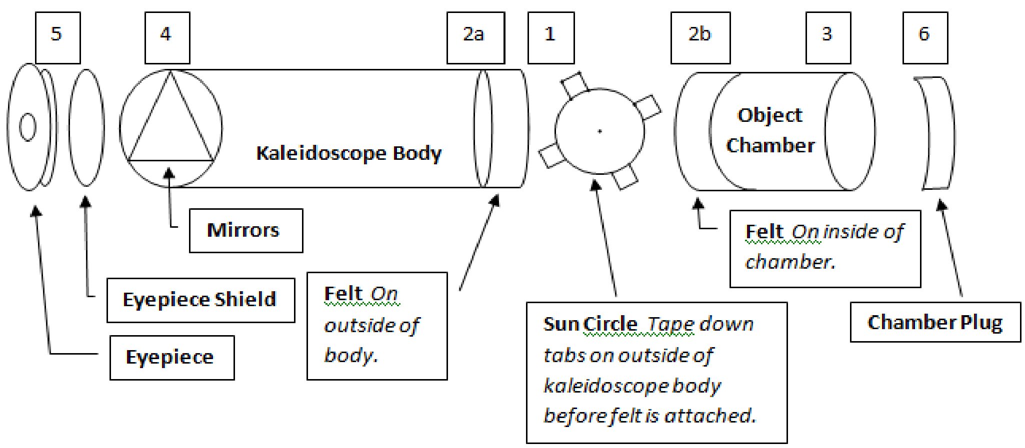
The length of the shape made is equal to the ________________ of the tube.

The width of the shape made is equal to the ________________ of the circle at the base of the cylinder.

The dimensions of the cylindrical tubes we will use in the construction of the kaleidoscope are in the table below. Determine the dimensions of the wrapping paper needed to cover both tubes. Do you want the width of your wrapping paper to be exactly the same as the circumference of the cylinder, or do you want to allow for overlap? Do you want the height of your wrapping paper to be exactly the same as the height of the cylinder? What would happen if the paper was too short? Too long?

	Height	**Diameter**
Long Tube	7.75 inches	1.5 inches
Short Tube	2.25 inches	1.77 inches

Part II: Construction Process



1. Fold down tabs of transparent plastic "sun circle" and tape down onto end of kaleidoscope body. This piece will keep beads in the object chamber from falling into the main body of the kaleidoscope.

2. Felt strips are adhered to the edges of both the kaleidoscope body and the object chamber to serve as a smooth cushion for rotating the object chamber on the kaleidoscope body and as "stops" to prevent the object chamber from falling off.
   a. Glue 2 felt strips (approximately ½" x 2 ¼") around *outside* of kaleidoscope body on the same end where "sun circle" is attached. There will be a gap between strips. Do not overlap the felt. Tape the strips down so that half the width of the tape is on the kaleidoscope body and half is on the felt strip.
   b. Glue and tape 2 felt strips to the *inside* of one end of the object chamber.
   c.

3. Slide the object chamber over the kaleidoscope body so that the felt strips on the object chamber touch the felt strips on the body. The object chamber should extend about 1 ¼" past the body.

4. Place the three mirrors into the kaleidoscope body so that they form an equilateral triangle. It may be a tight fit, so push firmly, but gently.

5. Drop the transparent eyepiece shield into the tube so that it will lie flat against the mirrors. Insert the eyepiece into the kaleidoscope body. Take your first glimpse through the eyepiece! At this point, you are looking through a **teleidoscope**.

6. Add beads to the object chamber until about half of the chamber is filled. Insert the chamber plug into the un-felted end of the object chamber.

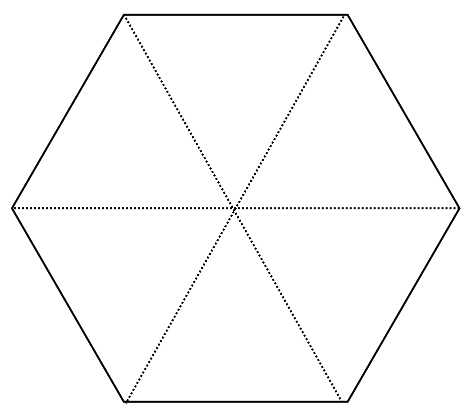
7. Hold up to the light, gently turn, and marvel at your creation!

8. Cut paper to fit around the kaleidoscope body and object chamber. Decorate and attach. If you choose, you can add stickers, ribbons, photographs, magazine cutouts, etc.

The Project
Part III: Analyzing Symmetry

1. We used **second surface mirrors** in our Kaleidoscopes. Second surface mirrors have an acrylic plastic coating over the mirrored surface for safety and durability, whereas first surface mirrors have no coating. From which type of mirror would you expect more vivid reflections? Why? (Think about the path a light ray would take to reach the mirrored surface.)

2. We will now analyze the symmetries of the image in your kaleidoscope.
   a. Make a rough sketch of the central hexagon. Rotate the object chamber until you see an image that looks easy to sketch. Sketching a few main features will be enough to analyze the symmetry. Don't sketch everything you see!



   b. If a shape has reflection symmetries, you could fold it in half and each side would be identical. Does your shape have reflection symmetries? If so, how many reflection symmetries does it have?

   c. To determine rotation symmetries, trace one of your triangles on tracing paper. Rotate the patty paper about the center of the hexagon. Does your shape have rotation symmetries? If so, how many rotation symmetries does your shape have? List the angles of any rotation symmetries.

   d. Draw two additional shapes with the same number of reflection symmetries and rotation symmetries as your picture.

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Suggested Readings

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