The Aesthetics of Scale: Weaving Mathematical Understandings

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Abstract

The use of visual arts applications to illustrate mathematical concepts is an old idea. Most instances, however, involve the observation and analysis of finished works and artefacts, rather than focusing on the *making* of them. We propose the idea of *making with rigour*, which incorporates the deliberate attending to mathematical structures into the process of making artefacts using specifically selected techniques. In this workshop, we suggest that there are additional insights to be gained by learners through the making process. Further, working in the same medium and technique at multiple scales can develop mathematical sensitivities. This enhances understanding by exposing mathematically essential properties that remain constant across multiple scales, yet are observed through the diverse perspectives afforded by the differing scales. The workshop will bring these ideas to light through participant experience and subsequent discussion.

Introduction

The significance of experiential learning in the teaching and learning of mathematical concepts has long been established [1, 2, 3]. In the case of visual and plastic arts, experiential learning will often involve technical considerations concerning the medium used. Multiple options are available to the teacher or curriculum developer. These include, for example, whether the learner will interact with a printed or virtual/digital (possibly three-dimensional) image, which largely limits the interactions to visual ones, or a tangible artefact, which allows more thoroughly sensory interactions such as flipping over, successively focusing on a part or the whole, changing the angle of view, touching and handling, etc.

A further, distinct mode of interaction, which is often overlooked, consists of artefact *making* for the learning of mathematics. Since its inception, the Halifax-based research group MathWeave (see also: textiles.teknollogy.com), has been investigating the mathematics inherent in textile making practices, and their potential in facilitating the learning of various concepts ranging from Elementary School to Graduate mathematics. In collaboration with mathematics educator Susan Gerofsky and the workshop participants, we propose to explore *making with rigour* at multiple scales to develop mathematical sensitivities.

We define mathematical sensitivities as the aesthetic awareness of the intrinsic mathematical structures of observable artefacts, processes and phenomena. In this context, the term "aesthetic" is used in the pre-Baumgarten [4], wider sense of *apprehension through the senses*, rather than its more recent meaning of subjective value judgment. [1] These sensitivities contribute to the establishment of the mathematical foundations of skills, concepts, intellectual understanding, proficiency and creativity [6].

The claims that will be investigated in the workshop can be expressed as follows:

- Exposure to artefacts embodying mathematical structures can develop sensitivity to those same structures via attentive experience.
- This attentive experience is provided by making with rigour, where the participants learn to engage with the technique but are also directed to observe and articulate the mathematical properties and relationships that make the technique.
- This process is valuable because analogies exist between weaving and mathematics at multiple levels, including:
  - Analogies in practices;
  - Analogies of concepts;
  - Mathematics as embedded in weaving practice;
  - Weaving as embedded in mathematical practices[2];
  - Mathematical sensitivities and choices informing aesthetic sensitivities and choices;
  - Aesthetic sensitivities and choices informing mathematical sensitivities and choices;
- The act of scaling can enhance the development of mathematical sensitivities by allowing the learner to:
  - Contrast the aesthetic and mathematical experiences at multiple scales;
  - Focus in and out of the artefact, thus observing structures locally and/or globally;
  - Physically interact with the artefact(s) in multiple ways (with the fingertips, with the hands, with the whole body, as a member of a collaborative unit);
  - Use multiple modes of apprehension and appreciation: visual, tactile, whole-body, social, etc.

The goal of the workshop is to provide this experience by working with the same intrinsic, woven structure at multiple scales, and articulating observations in group discussion.

Workshop

There are three making components in the workshop section of our presentation, each providing experiences of making with rigour at a different scale. After a short introduction, participants will be directed to one of two stations that will concurrently provide experiences with small and medium-scale weaving. Participants will be able to engage with both stations and choose which project to work on first. Later, the whole group will work together on the third, largest scale of weaving. Finally, participants will share their emergent mathematical observations.

We will provide weaving demonstrations and ask participants to notice how their understanding of both aesthetic and mathematical concepts may change from experiencing the various components.

Component A. Assuming participant's knowledge and experience with weaving will range from novice to professional, we will offer participants a basic experience weaving ribbon on cardboard (see Figure 1). For experienced weavers, this will be a reminder of weaving on a small scale, while for novices, it will get them familiar quickly with the basic terms, concepts and skills. Individuals weaving at this scale primarily engage

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[1] The beauty of mathematics is a topic of discussion in itself -- see for example [5].

[2] As Ada Lovelace said: "We may say most aptly, that the Analytical Engine weaves algebraical patterns just as the Jacquard-loom weaves flowers and leaves." [7]; see also [8].

their arms, hands, and fingers, while tracking with their eyes. The activity involved in this weaving scale is comparable to writing with a pen or typing text on a computer.

Component B. Participants will engage in a medium-scale weaving experience, weaving a larger piece with wider ribbons. The goal will be to make the same structure at a scale that engages more of the body, and may be accomplished more easily by two people working in concert, comparable to making a bed. Using the longer strips and working with a partner will allow people with different levels of experience to work together. Weaving on this scale has the potential to engender the creation and use of common language for design and rhythms of movement. Participants will need to use their full arms and spines, bending from the waist and reaching forward to pass materials back and forth. The body will need to stretch, bend and twist, and possibly walk around a table, creating a quality of movement and engagement and attentiveness quite distinct from the small-scale weaving.

Component C. Participants will cooperate on a large scale 'being the loom' and creating a woven piece through complementary and sequenced movements that will be orchestrated by observers. The "yarn" used at this scale will be larger than in the previous two components. Weaving on this scale may resemble forms of folk dance, both in terms of the full-body movements of the individuals and the coordinated choreography of the whole group. For individual participants, the whole body will be fully engaged in maintaining co-ordinated locomotion and balance as participants raise, lower, and step over and under warp threads, involving engagement of the spine and core of the body through reaching, bending, stretching and twisting. Such physical movement involves both external perception of the work (i.e., the collaborative group work to create this large-scale weaving) and proprioception (i.e., an individual perception of the position, movement and equilibrium of one's own body and the efforts it takes to achieve these). For the group as a whole, the collaborative movements that generate the woven piece will need to be coordinated, both from within the group, from the point of view of each individual weaver and the threads they control within the pattern, and from an outside observer who can perceive and direct the patterns being created by all the people and threads as they move. We expect that some kind of rhythm will emerge as this activity develops, as a way of coordinating the complex of simultaneous movement required by participants in charge of warp and weft threads as they move to create the weaving.

At the larger, social scales, two primary modes of communication—peer-to-peer and caller-to-group—characterise two different modes of approaching the making process. The caller-to-group mode of communication can be seen as analogous to the "recipe" or "algorithmic" approach to process, in which the individual weaving participants simply move their threads up or down according to the "call" of the caller. It is not essential that these participants have an overview or understanding of the woven interlacement itself, they need only move as instructed, and attend very locally to their own performance and perhaps that of their neighbours. On the other hand, for participant weavers, the peer-to-peer (or internal) mode of communication is more active and skill-laden. Peer participants confer with each other about the woven interlacement that they are trying to achieve, guided by their sensitivities to process, material, and patterns of movement that will achieve it in a satisfactory way. They may differ in skill and understanding, but they will inform each other through the process of conferring and achieving a consensus about what each will need to do. The process of conferring, as much as the actions of making themselves, will foster sensitivities and deeper understanding, by directing participants' attention to their process and its criteria for success.

For the final portion of the workshop, the participants will reflect on and discuss the impact of juxtaposing multiple scales of the same process and how communication, movement, making and finished product were affected.

Analysis

Pedagogical implications To think about this sequence of artefact-making activities in a pedagogical frame for teaching mathematics, we would first characterize pedagogy as the organizing of situations in order to foster attentive experience, emergent sensitivities, and analytical practices that are needed to sort and articulate concepts, to deepen understanding, and to envision new directions. Such situations are more likely to engage students positively if they involve activities that are enjoyable or interesting in themselves, in which mathematically salient ideas or concepts are perceptively embedded. Making such artefacts has proven to be this kind of enjoyable or interesting activity, while simultaneously involving learners with mathematical ideas and problems to resolve. Enjoyment, intrigue, or satisfactory results all foster the desire to repeat the making activity, often with different variables. Repeated exposure through making practice develops and internalizes the sensitivities that are the foundation of skills, concepts, intellectual understanding, proficiency and creativity [6]. Sensitivities not only provide more detail about where you are, they also indicate other directions in which you might go—they suggest new and deeper possibilities. Sensitivities direct attention, and attention engages and strengthens sensitivities. Sensitivities, because of their foundational character, can inform analysis.

Broadly speaking, our work proposes that we can develop mathematical sensitivities through the process of making and movement and the examination of the results of these processes. We conjecture that a change in scale changes perspective; it alters what one notices, attends to and can articulate. It alters the character of the control over the process and of the interaction between body, materials and techniques. Making with rigour at various scales allows for a fuller set of experiences to foster the development of mathematical sensitivities [6]. Furthermore, making by physically engaging with materials, at any scale, has the potential to take experience deeper than does visual observation; physical experiences of making allow us to internalize, articulate, and interrelate our understandings, both practical and conceptual [6]; it expands our capacity to imagine and invent.

Our position is supported by the work in the multi-year Graphs and Gestures research project [9, 10, 11]. In these studies, groups of adults and K-12 students were observed, videotaped and interviewed over the course of pedagogical activities in which participants gestured the shapes of graphs of mathematical functions on three scales:

- using just eyes, wrists and fingers (corresponding to our small-scale weaving);
- using full arm movement, but minimal movement of the spine, body core or lower body (corresponding to our medium-scale weaving); and
- using full-body motion, spine and core engagement, movement of the lower body, disequilibrium, and sometimes locomotion and coordinated movement with others (corresponding to our large-scale weaving).

Consistently across diverse groups and over a six-year time scale, participants demonstrated distinctly different kinds of attentiveness to mathematically-salient features of the graphs at these three different scales. Working at the smallest scale, learners described their experiences in terms of seeing something from a distance: the mathematical objects were seen as an overall visual phenomenon, which remained under the individual's control, 'at arm's length', and in this process, no feature emerged as particularly mathematically salient.

However, as learners moved to the medium and large scale gestures and movements representing the graphs, learners began to talk about 'being the graph' or 'riding on the graph' rather than 'seeing the graph'. It was observed that, at these larger scales, learners no longer eye-tracked their movements as they had at small scale. Voice and sound were more often engaged as a secondary modality to describe the shapes and 'movement' of the graphs. Participants consistently reported feeling closer to the graph, speaking about *being in or on the graph* or *the graph being in them*. From the point of view of mathematical pedagogy, the larger-scale gestures were reported as inducing awareness or attentiveness to a number of mathematically-salient features of the graphs: slope, maximum and minimum points, changes of direction, discontinuities, symmetries, increases and decreases in y-values, and intersection points, including roots.

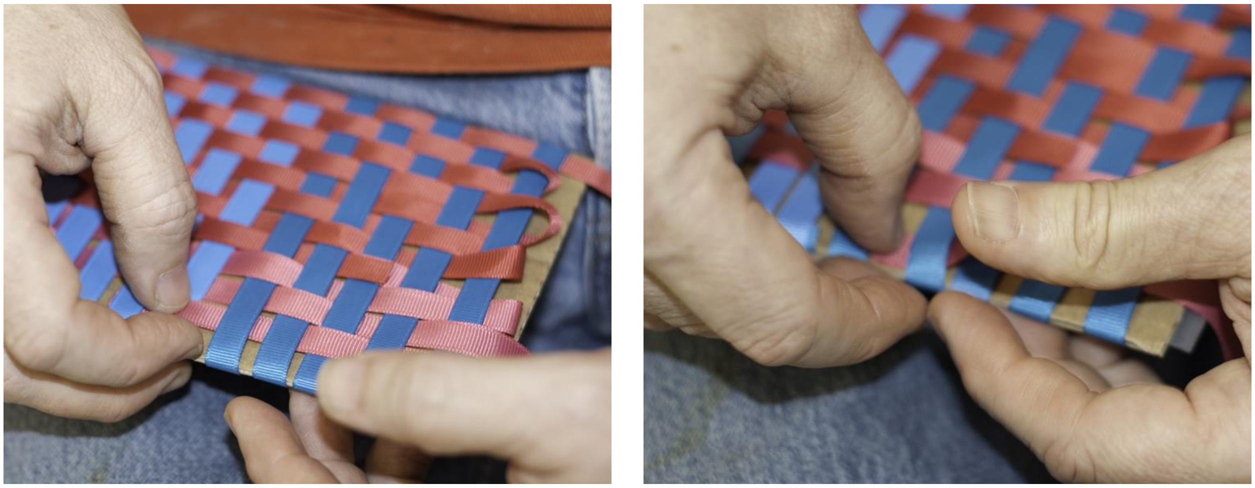

Figure 1: *Small hand-held weaving device made of re-used cardboard.*

Weaving at a small, hand-held scale (see Figure 1) is intimate and highly individual. While conducive to a general or overall perspective of the object as a whole, it is highly focused on controlling the patterns of hand and finger movement [9]. Moreover, at this scale individuals develop their own effective habits of movement, reliable procedures, observation, articulation, conceptualization, and criteria of proficiency [12], some of which may be mathematically-salient, even if the individual does not always recognize them as such.

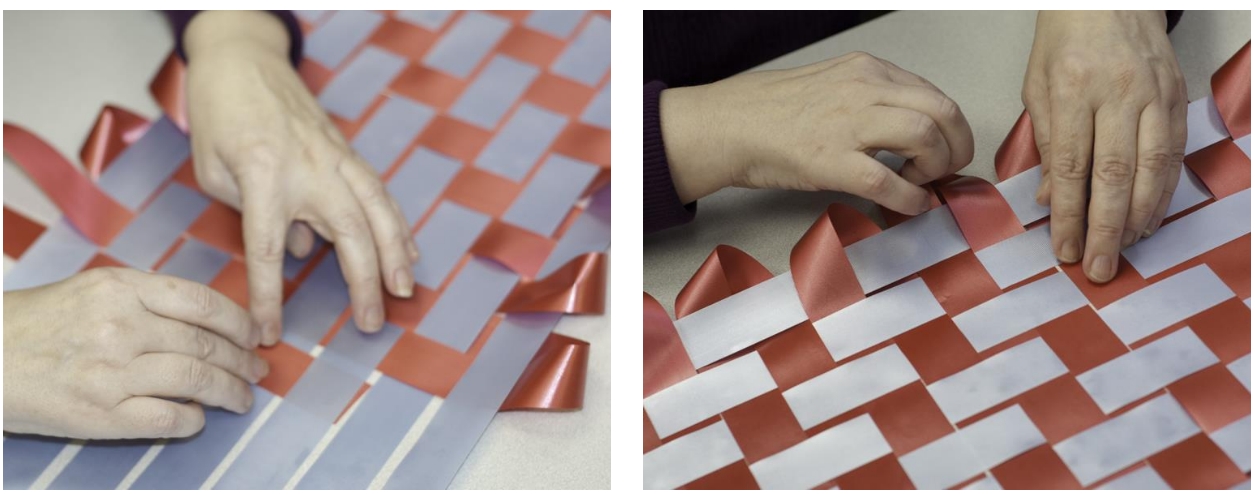

Figure 2: *Working at a larger, tabletop scale.*

Weaving at a larger scale that engages the whole body requires coordinating patterns of movement throughout the whole body; it may even call into play additional tools to hold elements in position or to move some elements around others. Larger scale thus requires attention to physical exertion and processes that might be well habituated and implicit with hand movement but are awkward and unfamiliar across the whole body. Bodily gestures are larger, more expressive, and require more room to manoeuvre, like a dance. Material elements are heavier, longer, perhaps less flexible and harder to control or push into place. It may be harder to spot errors in the woven pattern, because it is harder to see the whole form. From these features emerge new problems of movement and attention that require different patterns of movement to be created, refined, and internalized.

Figure 3: *Working together on a large weaving project.*

Weaving at yet a larger, social scale further reduces the participating weaver's perspective to a level of local detail, with less or no view of the whole. It requires that attention be paid to the participants adjacent to oneself. New issues arise of coordination of body patterns of movement with other participants. Each person's pattern of movement is reduced to a limited component of the entire pattern of movement across the whole set of participants, like a choreographed ensemble dance. It may require a master caller to orchestrate the process of movement globally, and/or it may require cooperation and negotiation with partners to ensure that the patterns of movement of the elements produce the desired effect over the whole structure. Sound and rhythm may be used to coordinate simultaneous group movements in time.

At each of these scales, different characteristics of the gestures, the properties of materials, the patterns of process and the patterns of structure are likely to emerge. What is suitable or possible at one scale (i.e. working with fine, pliable materials) is impractical or impossible at another scale. How the maker organizes her understanding of the resulting process of making at each level differs, especially in the change from individual to social. Different body and communication skills and perceptions come into play, which highlight different characteristics of the making process, and thus lead to new or alternate understandings of the final woven interlacement, which in principle is the same at each level. That is, each identifiable final woven interlacement is identical in the relationship of its component elements to one another, but the making process at each scale is neither physically nor cognitively so.

In the most 'traditional' math classes, the vast majority of work is done only at the small, individual scale. Designing pedagogical experiences for mathematical learning that integrate making at a variety of scales offers different qualities of engagement and attentiveness on the part of learners. Each scale engages and develops different sensitivities, which may be physical, intellectual, affective, and interpersonal. Sensitivities that may remain tacit or unspoken at an individual level must be articulated in some fashion and coordinated at the social level of making, so as to achieve consensus and cooperation towards the final goal. Some of these sensitivities and related concepts are mathematically salient. By working with multiple scales (and the movement among different levels of scale), we aim to stimulate further research and awareness related to changes of scale and related pedagogical and aesthetic effects of these changes, in a variety of mathematically interesting activities.

Mathematical analysis. There are many avenues for exploring the mathematical aspects of weaving. The workshop is set up to activate emergent directions, including:

- Number Theory: if each strip is assigned a number, then number properties can be explored through the patterns that emerge from their paths and relationships (e.g. adjacency). For example, if a pattern is repeated every three strips, then one can look at properties of multiples of three. These patterns are relevant for certain weaving designs like twill. The properties can be explored at all three scales. At the small scale, one can compare and contrast between different strips. At the large scale, one can focus on what happens to a specific strip.
- Modular Systems: one can consider the remainder after division by a fixed number (called the modulus). For a given modulus m, there are m possible remainders. The usual 12 hour clock system corresponds to arithmetic modulo 12. In this system, $8 + 9 = 5$ because if you start at 8 o'clock and add nine hours, you are at 5 o'clock. There are three modulo classes for the number three: integers with remainder 0, integers with remainder 1, and integers with remainder 2. The set of remainders $\{0, 1, 2\}$ together with the arithmetic operation of addition is an example of a group (see next bullet).
- Group Theory: a powerful field of abstract mathematics, group theory has many applications. The set $\{0, 1, 2\}$ of modulo classes for a modulus of three form a group under addition [13]. A group is a set together with a binary operation that satisfies several conditions: the operation has closure (answers are always defined), the operation is associative (brackets don't matter), there is an identity (an element that doesn't change things), and each element has an inverse (a way to get back to the identity).
  1. Closure: $0 + 0 = 0, 0 + 1 = 1, 0 + 2 = 2, 1 + 0 = 1, 1 + 1 = 2, 1 + 2 = 0,$
     $$2 + 0 = 2, 2 + 1 = 0, 2 + 2 = 1$$
  2. Associativity: For all a, b, c in $\{0, 1, 2\}$ we have $(a + b) + c = a + (b + c)$
  3. Identity: The identity is 0 (adding 0 doesn't matter), 1+2=0, 2+0=2,

  Inverse: Each element has an inverse: inverse of 0 is 0, inverse of 1 is 2, inverse of 2 is 1. Other examples of groups can be explored via weaving. Examples include groups of symmetries of an object. The symmetry of an object is a transformation that preserves the shape, size and location of an object. These transformations can be rotations, reflections or translations. A square has eight symmetries (four rotations and four reflections). For a specific object, the symmetries together with composition form a group. Woven artefacts can be analysed and compared using their groups of symmetries.
- Geometry: one can explore various geometrical concepts through weaving. These include shapes such as polygons; angles; topological features such as number of holes; and transformations such as rotations, reflections and translations.
- Variables: mathematics provides one way to describe relationships between objects that can change. The concept of variable is vital to many areas of mathematics, and particularly for applications of mathematics to real-world problems. In weaving, mathematical questions arise from wondering what happens if changes are made during the weaving process. These questions can arise at all scales.
- Fractals: the notion of scaling can lead to fractals. There is no strict mathematical definition of fractal, but one feature that is common to most fractals is self-similarity. An object is self-similar if it can be expressed as the union of scaled down versions of itself. Hence an exploration of different scales can lead to fractals. Moreover, the determination of an appropriate scale to make observations is an important mathematical method.

On a broader and deeper level, the experience of weaving at multiple scales provides a model for what a research mathematician actually does. One strategy that research mathematicians use is to scale up or down to obtain a different perspective. When a researcher is stuck on a problem (struggling to work out the details of a new theory, struggling to formulate a conjecture, struggling to construct a proof, etc.), one possible approach to get unstuck is to either look at a more general problem or a more specific problem.

Moreover, weaving offers a metaphor for mathematical research, in that it is usually possible to take a problem further or to change direction.

Conclusion

Traditionally, the teaching of mathematics is pulled between the pedagogical poles of teacher directed paper and pencil tasks and more open-ended student discovery methods. We propose a kind of embodied pedagogy for the teaching of mathematics, based on making artefacts and movement. In our pedagogy, we develop mathematical sensitivities through attentive, meaningful experience.

Along with seeking new ways to develop sensitivities, we advocate for the development of richer, deeper kinds of mathematical and aesthetic investigations. Such investigations have the potential to lead students to more profound disciplinary and cross-disciplinary understanding.

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