Math-Infused Art Lessons, Art-Infused Math Lessons

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Abstract

This paper begins with an explanation of why I am designing processes for teaching math-infused art and art-infused math. Then, I take the reader and workshop participants through a process for creating mirrored designs on the coordinate grid. Finally, I reference research and books that are influencing my choices as I continue to develop my methods and lessons.

Introduction

Enthusiasm is contagious. Beauty is inspiring. Math is beautiful to me and I hope my enthusiasm will inspire others to see the beauty in math.

I am a fiber artist. A fiber artist is one who makes art out of fibrous stuff such as fabric, yarn, handmade papers and basketry materials. I am also a geometric artist. I am addicted to patterns and structures. I am addicted to drawing on graph paper – to capturing an image in pixels and vectors. I see geometry everywhere. I frame my visual perception of the world within the x and y axis of the coordinate plane so that I may understand size, proportion and the way shapes and forms overlap, connect or relate to each other. I also consider how flat and curved planes come together to create three-dimensional forms. I use that information to make all sorts of things.

I am also a home schooling mother, art and craft teacher, and graduate student, taking courses in elementary education. In all of these pursuits I cannot help but see and experience the interrelatedness of math and art, or math and design.

I wish to pass my love and knowledge of the relationship between math and art to as many people as possible. I wish to inspire teachers to inspire their students, to open people's hearts and minds to see math as a creative endeavor, to equate numbers and mathematical processes to playing with shape and line in ways that create beautiful objects a person can hold in his or her hands.

I am making it my business to create processes for infusing art lessons with math lessons, and vice versa, to connect the two in as many ways as possible in hopes that thoughts of math will invoke feelings of joy in students, at all grade levels, as it is associated with crayons, paints, markers, colored pencils, collage papers, cardboard, wood, fabric, clay and any other materials that bring out the playful child within each of us.

Process for Creating Mirrored Designs on the Coordinate Grid

There's an art to drawing on graph paper. The more time you spend doing it, the easier it is to navigate the lines and squares - to use the lines as guides and to ignore them completely when necessary. My hand and eyes are trained to draw diagonals across a rectangular array without being distracted by the intersecting horizontals and verticals, to freehand draw a circle by recognizing equal distance from a center point, and to repeat a matching curve at a different location and orientation. For me, shapes jump off the nearly blank page. All I have to do is outline what I see in my mind's eye. How does one break this down into small steps in order to teach it to others? How does one communicate the math and art concepts that have long since become intuitive?

If you are new to drawing on graph paper, give yourself time to get used to your surroundings and be prepared for your eyes to become crossed and your hand to lose self-control from time to time. If that happens, just rest your eyes and hand and return later.

Graph Paper. Are you like me? Do you sit and contemplate the essence of graph paper? If you do, you can move down to the next topic. If you don't, follow along with me.

Graph paper is nothing more than equally spaced horizontal and vertical lines, forming rows and columns of squares. As a fiber artist, this beautiful structure allows me to utilize the squares as pixels for creating needlecraft designs, and the lines for drafting sewing patterns. Neat, huh?

When working with students, the structure of graph paper creates points of reference. Let me explain. Horizontal, vertical, perpendicular and parallel are already present. Anything you add can be related to those concepts. Points exist at every intersection of lines. Measuring becomes counting units of a predetermined length. Units and square units are already defined. Angles of 90°, 180°, 270°, and 360° have already been laid out. Everything you do from there builds upon and refers to the pre-existing structure.

Number Sets and Patterns. Designing woven structures for weaving on a loom is a game of number patterns and sets. Block designs, or creating square and rectangular areas of color, involves the intersection of horizontal and vertical lines set at various distances apart. These lines intersect to form distinct areas that may or may not vary in size. Stripes can be all equal in width or some can be more narrow or wide than others. It is the same with plaids, which are nothing more than stripes that intersect perpendicularly. Designs for woven structures are typically mapped out on graph paper. The number sets are chosen to reflect the proportion the designer desires. The numbers define the width of each stripe. The sets of numbers are repeated over and over again along the width and/or length of the fabric.

Tessellations. I drew tessellations for many years before I even heard the word "tessellations." What is a tessellation? It is a tile that repeats to cover a flat surface without any gaps or overlaps. A square can be used as a tessellation tile because it can be repeated over and over again to cover a flat surface in precisely the same way it does on graph paper. How convenient!

Coordinate Grid. The coordinate grid consists of the x-axis and the y-axis, or one horizontal number line intersecting one vertical number line perpendicularly. We designate the point where the two intersect as 0 (zero). We then put marks at equal distances and number them in order from 0, positive when moving to the right or upward, and negative when moving to the left or downward. These two intersecting lines, like the north, south, east, and west of a compass, create a little universe of their own with four quadrants. The upper right quadrant is quadrant I and contains all positive numbers. The quadrants are numbered counterclockwise. The upper left is quadrant II. The lower left is quadrant III. The lower right is quadrant IV. Quadrant III contains all negative numbers. Quadrants II and IV contain a mix of negative and positive numbers depending on whether you are referring to the x- or y-axis. Since the coordinate grid, also known as the coordinate plane, relies heavily on horizontal and vertical lines at equal distances apart, intersecting perpendicularly, or we can say at 90°, we tend to use a very special type of paper to draw them on. You guessed it! Graph paper.

Now we are ready to make geometric art on graph paper using all the concepts explained above, and more.

Creating a Tile. We will now create square tessellation tiles using number sets. First, we must choose a number. I have chosen the number 6. My square will be 6 units by 6 units. I will show the size of my square by drawing it on graph paper and numbering one horizontal and one vertical edge. I will make the bottom left corner, or vertex, 0 and count over six to the right and six upward. This is not only a square tessellation tile, but it also represents quadrant I of the coordinate grid and contains all positive numbers. (Figure 1)

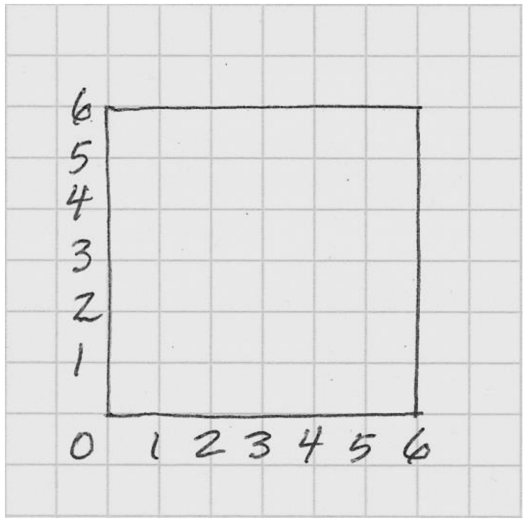

Figure 1: *6x6 square tile numbered as in quadrant I of the coordinate grid.*

Now we will use a set of numbers to create a block design within the square tile in much the same way a weaver defines square and rectangular areas of color, as previously mentioned. We can choose three numbers that add up to 6. We could use fewer or more numbers than three. I have just chosen three for the purpose of example. Three numbers that add up to 6 are 1, 2, and 3. 1+2+3=6. Using the commutative property of math, we can put these three numbers in any order and they still add up to 6. How many different orders can we put them in? Well, let's see.

1,2,3
1,3,2
2,1,3
2,3,1
3,1,2
3,2,1

We have six sets of numbers that each add up to 6. We can make six different tiles, one with each set of numbers. We can use these numbers to define square and rectangular areas within our square tile. The placement of the varying sizes of areas depends upon the order of the numbers. But, before we plot the numbers along the x- and y-axes we need to actually change them a bit. If we plotted numbers 1,2, and 3 on the axes they would all be one unit apart and end at 3 units, not 6. Therefore, we must add the second to the first and then the third to the first two. The number set for 1,2,3 becomes 1,3,6. We will also place a 0 at the beginning of each set of numbers so we can include each corner of the square in the design. Here are our new sets:

0,1,3,6
0,1,4,6
0,2,3,6
0,2,5,6
0,3,4,6
0,3,5,6

We can plot each set of numbers on the x- and y-axes of each tile, one set per tile. We can also plot points where the horizontal and vertical lines of the plotted points intersect. There will be 16 points plotted in all on each tile. To save space, we will just use one tile as an example. The tile created by the number set 0,3,5,6 will look like this: (Figure 2)

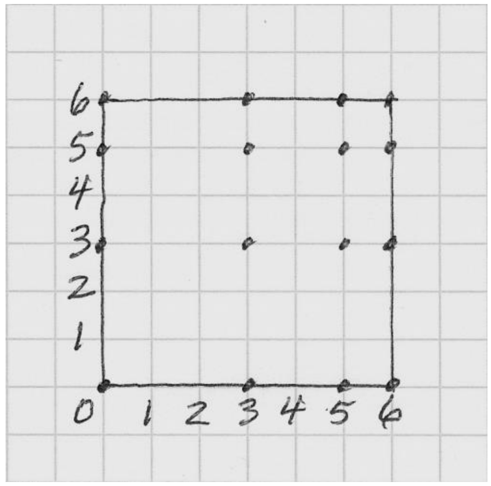

Figure 2: *Points plotted on the 6x6 tile using the 0,3,5,6 number set.*

These are the plotted points:

(0,6)	(3,6)	(5,6)	(6,6)
(0,5)	(3,5)	(5,5)	(6,5)
(0,3)	(3,3)	(5,3)	(6,3)
(0,0)	(3,0)	(5,0)	(6,0)

I recorded the coordinates in the order they actually appear in quadrant I, from upper left to lower right. This is important for when we begin to mirror in the other three quadrants.

We can now play "connect the dots" and create a design using vertical, horizontal and diagonal line segments connecting any two points. Here is the design I created using the 0,3,5,6 tile: (Figure 3)

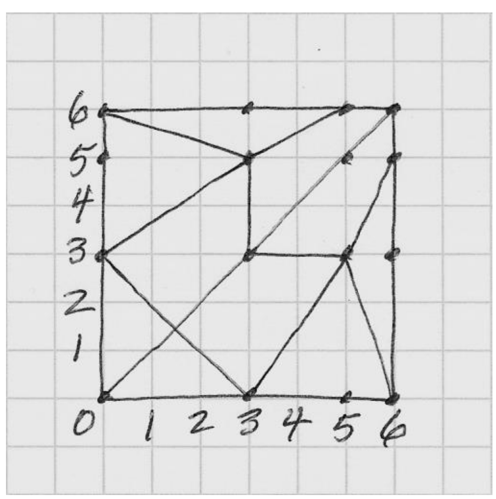

Figure 3: *Tile design drawn by connecting plotted points with line segments.*

This tile already represents mirrored symmetry. The line of symmetry runs along the diagonal line from (0,0) to (6,6). I did not have to create a symmetrical design. It could have been asymmetrical. Either type of design, symmetrical or asymmetrical, can be used to create mirrored designs using the next steps of the process. Draw several 6x6 squares with the same plotted points and create some designs.

Placing a Tile on the Coordinate Grid. You can choose a tile from your collection of designs to place on the coordinate grid for the purpose of mirroring. I will continue with the same tile we are already working with. First, draw a 12x12 box on your graph paper and divide it into four quadrants by drawing a horizontal and vertical line down the middle to represent the x- and y-axes. Then, plot the same points and draw the same line segments as in the tile you have chosen, in quadrant I. Simple enough? (Figure 4)

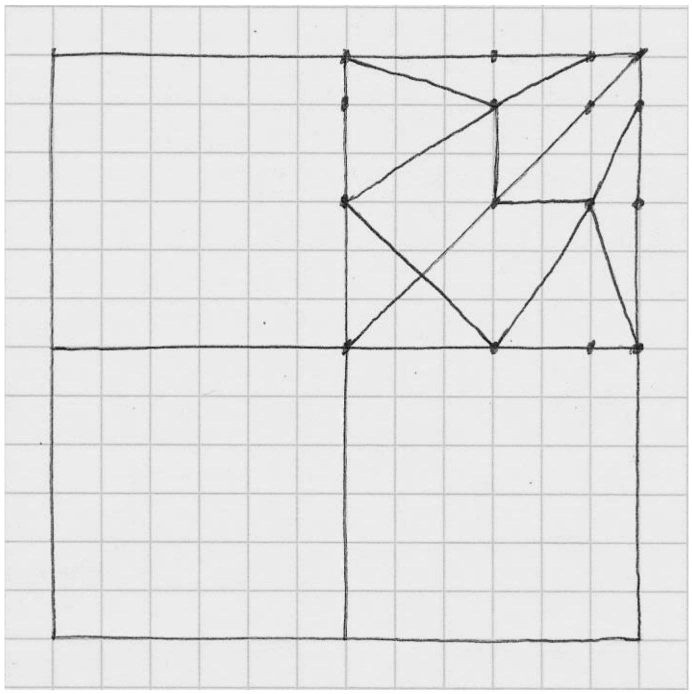

Figure 4: *Tile design from figure 3 placed in quadrant I of the coordinate grid.*

Mirroring Using Coordinates. Now we can mirror our design into quadrant II. If you have a small mirror, you can stand it up along the y-axis to the left of the design in quadrant I, with the mirror facing quadrant I. If you look into the mirror you will see the image you are about to draw in quadrant II.

We can also create a mirrored image by changing our set of plotted points from quadrant I. All of the numbers on the y-axis remain the same. All of the numbers on the x-axis become negative, except 0 of course. Reverse the order of the columns to represent the visual change. Is this as much fun for you as it is for me?

(-6,6)	(-5,6)	(-3,6)	(0,6)
(-6,5)	(-5,5)	(-3,5)	(0,5)
(-6,3)	(-5,3)	(-3,3)	(0,3)
(-6,0)	(-5,0)	(-3,0)	(0,0)

Plot the points and connect the dots as a mirrored image. For example, the line segment that connects (0,3) to (3,0) in quadrant I now connects (0,3) to (-3,0) in quadrant II. (Figure 5) Disregard the negatives and focus on absolute value when connecting line segments in all quadrants.

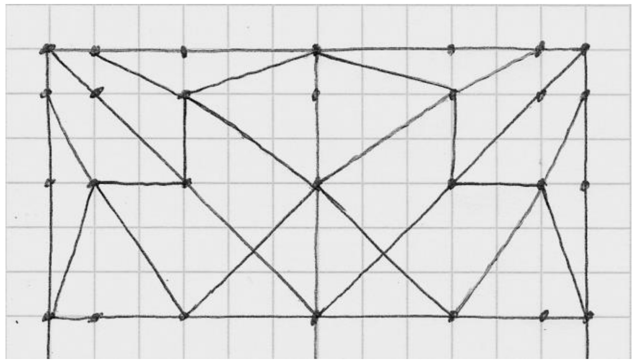

Figure 5: *Tile design from quadrant I mirrored in quadrant II.*

For quadrant III, all the numbers become negative and the rows are placed in reverse order.

(-6.0)	(-5,0)	(-3,0)	(0,0)
(-6,-3)	(-5,-3)	(-3,-3)	(0,-3)
(-6,-5)	(-5,-5)	(-3,-5)	(0,-5)
(-6,-6)	(-5,-6)	(-3,-6)	(0,-6)

For quadrant IV, the y-axis remains negative, the x-axis returns to positive, and the columns are reversed.

(0,0)	(3,0)	(5,0)	(6,0)
(0,-3)	(3,-3)	(5,-3)	(6,-3)
(0,-5)	(3,-5)	(5,-5)	(6,-5)
(0,-6)	(3,-6)	(5,-6)	(6,-6)

After each quadrant's points are plotted, the line segments can be drawn. (Figure 6)

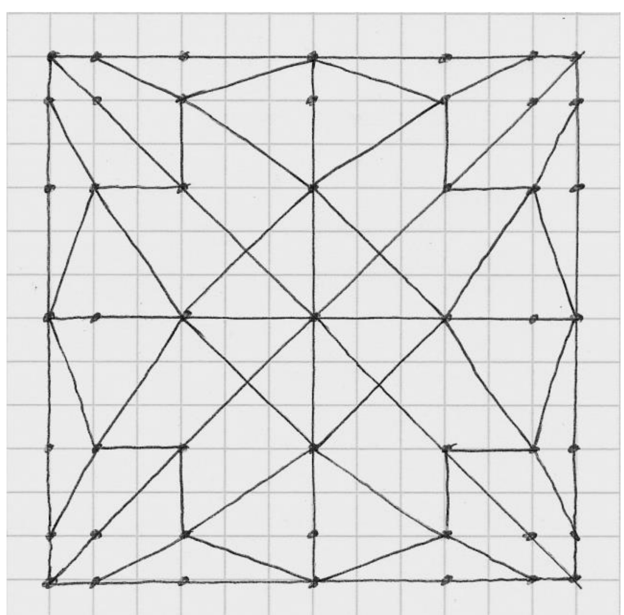

Figure 6: *Double-mirrored image of the original tile design from figure 4.*

Options for Creating a Finished Work of Art. The design produced in today's workshop is only the starting point for artistic exploration. The artwork that can be produced using this image is only limited by an individual's imagination and access to materials. Since the image is so conveniently drawn on graph paper, it can be reduced and enlarged quite easily to any size. The image can be repeated multiple times, if that is desirable. Being a square, it can be repeated in as many rows and columns as you choose, without any gaps or overlaps. Or, you can randomly place the image for a more dynamic effect. There are multiple ways of transferring the image to the surface you wish to work on, whether it be paper, wood, canvas or anything else. You can use any art medium you wish such as paint, markers, crayons, colored pencils, art papers, and fabrics. I have included an image of the same design we just produced, repeated multiple times, and colored with colored pencils. (Figure 7) Can you find the double-mirrored image and/or original tile?

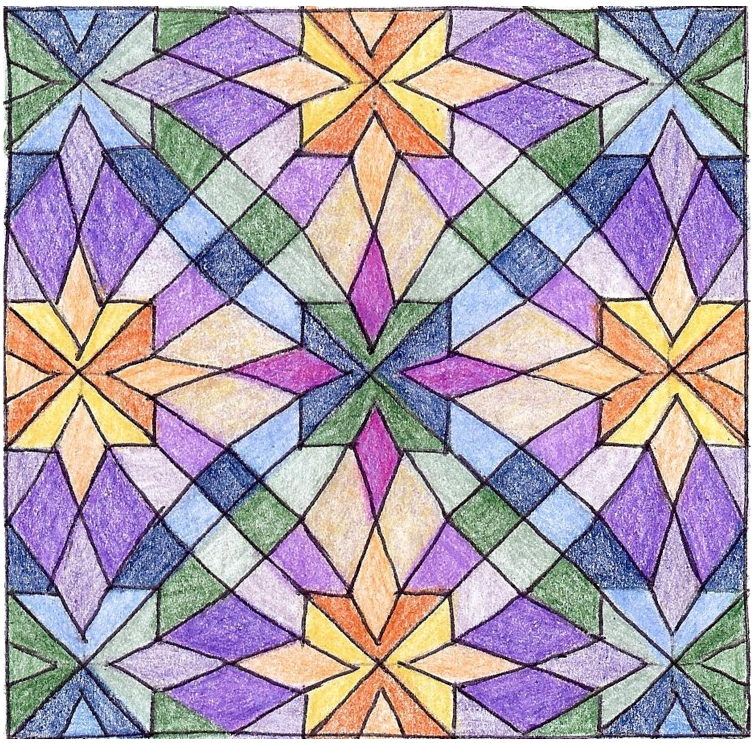

Figure 7: *Colored pencil drawing made by repeating the double-mirrored image.*

Reducing and Enlarging. You can reduce or enlarge an image drawn on graph paper by changing the size of the squares, or by multiplying or dividing the number of units, and redrawing the image. For example, if the original image is drawn on a graph of 4 squares per inch and you want to make it 2 times larger, just use a half-inch grid. The process of counting the units and plotting the points remains the same. Or, you can multiply the units by 2. The number set for the example we just did together would change from 0,3,5,6 to 0,6,10,12. And of course, there is always reducing and enlarging on the copier, but where is the math learning in that?

Transferring the Image. The final image can be projected on a surface such as a wall or a large sheet of paper taped to the wall. It can be transferred by use of a light table or light showing through a window. You can purchase transfer paper from an art store. Or, you can create pattern pieces and trace the individual shapes onto a surface such as a painting canvas. Enlarging each shape in the design individually to make pattern pieces creates more opportunities for learning.

Conclusions

The lesson in this workshop has been used with students in grades 5-8. I have been invited into these classrooms following professional development classes I have taught specifically to art teachers. My intention is to reach students who enjoy art and may or may not enjoy math. In the lessons I design, I like to move back and forth between concrete manipulatives, representational drawings, and projects that allow for creative decision making and personal expression to allow the students to use the newly-learned concepts abstractly. My primary focus is to develop spatial understanding in all students because spatial understanding is necessary for all career endeavors and very relevant for STEM careers, specifically.

Fifty years of research has shown that strong spatial skills lead to success in the completion of undergraduate and graduate degrees in STEM-related subjects and in the success of individuals in STEM careers. [1] Students interested in STEM careers often drop out due to lack of spatial skills. [2] Providing opportunities for students to develop their spatial skills is very relevant because it has been proven that these skills are learned through experience. [2]

People ask why I teach on graph paper instead of using computers. There are two reasons. First, like historical education reformers John Dewey and Maria Montessori, I am a firm believer in the value of having students work with their hands. Second, I teach professional development classes for teachers from a variety of classroom situations. I am quite aware that while it is desirable, many schools do not have a computer for every student. Graph paper is much more easily accessible for all. Even though the students are not using computers in my lessons, they are using concepts and terminology that will prepare them for using computers in the future.

Two inspirational books continue to inform my work – *Where Good Ideas Come From* [1] and *Creating Innovators* [2]. Both authors focus on how innovation occurs and how to help others become more innovative in their thinking and doing.

And finally, I would like to mention a college textbook created for future teachers, *Elementary and Middle School Mathematics Teaching Developmentally* [3]. This book contains countless examples of how to make math lessons interesting and accessible to many different learning styles. I use this book as a reference and for inspiration.

References

[1] Jonathan Wai, David Lubinski, and Camilla P. Benbow. Spatial Ability for STEM Domains: Aligning Over 50 Years of Cumulative Psychological Knowledge Solidifies Its Importance. *Journal of Educational Psychology*, 101(4):817-835, 2009.

[2] David H. Uttal, Nathaniel G. Meadow, Elizabeth Tipton, Linda L. Hand, Alison R. Alden, Christopher Warren, and Nora S. Newcombe. The Malleability of Spatial Skills: A Meta-Analysis of Training Studies. *Psychological Bulletin,* 139(2):352-402, 2013.

[3] Steven Johnson, *Where Good Ideas Come From: The Natural History of Innovation,* Riverhead Books, 2010.

[4] Tony Wagner, *Creating Innovators: The Making of Young People Who Will Change the World,* Scribner, 2012.

[5] John A. Van de Walle, Karen S. Karp and Jennifer M. Bay-Williams, *Elementary and Middle School Mathematics Teaching Developmentally,* Pearson, 2014.