Proceedings of Bridges 2015: Mathematics, Music, Art, Architecture, Culture

Exploring Ratios and Sequences with Mathematically Layered Beverages

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Abstract

We describe a workshop that teaches ratios and integer sequences such as the Fibonacci sequence through layered drinks. Liquids with different ratios of sugar to water have different densities and can be layered. The addition of food coloring gives drinks layered in this way a visually compelling effect.

1 Introduction

Everybody loves food, which makes it a perfect vehicle for conveying mathematical ideas to the general public. People rarely think of cooking as being particularly inherently mathematical. It can be surprising to realize that the mathematical concepts of ratios and proportions are integral to the process of cooking and baking and reflect on underlying mathematics [3]. Many recipes have fairly strict ratios of ingredients (for example, baked goods), while others, like those for beverages, allow for a great deal of flexibility.

This workshop takes advantage of the flexibility of ingredient ratios in beverages to explore and teach fractions and integer sequences. Unlike most examples of food-based mathematical art, where the math is purely visual with no effect on the actual flavor and experience consuming the food [2, 4, 5], the math in our beverages is conveyed entirely by the ingredients and flavor.

Liquids with different ratios of sugar to water have different densities and can be layered. The ratio of sugar and other flavorings between layers can be calculated and tasted as part of a teaching exercise on fractions. When a large number of layers are used, the proportion of flavorings can increase according to various monotonic integer sequences, for example, the Fibonacci sequence [8, 7].

2 Workshop Description

There are two main parts to this workshop that can be taught together or separately. The first is primarily a workshop on ratios and fractions, while the second goes on to explore integer sequences. This section is primarily concerned with the abstract and pedagogical aspects of this workshop. Specific instructions for creating layered beverages are given in Section 3.

2.1 Exploring Ratios

The workshop begins with simple two-layered beverages as in Figure 2.1. Beverages with two layers are great as a way to increase comfort with fractions and ratios. The amount of sugar in each of the layers can be expressed as a ratio. For example, if the bottom layer has 5 teaspoons of simple syrup per unit volume and the top layer has 3 teaspoons of simple syrup per unit volume, then the sweetness ratio is 3:5. This means the top layer is $\frac{3}{5}$ as sweet as the bottom layer, and the bottom layer is $\frac{5}{3}$ as sweet as the top.

Note that we specified an amount of sugar per unit volume. We encourage participants in the workshop to perform some more interesting fraction calculations by changing the relative unit volumes between layers.

Figure 1 : *Fractions can be taught with only two layers.*

For example, if one layer has 5 teaspoons of simple syrup per cup and another layer has 7 teaspoons of simple syrup per 2 cups, which layer is more dense? What is the ratio of sugar between the two layers?

Performing these calculations requires a basic understanding of fractions. Participants can test their calculations by creating layers and then comparing their sweetness. They can also try layering the two layers. If they are incorrect about which layer is denser, then the layers will mix when they pour the less sweet layer first.

Similar and more complicated calculations can be explored by varying the quantity of other flavorings in the beverage. We now get the ratios between flavoring and sugar, as well as all of the previous ratios between layers.

2.2 Exploring Sequences

Drinks with many layers may be used to explore integer sequences as in Figure 2.2. However, due to the requirement that each layer be less dense than the previous layer, only sequences that are monotonic can be explored. Sequences that work well for this exercise include basic arithmetic sequences, sequences that can be described by a recurrence relation, such as the Fibonacci sequence and other similar sequences, and generally any monotonically increasing sequence where the first several elements in the sequence are not too far apart. Sequences that grow large too quickly will tend to have almost flavorless layers followed by unpalatably sweet layers.

Participants can use the recurrence relation to calculate what the proportions should be for several layers and then create beverages for those sequences. It is interesting to have the sugar and flavor proportions vary according to different sequences or according to different elements of the same sequence.

Figure 2: *Drinks with many layers can be used to explore integer sequences. The lemonade shown here has layers that increase in sweetness according to the Fibonacci sequence.*

As an example, participants might make lemonade according to the Fibonacci sequence. In this case, they should find that the intensity of flavors as you go down the layers increases exponentially. If the proportion of sugar for layer n can be represented F_n per unit volume and the proportion of lemon juice is F_{n-1}, we find that the ratio of sugar to lemon juice approximates the golden ratio, with the approximation becoming better the further down the drink that you go. Indeed, Fibonacci lemonade may be the world's first tastable example of the relationship between the Fibonacci sequence and the golden ratio! The proportions for Fibonacci lemonade are given in Section 3.3.

Workshop participants might also explore other Fibonacci-like sequences, like the Lucas numbers. It can be interesting to observe that in all such sequences, the ratio between adjacent numbers approaches the golden ratio. In contrast, if students experiment with basic arithmetic sequences they might observe that the ratio between adjacent values of the sequence approaches 1 as the number of layers tends towards infinity.

Many other sequences can be turned into layered beverages with this technique. Another interesting sequence to experiment with might have the amount of simple syrup follow the sequence of triangular numbers and the amount of lemon juice follow the number of line segments between closest pairs of dots in the triangle.

In addition to integer sequences it is possible to explore sequences of fractions and have the amount of simple syrup and flavoring to volume correspond to these fractional sequences.

3 Layered Beverage Construction

There are many tutorials available for making layered drinks out of standard store-bought sugar beverages [1]. For this exercise we want more control of the ratios of flavor, sugar, and water in our drinks, so we mix each layer ourselves.

3.1 Ingredients and Materials

- Lemon Juice or other flavoring(s)
- Simple Syrup (1 cup sugar dissolved in 1 cup water)
- Water
- Food Coloring (optional)
- Ice

- Measuring spoons
- Tall clear cups
- Measuring cups (preferably liquid measuring cups)

3.2 Method

You must start with the sweetest and densest layer and work your way backwards up the drink to create the layers. Figure 3.2 shows this drink layering in action. See Section 3.3 for an example of specific ingredient proportions for Fibonacci lemonade.

First, fill tall glasses with ice. Then, do the following steps for each layer.

1. Add flavoring and simple syrup according to the desired ratio to your liquid measuring cup.
2. Add food coloring if desired.
3. Add water to the measuring cup until the desired total volume is reached.
4. Stir to blend all ingredients in your measuring cup.
5. Slowly pour a layer from your measuring cup into your drink glasses. You want to pour directly onto an ice cube: the ice cubes are there to slow down your liquid as it goes down the cup and to help keep the layers distinct.

3.3 Recommended Layers for Fibonacci Lemonade

These are the proportions for 1/2 cup total volume per layer. The extra volume should be filled in with water.

1. 0 tsp. lemon juice, 1 tsp. simple syrup
2. 1 tsp. lemon juice, 1 tsp. simple syrup
3. 1 tsp. lemon juice, 2 tsp. simple syrup
4. 2 tsp. lemon juice, 3 tsp. simple syrup
5. 3 tsp. lemon juice, 5 tsp. simple syrup
6. 5 tsp. lemon juice, 8 tsp. simple syrup

Figure 3 : *The author running a mathematically layered drinks workshop. Photo courtesy of the University of Arkansas Math Club.*

4 Discussion

Many students struggle with understanding fractions even though they are a crucial skill that is generally taught fairly early in the mathematics curriculum as a precursor to later skills [6]. Fractions can seem illogical and hard to conceptualize. This workshop gives a fun way to for students to practice calculating fractions in an unorthodox setting using a sense almost never used in mathematics classrooms.

Layered beverages are a particularly nice food to play with for this exercise. The layering caused by density is somewhat counterintuitive and can be a nice science lesson in addition to the math lesson described. While this workshop describes an exercise intended to be fun and educational for all ages, it is also possible to make adult layered drinks where the density of the layers is adjusted by the fraction of alcohol in the drink.

Many other food items, like layered snow cones or popsicles, could be used to teach the same ideas. Recipes in general are a natural environment to find fractions. Lots of recipes allow for a level of ingredient flexibility that would let you play with the proportions to achieve interesting ratios between them while still ending up with a delicious result. For example, Evelyn Lamb has described an ‘e-1’ salad dressing [9] and Nick Couch has created a recipe for golden ratio flapjacks [10]. We hope that others are inspired by the basic ideas of teaching math through layered drinks and try creating other mathematical foods.

5 Acknowledgements

The author would like to thank Henry Segerman for inspiring her exploits into mathematical cuisine that you have to eat and taste in order to experience its mathematical nature, Scott Kominers for encouraging her to write this paper, Edmund Harris for inviting her to run this workshop at the University of Arkansas, several anonymous reviewers for helpful comments, and SAP for funding her research.

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