Gallery Layout in Borges' Library of Babel

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Abstract

Jorge Luis Borges imagined the infinite *Library of Babel* in an eponymous short story. The Library fills space with hexagonal galleries, containing all possible books of a certain size. Borges' description of the connectivity between galleries is both incomplete and inconsistent. But, by relaxing two troublemaking constraints in a natural way, there is a way to connect galleries so that every hexagon contains a gallery, and every gallery is reachable by foot in a time proportional to the geometric distance from the starting gallery. A virtual 3-D version of such a Library is available on a Web site.

Background

Jorge Luis Borges is an Argentinian writer famous for his imaginative short stories. One of the most intriguing is *The Library of Babel*, written in 1941, and currently available in translation in reprinted collections and online [1,2]. As described by Borges, the Library has an "indefinite, perhaps infinite" number of hexagonal galleries, four sides of which have five shelves, thirty-two books to a shelf, each book having 410 pages. The galleries stretch endlessly in all directions, including up and down on infinite spiral staircases passing through the small vestibules between galleries. Borges also calls for a large ventilation shaft, surrounded by a low railing, in the center of each gallery, with an infinite view up and down.

The books contain all possible sequences of alphabetic symbols: 22 letters plus the comma, period, and space. Borges does not specify which 22 letters; subsequent speculations have not settled the matter. Most such sequences are nonsensical, and much of the story is devoted to the efforts of librarians – all Library inhabitants are called librarians – to find interesting books in the apparently random ordering of the shelves. In the story, one librarian of genius posits that no two books are identical. The story also mentions the fact that there can be only a finite number of such books, and speculates that the Library is unbounded but periodic.

One problem for mathematically inclined readers is in deducing the floor plan of the Library. Borges states that one of the two free sides of each gallery opens onto a vestibule leading to another gallery. A staircase is located in the vestibule. Why is this a problem? Because, without a second exit from a gallery, a librarian will have access to only two galleries on each floor. Perhaps Borges meant to say that *both* free sides of a gallery lead to other galleries. A friend of mine whose native language is Spanish says that the original Spanish text has been accurately translated, leaving us in doubt as to Borges' intent for the other free side.

A penetrating discussion of this problem is given by Bloch in Chapter 5 of a recent book that presents the history of efforts to understand the nature of Borges' Library, and offers some useful new insights [3]. It seems that all students of the story agree that Borges intended that all galleries should be accessible from any one of them, and the galleries should fill space in all directions. In an interview

reported in [4], Borges said that he chose the hexagonal shape for galleries because hexagons, unlike circles, can be packed without leaving spaces in between. Bloch points out that even if the other free side of a gallery also leads to another gallery, there are still some difficulties. To clarify the difficulties, it helps to specify the goals we are trying to achieve more precisely.

Objectives

We take it for granted that galleries are placed in a honeycomb pattern. Further, gallery walls are aligned vertically, because of the large uninterrupted central ventilation shafts. A satisfactory floor plan should have the following properties relating to the placement of exits and staircases:
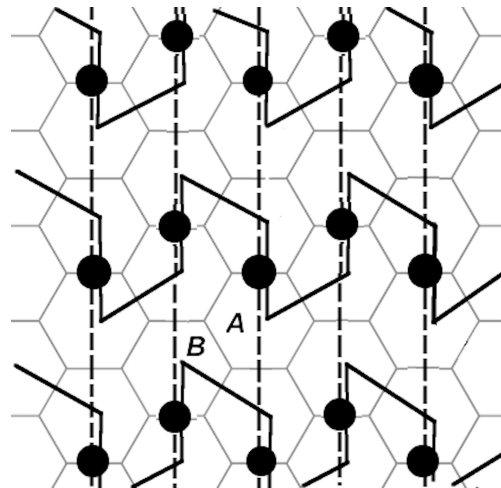
a) On each floor, every hexagonal cell is a gallery – there are no empty hexagons.
b) The two free sides of each gallery lead to another gallery through a small vestibule.
c) At least one of the two adjacent vestibules contains a staircase.
d) Every gallery is efficiently reachable from any gallery.

For property d), we will explain presently what "efficiently" is intended to mean. In property b), we do not assume that exits will always be in the same two sides, even up to symmetry, despite a passage in the Borges story to the effect that all galleries are identical.

With two exits from each gallery, galleries on the same floor can be chained into a path that visits every gallery on the floor. Bloch gives examples like the double spiral in Figure 1. So what's wrong with that? Note that as one moves out on a spiral arm, there are pairs of hexagons that are neighboring (they share a wall), but because they are on different arms of the spiral path, the walking distance between them can be arbitrarily large (in an infinite library). This problem has been shown by Bloch to be unavoidable. Bloch has proved a nontrivial property he calls "strong inaccessibility": paraphrased, one can find pairs of neighboring hexagons on a path-connected floor such that the shortest path between them on the same floor can be arbitrarily large (in an infinite library). Efficiency requires an escape from this property, and it succeeds by permitting paths to visit multiple floors, to get between galleries on the same floor.
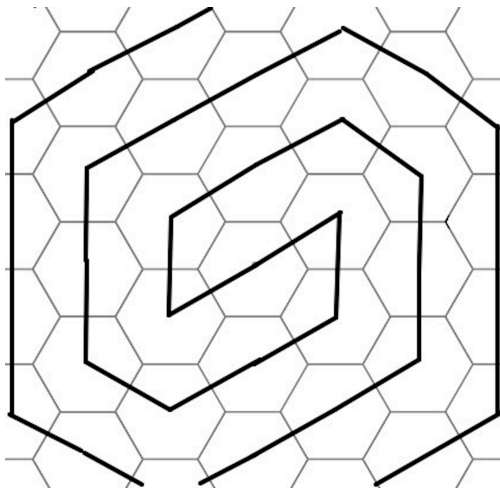
**Figure 1**: *A spiral path.*   **Figure 2**: *Bloch's efficient construction.*

We will say that a Library floor plan is *efficient* if the walking distance between neighboring galleries on the same floor is bounded. The *walking distance* is defined here as the number of vestibules a librarian must pass through. If a floor plan is efficient, the walking distance from a gallery to any other is *linear* with respect to the geometric distance. That is, it is bounded by some constant multiple of the latter distance. This is true even between galleries on different floors, given the convention that the walking distance up or down a floor is 2 (since there are two vestibules to pass through), and applying property c) to ensure that there is always a staircase nearby.

Examples

Here are two examples of efficient floor plans, one in Figure 2, due to Bloch, and one in Figure 3, by the Rice+Lipka architectural firm. Bloch's construction, in Figure 2, requires that adjacent floors have regular but different floor plans. Galleries on both floors share the same hexagonal grid and the same staircase vestibules (the dots). The other gallery exits and the paths through them are indicated with the solid lines on one floor and vertical dashed lines on an adjacent floor. Figure 2 is efficient – the maximum necessary walking distance between two adjacent hexagons is 6. For example, to get from *A* to *B* on the solid-line floor, one might go up one floor exiting south on the dashed-line floor, south, back down to the solid-line floor northward, and west .

The second example satisfying property d), due to Rice+Lipka, is diagrammed in Figure 3 and shown in an artist's conception in Figure 4, borrowed (with permission from the architects) from an article in the *Places Journal* [5].
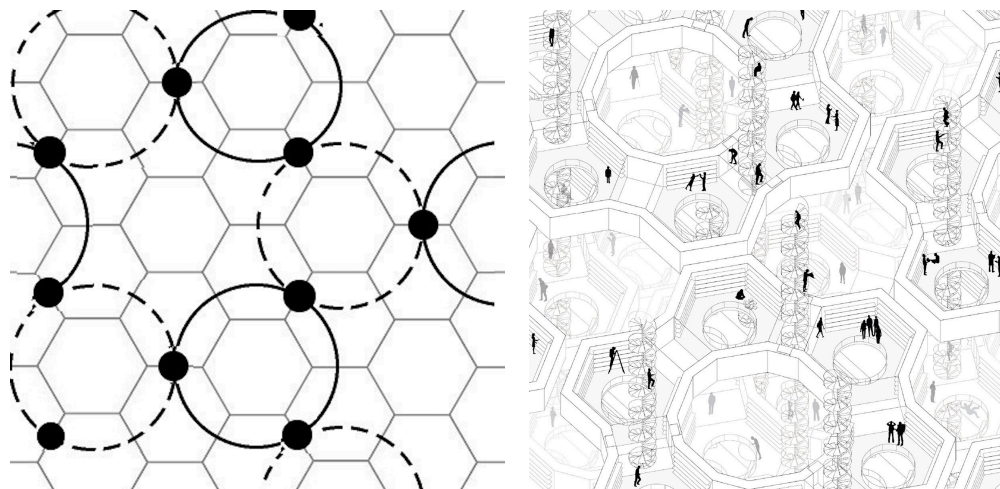
Figure 3: *Rice+Lipka model.*   **Figure 4**: *The Universe © Rice+Lipka Architects.*

In the Rice+Lipka model, galleries are grouped into separate rings. A librarian can walk from one ring to another by ascending or descending to an adjacent floor, which has the same floor plan, but shifted so as to join the upper and lower rings at the staircases. This plan is efficient, and more uniform, since the galleries are identical in door placement, up to rotation, even across multiple floors. However, it does not satisfy property a), since there are empty hexagons on each floor within and outside the rings of galleries.

A New Floor Plan

The plan in Figure 5 is as efficient as the Bloch pattern, but might be seen as simpler. Again, even and odd floors have a different layout, enabling path connectivity for the Library. The maximum walking distance from one gallery to an adjacent one is 6. There is another benefit associated with this floor plan: it has been modeled in the form of a computer game. The game is just a simulation with a 3-D model of the Library, on the author's website [6]. Figure 6 is a screen shot from the game. This Library model can be viewed and explored with most popular browsers on Mac OS X or Windows, using a webplayer plugin downloadable from the Unity3D website. A standalone version of the model can be requested from the author, including one for Linux.

The game has a third-person viewpoint. Any book can be opened, but, as expected for the Library of Babel, most of them contain random nonsense. However, the content of books is generated with a preference for English digraphs and other language characteristics, so interesting snippets are easier to find. The game Library has also been "salted" non-randomly with a few interesting books, and other features may be added as time goes on.
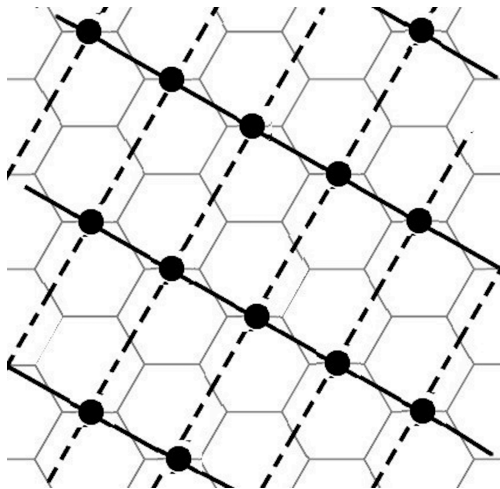

Figure 5: *A new Library model.*   **Figure 6**: *Screenshot of the virtual Library.*

References

[1] Borges, Jorge Luis, *Labyrinths: Selected Stories and Other Writings*, Edited by Donald A. Yates and James E. Irby, London: The Folio Society, 2007, and New York: New Directions, 1964.

[2] Borges, Jorge Luis, *The Library of Babel,* Internet Archive, Online: `https://archive.org/details/TheLibraryOfBabel` (as of 4/9/2015).

[3] Bloch, William Goldbloom, *The Unimaginable Mathematics of Borges' Library of Babel*, Oxford University Press, 2008.

[4] Sarlo, Beatriz, *Borges, a Writer on the Edge,* Ch. 5, Borges Studies Online, J. L. Borges Center for Studies & Documentation. Online: `http://www.borges.pitt.edu/bsol/bsi5.php` (as of 4/9/2015).

[5] Bernheimer, Kate*, Fairy Tale Architecture: The Library of Babel*, Places Journal. Online: `https://placesjournal.org/article/fairy-tale-architecture-the-library-of-babel` (as of 4/9/2015).

[6] Millen, Jonathan K., *Library of Babel*, Online: `http://www.jonmillen.com/babel/BabelWeb.html` using the Unity webplayer at `http://unity3d.com/webplayer`.