Programmable Mathe-Musical Boxes

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Abstract

Programmable mechanical musical instruments, such as the player piano, have a long history. The twentieth-century composer Conlon Nancarrow employed the player piano to realize compositional ideas that would have been impossible for a human pianist. More recently, inexpensive programmable music box kits have inspired several composers to create works expressing mathematical, as well as musical, ideas. I discuss compositions by Vi Hart and Karlheinz Essl and provide resources for using music box kits in the mathematics classroom.

Programmable Musical Instruments

Although mechanical musical instruments such as hydraulic organs have been in use since the time of Archimedes, the first programmable instrument was a water flute described in the ninth century *Kitab al-Hiyal* by the three Baghdadi brothers known as Banu Musa [5]. Many programmable instruments, including the water flute, make use of a cylinder that has pins protruding from its surface, as in Figure 1, left. As the cylinder is turned, the pins trip a mechanism that causes a note to sound. Such instruments are "programmable" in the sense that each cylinder stores the instructions for playing a particular musical piece. The player piano uses an innovation that allows much longer pieces to be performed: a long roll of paper is used rather than a cylinder, and a pneumatic device "reads" the paper by passing air through perforations that encode musical notes and even dynamics [5]. The invention and mass production of the player piano in the late nineteenth and early twentieth centuries brought programmable instruments into many households. It also inspired the composer Conlon Nancarrow (1912-1997) to write dozens of pieces for player piano and painstakingly hand-punch the rolls himself. In this way, he was able to write music with mathematical structures, such as irrational tempo relationships, that were too complicated for human players to reproduce [12].

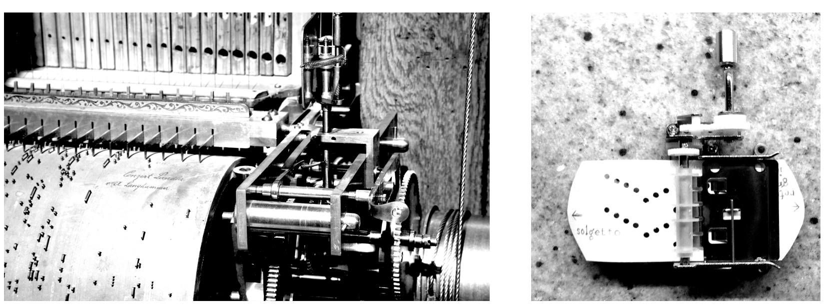

Figure 1: *Left: Cylinder and player piano mechanism, Vienna, Austria © Jorge Royan, http://www.royan.com.ar, CC-BY-SA-3.0. Right: The Kikkerland Mechanical Music Box.*

A programmable mechanical instrument, in the form of a music box, has only recently become available inexpensively. Kikkerland Design imports a small music box with a fifteen-note, two-octave diatonic scale (Figure 1, right). In addition, 20-note diatonic and 30-note chromatic models may be ordered from China. Each music box comes with a supply of paper strips and a hole punch. Turning the crank draws the paper strip through the box; any hole in the strip causes a pin to strike a small metal bar that sounds a note. The ingenious design of these little toys has even inspired an award-winning chemistry project [1].

In a world accustomed to digital technology, programmable musical instruments are hardly novel. Yet the toy music box kit has inspired several composers, including Karlheinz Essl and Vi Hart, who seem particularly attracted by the music box's potential to express mathematical relationships both visually and aurally. Essl's piece "Listen Thing" [3] has four parts, each produced by the same punched strip and corresponding to the four different ways the strip may be inserted into the music box. His "Pandora's Secret" [4] is a performance art piece for instruments, including a music box, and electronics. The music box strips are punched in geometric patterns, producing striking musical effects that are amplified by looped electronics. Hart's "Theme from the Harry Potter Septet" [8] is played by cranking the same strip twice through the music box. Unlike Essl in "Listen Thing," Hart does not take out the strip after playing, turn it, and re-insert it. Rather, after inserting the strip halfway, she twists it and tapes together the ends to form a Möbius band, so that the strip loops through the box twice to return to the starting notes of the piece. Her "Pachelbel's Music Box Canon" [9] for four music boxes is both a playful performance of Pachelbel's Canon in D (c.1680), originally scored for three violins and continuo, and a visual demonstration of what a canon actually is. Rather than using three separate strips, the performers of the violin parts crank the same strip through three music boxes simultaneously, so that each box plays a different point in the canon. A fourth box repeats the continuo part in, literally, a loop.

Classroom Exploration

With the music box kit, the relationship between seeing and hearing is unusually direct. I use music boxes in my university level, general education mathematics course "Sounding Number: Music and Mathematics from Ancient to Modern Times" because my students, many of whom have no musical training, have difficulty connecting heard music with its visual representation. Mathematical topics covered using music boxes include plane symmetry, frieze patterns, and elementary group theory (a math text for a general education course covering these subjects is [13]).

Every music box kit comes with one strip pre-punched to play "Happy Birthday." Immediately, everyone realizes that there are four ways to insert the strip into the box. Which way produces "Happy Birthday?" What about the other three ways? This leads to the musical definitions of *retrograde* (playing backwards in time), *inversion* (exchanging low and high pitches), and *retrograde inversion* (doing both of these). Students investigate the questions: Which musical transformations are produced by the rigid symmetries of the strip (horizontal or vertical reflection and 180° rotation)? Is the retrograde of the inversion the same as the inversion of the retrograde? Does the way you put the paper into the music box make a difference in every composition? The answer to the last question is no: my favorite example is Nancarrow's "Canon X" [11], which is a three-plus minute piece whose piano roll appears to have 180° rotational symmetry.

Composers may use retrograde and inversion to take a short melody and elaborate on it. Vi Hart's "Folding Space Time" [7] represents the process of symmetric transformation by literally folding or looping the paper strips. In general, a two-part canon involves two copies of the same melody playing simultaneously, with one or both copies transformed in some way. We study four of J.S. Bach's canons on the Goldberg ground (Figure 2) and discuss the use of retrograde and inversion. In canons 1 and 2, the eight-note motif is meant to be played simultaneously with its retrograde, as indicated by the backwards time signature at the end of the staff. Note that canon 2 is an inversion of canon 1. In canons 3 and 4, the second entrance of the ground, this time in inversion, happens at the *segno*. A student who has studied music theory might be able to explain why the Goldberg ground makes a good canon subject.

Figure 2: *Excerpt of J. S. Bach's "Fourteen Canons on the Goldberg Ground" (BVW 1087). http://commons.wikimedia.org/wiki/File:Verschiedene_Canones_BWV_1087.JPG*

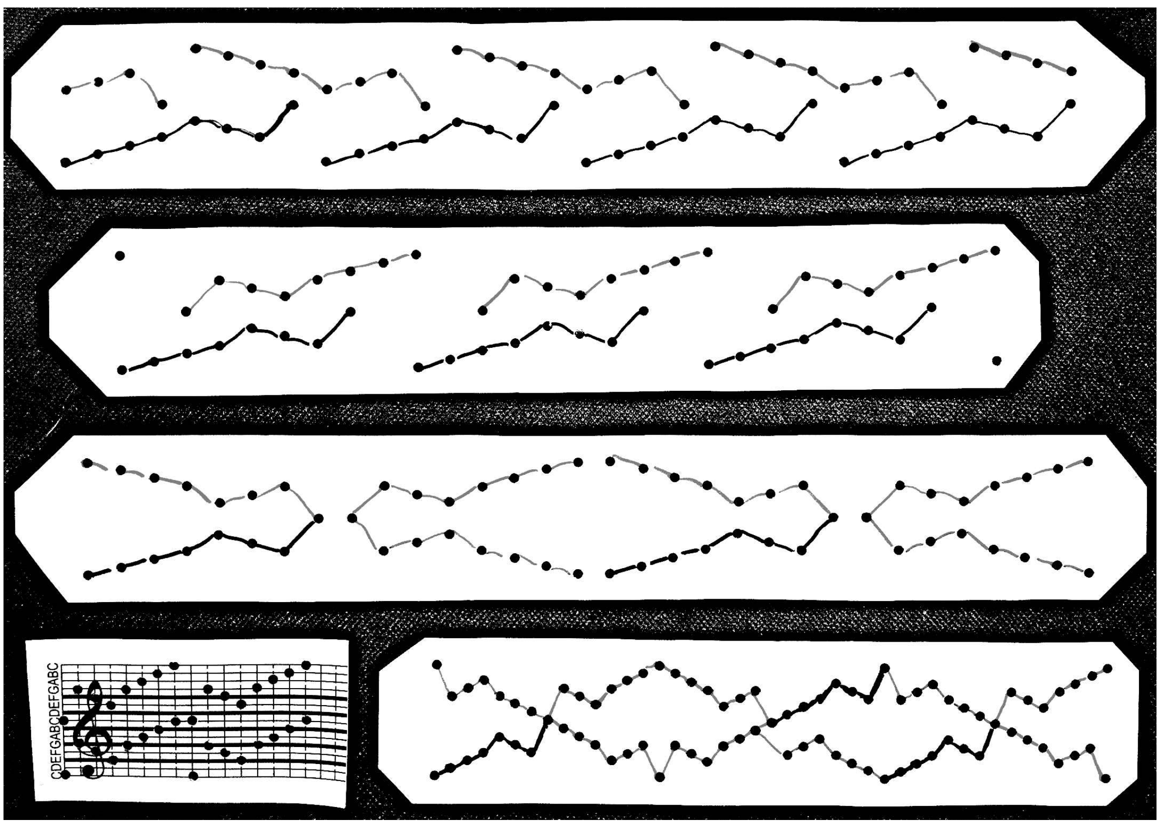

Figure 3: *The Goldberg ground strip, bottom left, and some canons based on it.*

Figure 3 illustrates some of Bach's canons on the Goldberg ground, as realized with the music box, and introduces new variations. The short strip on the lower left can be inserted into the music box in four ways, producing the original Goldberg ground, its retrograde, its inversion, and its retrograde inversion. The first strip (top of figure) can play numbers 3 and 4 of Bach's fourteen canons, if the left-hand end of the strip is inserted into the music box first. Because the punch pattern has glide reflection symmetry, the strip can only play two essentially different canons. However, Bach's canons 3 and 4 are essentially the same, modulo the order of entrances. The other three strips are my own attempts to use the Goldberg ground in canons with symmetries not represented in Bach's work. My goal is to realize the seven classes of frieze patterns as canons on the Goldberg ground.

I have written compositions myself as demonstrations, and my students do the same. Each of the two pieces in Figure 4 is formed of transformations of a single motif. The entire top strip has strict 180°

rotational symmetry, while reflections and augmentation (stretching) of the motif are also used. In the second, stacked rigid transformations of one motif create patterns that have different symmetries.

There are other topics that may be developed, such as polyrhythms, as in Figure 5. In this music box strip, inspired by Jim Bumgardner's "Whitney Music Box" [2] and Nancarrow's experiments with complex tempo relationships [12], the *n*th note from the top plays *n* times per measure, where the entire length of the piece is one measure. Classroom exercises and printable templates for music box strips are available on my course web site [6].

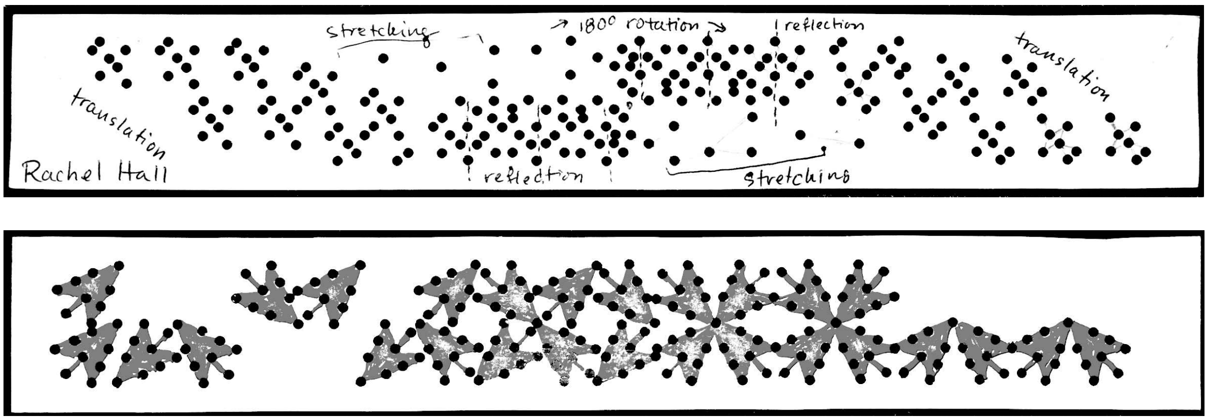

Figure 4: *Music box compositions by the author.*

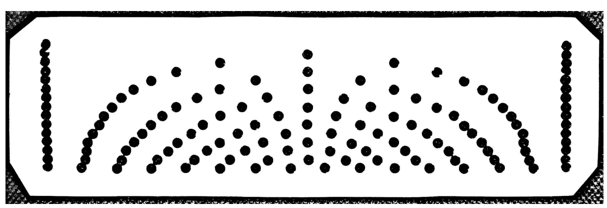

Figure 5: *A music box strip exploring polyrhythmic relationships.*

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Note: All web sites were accessed on March 13, 2015.

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