Galaxies Containing Infinite Worlds: Poetry from Finite Projective Planes

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Abstract

At first glance, poetry and the finite geometry of projective planes seem far apart. However, there is a history of overlap between combinatorial mathematics and poetry, stretching back at least to the twelfth-century sestinas of poet Arnaut Daniel. This intersection remained vibrant during the twentieth century through the works of OULIPO, and we will describe our own recent exploration of the Rubáiyát of Omar Khayyám using graph theory. After a short introduction to finite projective planes, including a brief discussion of questions of existence, we will introduce the process of composing poetry using a finite projective plane as a guide. We discuss a classroom activity we implemented in a creative writing class in which students composed poetry based on the Fano plane, and we present a plan for expanding upon this activity. Included here are several examples of such poems, including students' work, and some suggestions for anyone interested in writing poetry in this style.

Background

A certain kind of mathematician always keeps an eye out for unusual ways to make connections to other areas not generally associated with mathematics. There is a long history of interplay between mathematics and poetry, and one early example is the algebraic system of equations described in Archimedes' Cattle Problem which was originally expressed by Archimedes in verse [5]. Combinatorial mathematics has informed areas of poetry too, as early as the twelfth-century sestinas of Arnaut Daniel [9], and extending into the twentieth century with Raymond Queneau's *A Hundred Thousand Billion Poems* [8] from 1961, and other works of the OULIPO community [6].

Inspired by these approaches, we recently decided to try our hand at exploring poetry through incidence geometry and used graph theory to analyze the Rubáiyát of Omar Khayyám. Khayyám was a twelfth century Persian poet, astronomer and mathematician. He is perhaps best remembered for his poetry, which survives as a collection of four line quatrains, which are generally anthologized under the name of the Rubáiyát [2]. Recurrent themes throughout the Rubáiyát include life, death, fate, love and drink.

Broadly, graph theory is the study of the connections between objects, such as highway systems between cities or computer networks. Graph theory is concerned with whether one object is connected to another, and the actual nature of the objects themselves is not relevant. A *graph* is defined as a set of *points* and *edges* such that if two points are *adjacent* then the two-set containing them is an *edge*.

We constructed a graph on the Rubáiyát by defining two quatrains of the Rubáiyát as adjacent if they contain the same word from a list of all interesting words appearing in the Rubáiyát (a distinction which was admittedly somewhat arbitrary; for more infomation on word choice see [7]). For example, quatrains three and 54 are adjacent because they both contain the word "door", but quatrain three is not adjacent to quatrain 37, even though they both contain the word "the" because "the" has not been deemed as interesting.

We then searched this Rubáiyát graph for complete subgraphs, known as *cliques*, where each edge in the clique is defined by a different word. So we excluded cliques merely arising from several quatrains all containing the same word. This is a variant of the graph clique problem. Given these search parameters, we were able to identify all cliques in the Rubáiyát, and the largest is a clique on six quatrains. There are three different such cliques. Figure 1 illustrates one such clique, where 3(i) indicates the third quatrain from Edward FitzGerald's first translation, 38(ii) indicates the thirty-eighth quatrain from the second translation, and so forth.

We chose the Rubáiyát mainly because of the mathematical talents of its author. It is clear that Khayyám did not compose his work with this kind of analysis in mind, but reading it in this way does provide a fresh perspective which has likely not been applied to other works.

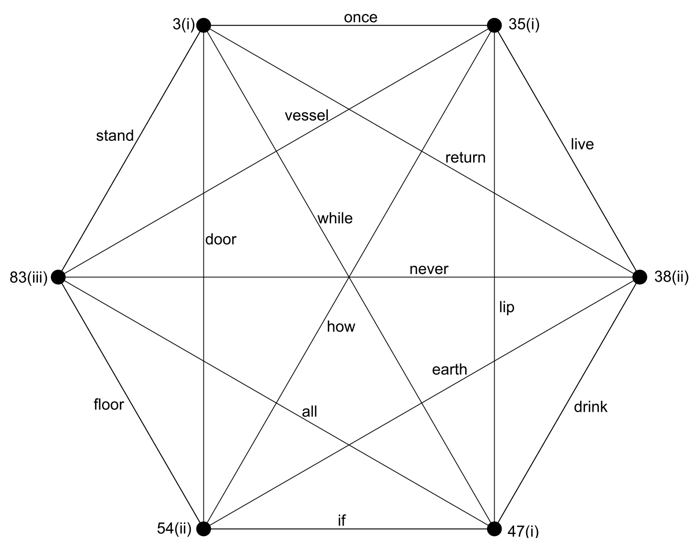

Figure 1: A complete graph within the Rubáiyát.

Projective Plane Basics

While we eventually found interesting graphs in the Rubáiyát, our initial hope had been to find a projective plane. A finite *projective plane* is an incidence structure consisting of a finite set of points, with certain subsets of the points defined to be lines [3]. Specifically, a projective plane is a collection of points and lines such that the following axioms hold :

- For any pair of distinct points, there exists precisely one line containing both points.
- For any pair of distinct lines, there exists precisely one point contained in each line.
- There exist four points such that no three of them are contained in the same line.

The smallest example of a finite projective plane is the well-known Fano plane, shown in Figure 2. It is an easy matter to verify each of the projective plane axioms on the Fano plane, where the seven lines are represented by the sides and altitudes of the triangle, along with the circle.

Some immediate consequences of the projective plane axioms are that each line must contain the same number of points, and that each point must be contained in the same number of lines. Further, these numbers are equal. It is convenient to call this number $n + 1$, and the *order* of the plane is defined to be n. A further

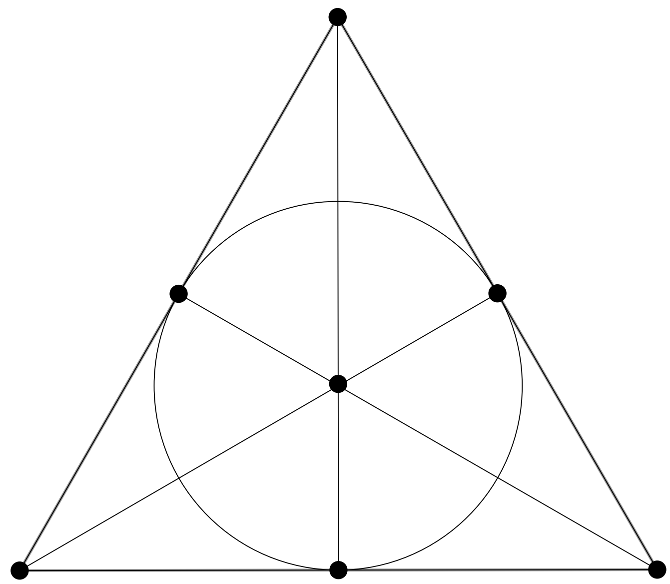

Figure 2: The unique projective plane of order two, also known as the Fano plane.

consequence of the axioms is that a finite projective plane of order n contains the same number of points and lines, which is $n^2 + n + 1$. So the Fano plane is a projective plane of order two, and is in fact the unique projective plane of order two.

There is a standard and well-known construction of projective planes for prime power orders $n = p^r$, where p is prime and r is a positive integer. This construction is based on the finite field of the given order, and in the cases of prime values of $n = p$, this construction gives the only known projective plane of order p. In other prime power orders the standard construction gives one of several known projective planes. Conversely, the only known finite projective planes have prime power order. The well-known Bruck-Ryser-Chowla Theorem [1] states that if n is congruent to 1 or 2 modulo 4, then n must be the sum of two squares in order for a projective plane of order n to exist. So no projective planes of order six or 14 exist, but the theorem does not rule out the existence of planes of order 10 or 12. A much-celebrated result, Clement Lam proved in 1989 that no projective plane of order 10 exists, and two years later he provided an engaging history of the problem and exposition on his computer search [4]. The case of $n = 12$ is the smallest in which it is unknown whether a projective plane exists, and the search for either the existence of a projective plane of non-prime power order or a general result ruling out such a plane is one of the major open questions in projective geometry [10].

Fano Plane Poems

To search for a Fano plane embedded inside the word-quatrain incidence structure of the Rubáiyát, we defined certain interesting words as points and the quatrains as lines. In this sense, a quatrain is a line containing the points corresponding to whichever interesting words it contains. Under that description, a Fano plane would be a subset of seven quatrains of the Rubáiyát along with seven words contained in those quatrains such that the projective plane axioms hold. Specifically, any two of the seven quatrains would share

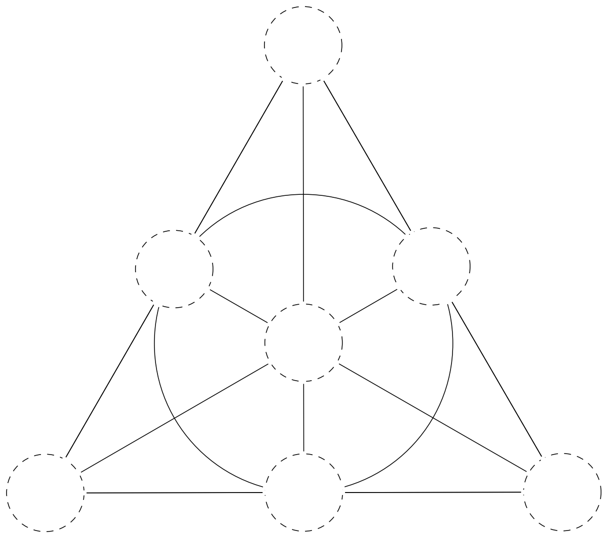

Figure 3: Fano plane poem template.

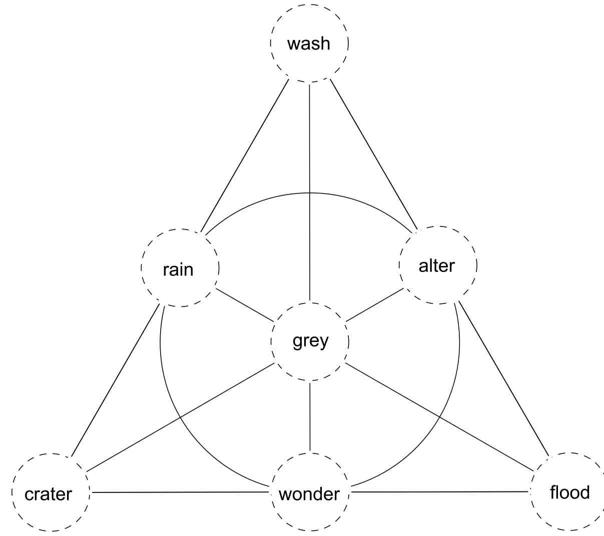

Figure 4: Filled in template for the poem ***As It Is***.

exactly one word, and any two of the seven words would both be contained in exactly one of the quatrains.

Given the list of 132 interesting words over the 114 quatrains, searching for a Fano plane in every possible subset of seven quatrains and seven words in the Rubáiyát is a computationally intractable problem. We attempted several strategies for searching over more likely subsets of the quatrains, but were never able to find a Fano plane in the Rubáiyát. We decided to abandon the search for a projective plane in the Rubaiyat, and focused on the more easily-realizable complete graphs described above.

And yet the hope of realizing a projective plane through poetry remained alive. We decided that using the Fano plane as a template for writing poetry would be fun – if we couldn't find it, we'd create it. The format of a Fano plane poem is relatively simple. The poet selects seven words to be represented as the points of the plane and composes a poem of seven stanzas, in which each stanza is represented by one line of the plane, so that the words appear in the stanzas according to the projective plane axioms. Figure 3 is useful in composing the poems, and four examples are included in this paper, two of which were written by students in our class. A filled in template for one of the student's poems is Figure 4, and the repeated words in this poem appear in bold below.

Obeying the rules of the geometry is of course much easier than writing a good poem. While some of the poets involved in this project prefered to choose the seven words first, others found that choosing precisely the seven words was not the best way to start. An alternate technique is to build a larger list of ten to fifteen words related to a theme, where the theme is chosen to be well-suited to repetition. Then, an opening stanza can be more organically built around three words from the larger list, and a closing stanza also written using three of the words, being careful that these two stanzas share exactly one of the words. At this stage, five of the required seven words will have been chosen.

When selecting words,it is important to use words that are not so obscure or specific as to be distracting or artifical in their repetition. It is also helpful to use words which have multiple meanings, can be used as different parts of speech, or even have various spelling. This helps to enliven the poem and keeps readers interested even when they knows the repetition is coming. The poet must "rise to the challenge of delivering surprise" [11], in the way Daniel Tammet describes the challenge of the sestina.

Additionally, some of us introduce further structure into each of the seven stanzas of the poem. For example, each stanza in the poem ***adore*** included here is a haiku. This added structure can help to unify the poem, but some of us have found that idea overly restrictive when taken in conjunction with the necessary word repetition. Most of us, however, agree that the projective plane restrictions can be useful in cutting down on creative noise.

As It Is
by Michelle Stampe

You blink at it, curious,
And trace the chasm under its lip and
Craters in its cheeks.
You **wonder** if they **flood** often.

It blinks at you, tense
A cat catching its shadow on a polished door
Its visage **altered** by bits of Crest toothpaste.
It follows your eyes out the window,
And you **wonder** if the **rain** will last.

You notice it caged behind the wire mesh,
Miraculously dry in the **flood** of **rainwater**,
And lean in close to study how the drops
Muddle its colors to **grey**.

Drops creating synapses between drops.

You glimpse it in puddles in the yard,
Altered and rippling in **craters**
Made from a **grey** sky and a nihilistic lawnmower.
You look up to see if nature's an honest painter,
And are disappointed by her depiction.

It stretches over the oval glass door of the **washing** machine,
A warped canvas adorned with
Green sheets and a blue flannel.
A **flood** of Downy **alters** the painting.

Soap becoming soap.

You see its dull eye in the glass of water
On your nightstand, and wish you could
Wash the film from its iris (inverted craters of rain).
You picture the wild iris in your mothers garden,
And romanticize a blossoming hillside (**craters** of inverted **rain**).

You see it when you close your eyes,
And **wonder** what it dreams about.
It sees you, and wonders if the **grey** in your eyes
Can be **washed** out with your blue flannel.

adore
by Dan May

we adore weather.
january silver sky
and leaves every spring.

you adore the leaves –
red, yellow, finely brown keys.
circles in the mud.

i adore the drone –
sound that circles forever
through my silver brain.

we adore the keys
to the dark drone overhead
which sets our weather.

i adore all beats,
anything that leaves me numb,
a drone in a hive.

you adore silver
and long gold keys on a chain,
unlocking your beats.

we adore circles.
we can weather it each time –
it never beats us.

In the Classroom

We introduced this poetic concept in an introductory creative writing class as the last assignment of the fall semester. Students ranged from freshmen to graduating seniors and only two of the twenty-three students had substantial experience with poetry, save for the previous six weeks of training. Because we wanted to study how organically these poems could be devised, we only previewed the project by telling students that the final assignment would focus on intersections between math and poetry. Their homework over the weekend was to generate a list of seven words for distinct topics within the three categories of theme, experience, and setting. For example, theme might be something like *family*, *death*, or *nature*. The experience category could contain anything from *first heartbreak* to *chronic illness* to *first day of school*. Likewise, setting could be just as diverse: the *moon*, an *institution*, or the *ocean*, for example. So students had an idea of how to craft these lists, we shared seven words for the setting of *playground* and the theme of *winter*. The *winter* list was used in the poem ***Pine For Me*** included here.

WINTER	PLAYGROUND
wing	swing
pine	run
winter	laughter
sleep	chase
bullet	recess
silence	climb
heart	tire

Overall, the students were excited for this secret project. At this point in the semester, they were experienced with writing exercises, some successful and some not, and so they were less concerned with a “right” answer (although some did send sample lists via email over the weekend) and more with the opportunity to play and be creative.

On that Monday, we delivered the introductory lecture on the Fano plane and discussed some sample poems. Students were then given twenty minutes to work with their lists and develop a draft or partial draft, with the help of Figure 3. The students immediately went to work using the allotted time while we circulated the room and answered individual questions. With the remaining class time, students read their drafts aloud and the class attempted to identify the word patterns as they listened.

This exercise was very well received by the students. We spent the following week conferencing individually with the writers as they worked on compiling their final poetry portfolios. Nearly every student had positive responses for the exercise and was either continuing to work on the in-class draft or had moved on to new Fano plane poems. Approximately half of the writers were considering this poem as a selection for the portfolio.

While we were very pleased with our first attempt, when we teach this project again we plan to dedicate at least two full periods to it and will restructure our delivery according to the plan below.

DAY 1

1. In small groups, students will analyze a sample poem for any obvious patterns and what said patterns do to the poem’s meaning and rhythm.
2. Once they identify the seven repeating words, we will input them into corresponding slots in Figure 3 as a visual of the poems structure.
3. Next, we will provide the background information on the Fano plane and then analyze a second example.

4. Once students have a solid idea of expectations, they will discuss their pre-determined lists in small groups to determine which will yield stronger poems.
5. The rest of the class time will be dedicated to drafting.

DAY 2

6. A second day will be dedicated to whole-class and small-group workshopping. Ideally, we will draft on a Friday and workshop on a Monday to give students a solid block of time to work on their piece. We will workshop a sample poem as a group, discussing its strengths and weaknesses in imagery, voice, and internal logic. Once students have a sense of how to discuss these drafts and what to look for, they will work in small groups of 3-4 to help the writer determine what is working well and what needs revision.

We found through trial and error that a poem can be written with a arbitrary collection of words, but it is very difficult to write anything good. Thus, we will encourage students to choose words that resonate for them, ones that have strong sense imagery and/or strong visual images for them. In this way, the words take on an atmospheric quality and the writer has a clear sense of tone and mood – and perhaps even a sense of the speaker – before writing.

In our conferences, students reported that they were at first apprehensive about the limits of the assignment. They were concerned that the restrictions on each stanza would be too confining and would hinder the organic development of the poem. What most reported, however, is that they felt more inspired by these rules. Though we created structural limits by using a repeating pattern, it did not limit the creativity nor life of the poem. They found it challenging to weave together the words in an organic way and to create an internal comprehension with the repetition while avoiding redundancy. The exercise required a balance between logic and creativity, a perfect intersection.

Existence
by Richard Walbe

Our planet earth shaped through wind and water,
though only by appearance,

its internal fire providing the warmth to create life,
powering earth as it is on all like worlds—

worlds carried by unseen winds
through space immeasurable,

the same wind that stokes the fires
of infinite galaxies—

galaxies containing infinite worlds, circling endless stars.
Maybe one contains water and life.

In space, life is a simple equation
of fire and water,

an equation to provide hope, that earth is not alone
in this infinite space.

Pine for Me
by Courtney Huse Wika

When the turkeys bullet from their sleep,
the pines feel their absence.

They winter their silence,
but I can feel their longing,
the pining for something that once was,

like the lost thrum of wing beats
when winter skies sleep.

My fickle heart is a red winter apple
that has known at least one bullet.

My fickle heart has slept in silence
for at least as long as

the pines have sympathized
with the fracture of grackle hearts
and the sincerity of nuthatch wings.

While the birds wing in the silence,
all I can hear is the crack of the bullet.

Future Ideas

This project started with the idea of finding a projective plane inside of existing poetry, and we have not succeeded at doing so. In general this task is computationally very difficult, but perhaps a Fano plane is waiting to be uncovered in some collection of poetry.

We could of course also use other projective planes to write poetry, and the smallest example larger than a Fano plane poem would use the projective plane of order three. In that case the poem would have thirteen stanzas with thirteen repeated words, which would present an even greater poetic challenge.

Another avenue of exploration would be to write poems using complete graphs or other classes of graph as a template. Writing on a complete graph would require that each stanza shares a word with every other stanza, which leads to even more word repetition than does a Fano plane poem. But perhaps another class of graph might serve well as a guide. Some of us have also discussed the idea of creating collections of longer writings based on incidence structures, where the connections between the pieces would be defined through broader themes or images as opposed to merely individual words.

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