Soccer Ball Symmetry

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Abstract

Among the most recognizable sport ball designs are soccer balls with twelve black pentagons and twenty white hexagons. However, soccer ball manufacturers are now exploring a wide variety of new patterns for both their panel designs and graphics. This paper surveys many existing soccer ball designs; it hopes to show how varied they have become and suggests that these designs may serve as an inspiration for other spherical art. Also, this paper promotes modern soccer balls as ideal toy examples to learn and teach various branches of spherical mathematics such as spherical symmetry, group theory, and tessellations.

Introduction

Many sports balls are assembled from flat panels of material in such a way as to achieve a desired curvature and they often have iconic panel arrangements. For instance, the panel layouts in Figure 1 can effectively be described as 'like a baseball', 'a beach ball', or 'a volleyball' respectively. And certainly when one says 'like a soccer[1] ball', the iconic soccer ball (Fig.2) with twelve black pentagons surrounded by twenty white hexagons comes to mind. Yet a look at soccer balls today will show a proliferation of different soccer ball designs, so many that it is difficult to even spot this classic soccer ball. It would seem that graphic designers and ball manufacturers are striving to outdo one another by exploring the geometric patterns that can be made on a sphere and by searching for the next unique spherical pattern. You may look ahead right now to Figure 5 to see this wide variety of designs.

Figure 1: *Iconic panel layouts of a baseball, a beach ball, and a volleyball.*

Figure 2: *The classic soccer ball design.*

This paper aims to showcase these new patterns. In order to focus on the artistic and mathematical aspects of modern soccer balls, this paper concerns itself with *visual* developments only: namely the designs of the panels, grooves, and printed graphics. Other design considerations such as how a moving ball appears to a player or how it behaves when it is kicked are not discussed.

[1] Although the sport is more often called 'football' outside of the United States, this paper uses the unambiguous (and originally British) term 'soccer'.

This paper also aims to promote soccer balls as mathematical objects for study. Just as the classic soccer ball can be used as shorthand for a ‘truncated icosahedron’ or ‘icosahedral symmetry’, modern soccer ball designs can also serve as suitable mental models for their own spherical patterns and mathematical concepts. In a workshop at Bridges 2012 [8], Yackel discussed the pedagogical benefits of temari balls as objects of mathematical investigation. Modern soccer balls can also be studied in this same vein conveniently and at little cost. For instance, a visit to a local soccer field or store with a camera in hand might be enough.

This paper is organized as follows: the next section will discuss the history of soccer ball designs and give some context as to what soccer balls have looked like in the past. Following that, we discuss various aspects of spherical mathematics, including especially a discussion of spherical symmetry that features a collection of many different designs. We finish by showing how soccer balls have a role as artistic objects as well as mathematical, and how they can serve as inspiration for some new spherical designs.

Background

It is of course impossible to describe every soccer ball development here. So what follows is an abridged history of the visual aspects of soccer balls. Determining the priority of each graphical innovation would also be a difficult task and so the discussion in this section is limited to soccer ball designs with wide exposure due to major leagues and tournaments. Interested readers may see a visual history and many more examples by visiting online resources like the one maintained by Pesti [7].

By the early 20th century, soccer games were played with balls made of leather wrapped around an inflatable rubber bladder. There was no standard panel design but you could often see panels stitched together using the kind of panel arrangement that you might expect on a volleyball (Fig.3a).

(a) *1938* (b) *1970* (c) *1978* (d) *2000* (e) *2006*

Figure 3: *Some notable designs of the last few decades.*

What many people find surprising is that the black pentagon / white hexagon design (Fig.3b) only became prevalent in the seventies, not before. Adidas used this design for the official soccer ball (called Adidas Telstar) for only two FIFA World Cup tournaments (1970 and 1974) before switching to a different graphic pattern. Despite this short tenure, the design became the iconic pattern that it is today, and the truncated icosahedron was established as the customary panel layout. It was customary but not completely ubiquitous; brands such as Mitre continued to use a volleyball-like panel arrangement.

Although the truncated icosahedron panels were standard, manufacturers started printing different graphics on these panels which deviated from solid black and white. Different graphics were used for branding purposes: to distinguish different manufacturers, or to signify balls meant for specific tournaments or leagues. For instance, Adidas used the triangular “Tango” motif (Fig.3c) on their hexagons for many years.

Soccer balls like the Nike Geo Merlin (Fig.3d) and the Adidas Fevernova (Fig.5f) appeared in 2000 and 2002 respectively. These were a departure from earlier soccer balls by featuring differing, orientable panels on the same ball. The resulting graphics exhibited different kinds of symmetry than the underlying panels. (We discuss this way of reducing symmetry more in the next section).

The latest 'era' begins with the official ball for the World Cup in 2006: the Adidas Teamgeist (Fig.3e). The Teamgeist was the first World Cup soccer ball since 1970 to feature a different *panel* design; the panel designs have been changing for World Cup soccer balls since.

Other than some practical panel design considerations such as creating a ball's curvature or affecting a soccer ball's flight, modern soccer ball designers now have a much freer rein to design panels and graphics. For example the Adidas F50 ball (Fig.5n) has eight unique panel shapes. So with new panel shape possibilities – and with them more new graphics – there has been a creativity arms race of sorts, which gives us a perfect opportunity to appreciate these designs through a mathematical lens.

Mathematical Ideas

The Truncated Icosahedron. One way to think of dividing the surface of a sphere is to start with a polyhedron and then inflate the faces to a sphere with a mathematical projection. The faces of the original polyhedron correspond to the panels of the ball. In the classic soccer ball's case, we start with a truncated icosahedron (Fig.4). Thus the geometry of the classic soccer ball is neatly described as a *spherical truncated icosahedron.*

Figure 4: *A truncated icosahedron.*

The truncated icosahedron is one of the Archimedean solids which, as the name suggests, date back to antiquity. The shape belongs to another fascinating family of polyhedra: Goldberg polyhedra, which are well explained by Hart [4]. Goldberg polyhedra have twelve pentagons surrounded by a number of hexagons and can be classified by starting at one pentagon, counting the number of hexagons out, turning 60° and then counting to the next pentagon. Thus a dodecahedron is a "1,0" Goldberg polyhedron while the truncated icosahedron is a "1,1" Goldberg polyhedron. In this paper, we will use Hart's notation by writing these as GP(1,0) or GP(1,1).

A Collection of Spherical Symmetries. We now discuss the collection that we see in Figure 5. Like any collection, the criteria for including each item can be subjective. It would be completely valid, for instance, to collect soccer balls based on which spherical polyhedron they happen to be. It would also be valid to discuss soccer balls in terms of possible molecular analogues as Fan and Jin have done [3].

As it is, our collection is classified according to different *spherical symmetry classes*. This classification is well suited to characterizing and understanding the kinds of patterns we can see on spheres and it affords us with many opportunities to discuss some interesting mathematics

For the purposes of classifying the symmetry of soccer balls, subjective criteria include matters such as whether to ignore or to pay attention to: logos that are not part of a sphere-wide pattern, minor variations in the graphics, valves, seams, and grooves. At least for Figure 5, we ignore all of these matters.

There are more choices to make for our spherical symmetry collection as there is a variety of notation we could choose from. The orbifold notation described by Conway, Burgiel, and Goodman-Strauss [1] is accessible to anyone with a surface-level (pun intended) understanding of mathematics. We will use this notation to be precise more than to elucidate. I encourage anyone who wishes to learn more to look up this text. For now, we can describe some of the basics.

A brief, but far from complete description regarding the classification of spherical symmetries follows. The first things to look for are *planes of reflections* and to identify where these planes meet (called

(a) **532* (b) **432* (c) **332* (d) *532*

(e) *432* (f) *332* (g) *3*2*

(h) **22N, (N=4)* (i) **NN* (j) *2*N, (N=3)* (k) *N*, (N=2)*

(ℓ) *N×* (m) *22N, (N=5)* (n) *NN, (N=3)*

(o) *O(3)* (p) D_∞ (q) C_∞ (r) *1*

Figure 5: *(Mostly) soccer balls showing: twelve of the fourteen finite spherical symmetries (a-n); three types of infinite symmetry (o-q); and one with no symmetry except the identity (r).*

kaleidoscopic points). The next thing to look for are the *centers of rotations* that are not kaleidoscopic points. These are points or axes of the sphere about which a pattern can be rotated onto itself. Finally we look for *glide reflections*. These are symmetries that combine both a rotation and a reflection in one step. Orbifold notation starts by listing the order of any unique rotational symmetries. Then, if there are any, it lists the order of any unique kaleidoscopic points after a * symbol. Finally, a × symbol at the end denotes that there is a glide reflection.

Kaleidoscopic points, centers of rotation, and glide reflections combine together on the sphere in a finite number of ways, fourteen in fact. We can describe each while referring to its representative spot in Figure 5. We start off with three symmetries that correspond to the icosahedron, the octahedron and the tetrahedron, each printed with a face with kaleidoscopic symmetry (Fig.5a-c). The next three are the same but with only rotationally symmetric faces (Fig.5d-f). Next, there is a special tetrahedral symmetry with both kaleidoscopic points and centers of rotation known as pyritohedral symmetry (Fig.5g). There are seven spherical symmetries that correspond to the seven frieze patterns wrapped around a sphere, (Fig.5h-n). These are the fourteen, but I have also included three more infinite symmetries which correspond to a blank sphere, a cylinder, and a cone (Fig.5o-q). Finally, we see a soccer ball with no symmetry other than the identity (Fig.5r).

Such a set of soccer balls can serve well as a visual cheat-sheet for the various types of spherical symmetry. The personality of each ball design can help with the memorization. For instance: “What did the spherical symmetry *332 look like again? Oh yes, it is the Adidas Jabulani.”

The collection is not perfect or complete. First of all, there is a non-soccer ball: a volleyball design with symmetry N* (Fig.5k). Also, you may have noticed that I have not found good candidates for spherical symmetries of the form N× or *NN (Fig.5i,ℓ). So if you see any soccer balls with these symmetry types in the wild, please let me know! In the meantime, Figure 11 shows some mock examples.

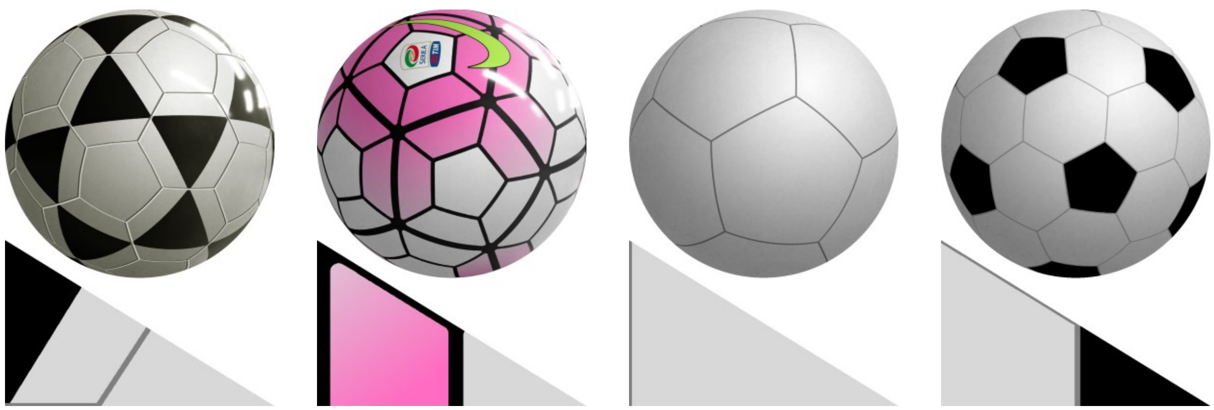
Icosahedral Soccer Balls: Manufacturers can keep their existing panel designs but still have a new look by just changing the graphics that are printed on the panels. As a consequence, many designs continue to exhibit icosahedral symmetry. To design a soccer ball with icosahedral symmetry, a designer merely has to create the graphics of the *fundamental domain*. A fundamental domain is a smallest region that can cover the entire sphere after reflecting (or rotating etc.) through any of the available symmetries.

Figure 6 shows eight different icosahedral patterns along with their corresponding fundamental domains. Let us look at these various designs with a mathematical eye. For instance, the Adidas Tango (Fig.6b) uses the classic panel design (Fig.6a) but the negative-spaces are white circles in a dodecahedron pattern. Thus the Tango could make a good model for a classroom demonstration about the connections between dodecahedrons and icosahedral symmetry.

A Puma King II (Fig.6d) shows a panel design achieved by enlarging the pentagons of a truncated icosahedron. The hexagons are no longer regular, but the seams allow for a fascinating Temari-like graphic with six great circles each touching ten pentagons. The Mikasa ball (Fig.6e) prints an equilateral triangle on each of its hexagonal panels and also displays six great circles touching ten pentagons. On the other hand, the Adidas Champions League ball (Fig.6c) features a star motif overlaid on top of each pentagon. However, here we see *ten* great circles each touching *six* stars!

The last icosahedral pattern to discuss is the black outlines of the Nike Ordem 3 after ignoring its white-to-pink gradient (Fig.6f). Although it *appears* to be made from 72 small panels, there are grooves in the panels that only look like seams. The *seam* pattern is the spherical dodecahedron (Fig.6g) also known as GP(1,0). Recall GP(1,0) denotes a “1,0” Goldberg polyhedron. Additionally on the Nike Ordem, we can see that the *groove* pattern is a GP(2,0) (Fig.6h). As you can see, this GP(2,0) design could easily be mistaken for the GP(1,1) classic soccer ball design. We can only wonder if the designers were aware of these connections. Whatever the case, from now on, taking the GP(2,0) for a soccer ball is no longer a mistake!

(a) *Classic* (b) *Adidas Tango* (c) *Adidas Champions League* (d) *Puma King II*



(e) *Mikasa* (f) *Nike Ordem 3* (g) *Dodecahedron* (h) *Goldberg 2,0*

Figure 6: *Icosahedral patterns and their fundamental domains.*

Symmetry Subgroups: Some soccer balls are able to demonstrate yet another mathematical concept: the idea of subgroups; especially how subgroups might be called out using color symmetry. This section presents a handful of specific soccer balls in order to show this ability.

In the discussions above, we have mostly looked at the printed graphics of a soccer ball. But a soccer ball can demonstrate that one class of spherical symmetry is a subgroup of another by looking at its panel design first, and then either considering or not considering the graphics. Take for example the Adidas Fevernova (Fig.5f). Its panel design has a *532 symmetry. Yet, when you take graphics into account, the symmetry type becomes 332, showing that 332 is a subgroup of *532. As another example, the Nike Ordem 2 (Fig.5m) shows that 225 is also a subgroup of *532. An important condition in both of these examples is to ensure that the panels (the group) follow at least the same symmetries as the graphics (the subgroup) – otherwise any two symmetries could be superimposed to get this result.

The Adidas Brazuca (Fig.7) adds an additional subtlety, that of color symmetry. Readers of a black and white version of this paper may want to find a colored image of one. The Brazuca's panels are identical in shape and details except they are colored differently. Ignoring color, its graphics have a 432 symmetry. If we were to consider the colors as unique from each other, we get 222 symmetry. *Color symmetry*, on the other hand is a symmetry that would allow all the panels of one color to map completely to another color, essentially permuting sets of colors. With this definition, the color symmetry of the Brazuca ball is back to 432. That is, no symmetry from the original group will break the color symmetry condition. Try it.

A new soccer ball design, the Adidas Conext 15 (Fig.8) reuses the seam pattern of the Brazuca, but has different graphics. What is different in this case is that the graphics themselves (not merely their color) lead to a 222 symmetry group. I encourage the reader to find this ball in person because photos of spheres with 222 symmetry do not show the symmetries very well and are more difficult to identify. Figure 9 shows more examples of these and I find that I am always initially stumped, then delighted when I recognize one.



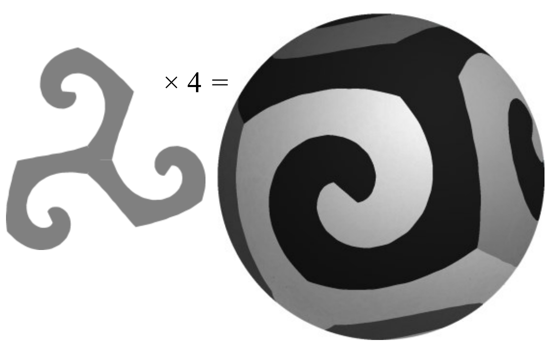
Figure 7: *Brazuca.* **Figure 8:** *Conext 15.* **Figure 9**: *More soccer balls exhibiting 222 symmetry.*

Spherical Tessellations. The Brazuca ball shows another mathematical avenue for exploration, that of tessellations on spheres. This ball can be thought of, in fact, as a modified spherical cube where each edge has been replaced by an S-shaped curve. Replacing the straight edges of polyhedra with S-shaped ones when done properly, results in better curvature. This idea is well described by Delp and Thurston [2]. There is more to be said about spherical tessellations as they relate to soccer balls, (more to be said, for that matter, about spherical mathematics in general), but for now, the mathematical discussion will have to be concluded with the suggestion that many properties of tessellations in the plane apply very well to tessellations on the sphere.

Artistic Ideas

It would be a shame to gain a better understanding of spherical mathematics and not be able to put it to some creative use. This section shows some projects that soccer ball mathematics have inspired.

To start, the most natural art project I can think of is to actually design a new soccer ball. The spherical tessellation topic above suggests an opportunity to make a tongue in cheek prediction about the next World Cup ball. Since the number of panels on World Cup soccer balls has decreased from 32 to 14 to 8 to 6, I thought a 4 panel spherical tetrahedron where each straight edge is replaced with a double-spiral could work. The result is a ball made of four triskelion shaped panels (Fig.10a). As an added bonus, the space-filling nature of the spiral edge does well to provide the needed curvature for non-stretchy materials such as paper for a nice papercraft project. Figure 11 shows two new *graphical* designs overlaid on the classic panels. They are included here as suggestions for the missing soccer ball symmetries of Figure 5.



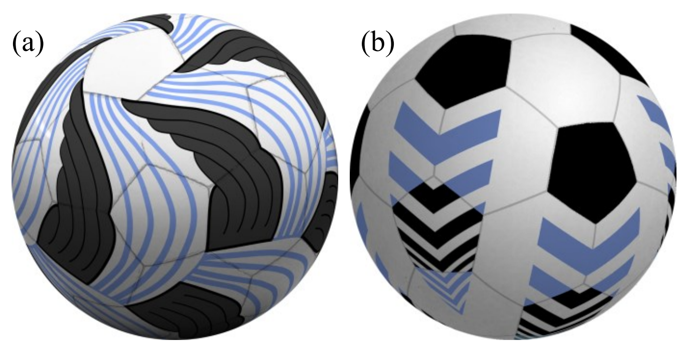


Figure 10: *Four triskelion shaped panels.* **Figure 11:** *(a) N×, (N=5); (b) *NN, (N=3).*

I have been inspired by the TSP art of Bosch and Kaplan [6] enough to create my own spherical version using the classic soccer ball pattern as a base image (Fig.12). I delight in thinking that every time the travelling salesman visits one of the twelve pentagonal islands, he is faced with a Bridges-of-Königsberg-like problem.

Many of the topics raised in this paper can also be explored on Temari balls. I have not seen any Temari balls directly inspired by a non-classic soccer ball yet, but I believe this is an idea that is well worth exploring in the future.

I finish by mentioning an opportunity to have some fun similar to that of a workshop by Hart [5]. Choose a soccer ball with a symmetry that you like, and mark out the spherical symmetry's fundamental domain. Then draw some graphics or put some imagery on it (virtually or not). If you can find the means, repeat the pattern according to the soccer ball's symmetries. Doing so will showcase some beautiful, orderly repetitions – in much the same way paper snowflakes do. I've written a program to duplicate a drawing by Evan Swart of a monster named Miro onto a virtual soccer ball using *532 symmetry (Fig.13).

Figure 12: *Spherical TSP Art.*

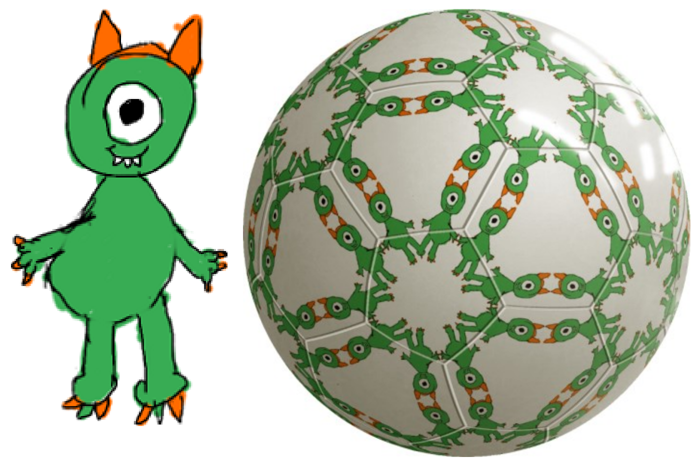


Figure 13: *"Sphero Miro" by Evan Swart.*

Concluding Remarks

While this paper has not revealed any new mathematics, I hope to have conveyed to the reader that soccer balls are ideally suited for many kinds of investigations and studies regarding spherical mathematics.

Some figures were cropped from photos by Warren Rohner (Fig.5d,f) and Christos Vittoratos (Fig.3a) and used under a Creative Commons license (CC By-SA 2.0). Digital illustrations were used instead of photos when image rights for existing designs were not available (Fig.5k,6e,f).

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