Geometric Constructions of Korean *Danchong* Patterns and Building Platonic Solids

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Abstract

Danchong is drawn patterns or paintings on wooden buildings to protect the structures from being weathered and decorate them. Five basic colors of Wu Xing and compound colors of them are used, and patterns are drawn under certain rules. It is found from most Korean palaces and temples. The purpose of this workshop is to understand *danchong*, try to regenerate some of its patterns using geometric constructions, and finally apply generated patterns to cover faces of Platonic solids.

1. Introduction

When a wooden building of a palace or a temple was built, the final step to finish the construction was applying *danchong* on its wooden surface such as ceilings, rafters, and columns. The purpose of *danchong* was to protect the structure from being weathered, hide coarse wooden surface, and decorate them by drawing meaningful patterns and objects.

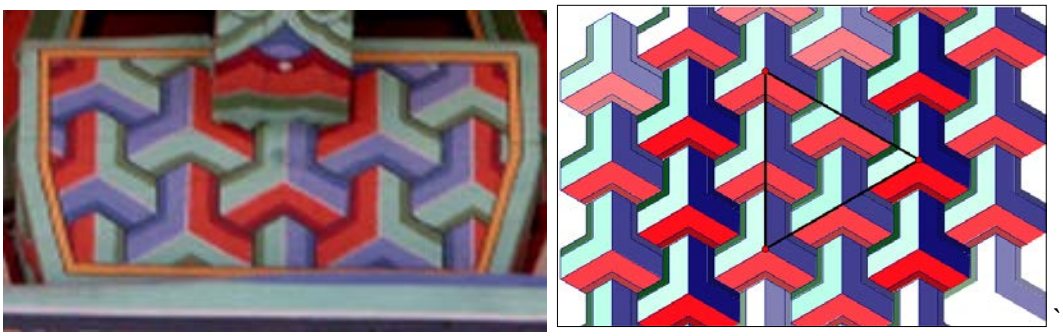
They used five colors of Wu Xing (blue, red, yellow, white, and black) and mixed colors of them. We can easily find beautiful *danchong* from structures of prestigious palaces or temples. For each part of the building there are different rules to create and apply paintings, patterns and their repetitions [1]. Some of them are based on very interesting mathematical pattern and tessellation structures [3]. However since they were drawn manually by an experienced artisan called a *Dochaejang*, it is not always possible to regenerate the pattern using geometric construction.

In this workshop, we will try to regenerate some *danchong* patterns using a geometric construction, and apply them on faces of Platonic solids to experience the encounter of Korean tradition and mathematical beauty.

2. Triangular Danchong Pattern (Silk Pattern I)

Figure1: *Danchong from Chungdo WoonMoonSa Temple*

Danchong pattern is divided in many different ways, such as by its location and by its theme [3]. One of which is called silk pattern and it is normally located in the middle of a pillar or a ceiling as a base pattern beneath any drawings. Because of its geometric beauty and various colors, it is named as silk pattern. We will focus on one of these silk patterns in Figure 1.



(a) (b)

Figure 2: (a) *Example of Silk Pattern from WoonMoonSa, Kyungsang province, Korea,* (b) *its construction using tessellation.*

Figure 3 illustrates steps taken to construct this silk pattern. It is rotated 30° clockwise around the center of the shape to find the unit as an equilateral triangle. First, by putting three hexagons together we can get the shape. And this unit can tessellate plane. During the workshop we will use this unit to generate a face of an octahedron as shown in Figure 4.

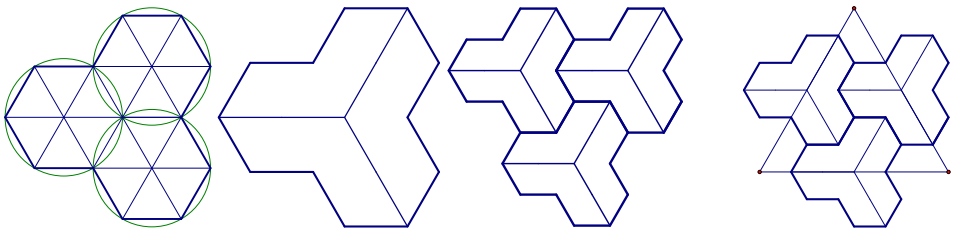


Figure 3

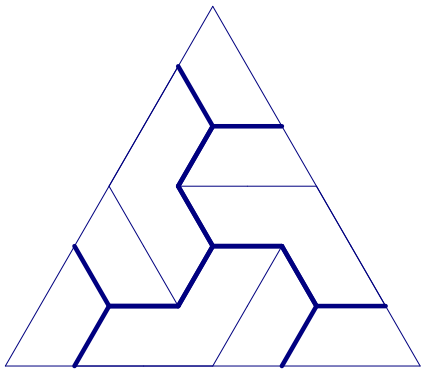


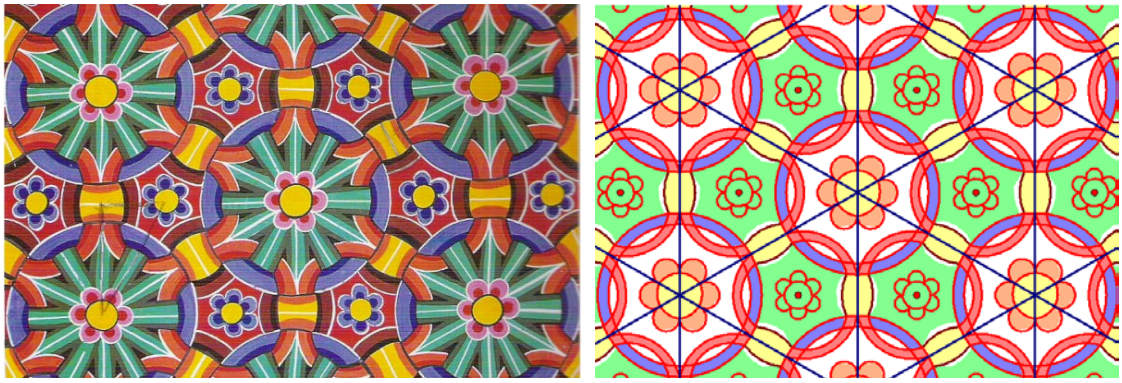
Figure 4

3. Water Wheel Pattern (Silk Pattern II)

Another silk pattern that is often found from Korean palaces and temples is water wheel pattern. It also has a geometric pattern and uses circles, hexagons or triangular shapes and due to its geometric beauty and pretty colors, it is classified as silk pattern (Figure 5).

Figure 5: *Gogyeonsa Temple, Gyeongsang Province, Korea.*

There are many variations of water wheel patterns and also there are similar patterns are called chain patterns. In this paper, we focused on one of those water wheel patterns. Figure 6 shows a popular water wheel pattern, which is applied to many designs of modern products and structures.

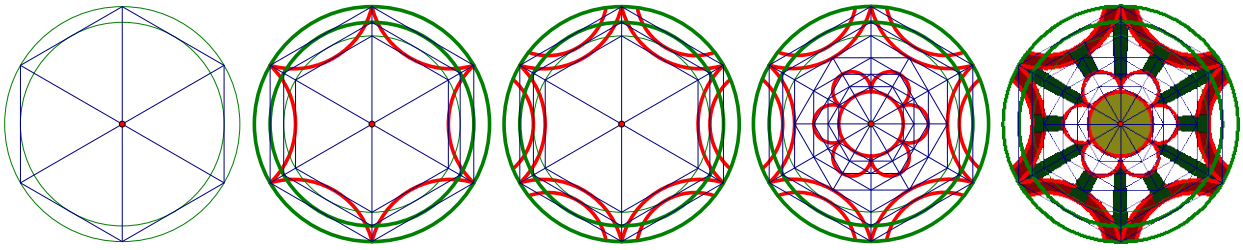


(a) (b)

Figure 6: (a) *Water wheel pattern on the wall of Korean temples and* (b) *its geometric construction*

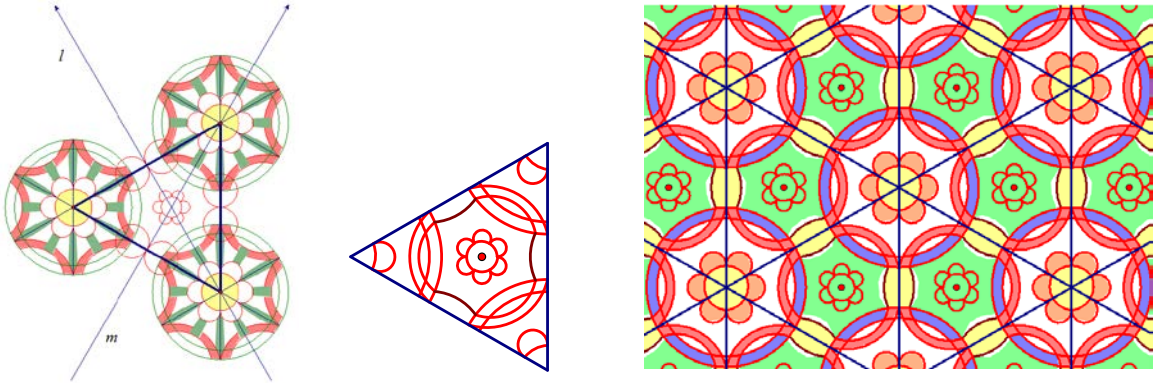
Figure 7 illustrates the steps taken to construct the water wheel pattern. First draw a circle and construct an inscribed hexagon inside it. After that, draw a smaller circle so that the circle is inscribed in the hexagon (Figure 7a). Repeat these steps two more times and using the third inscribed hexagon, find the midpoint of each side. Using *Geometer's Sketchpad,* you can draw an arc if three points are chosen. Three points, the vertices of the largest hexagon and the midpoint of a side of the smallest one, draw six arcs (shown in red, Figure 7b). The smaller arcs can be drawn to have the same thickness by choosing three points including midpoint of the second hexagon (Figure 7c). Then by connecting the midpoints of a side of hexagons, draw smaller hexagons repeatedly and draw a flower with six petals in the middle (Figure 7d). Last picture shows the completed water wheel pattern (Figure 7e).

In Figure 8, three water wheels are constructed to create the unit of tessellation for this silk pattern.



(a) (b) (c) (d) (e)

Figure 7: *Construction of water wheel using hexagons and circles.*



(a) (b) (c)

Figure 8: (a) *Reflection over a line to create two more identical shapes*, (b) *a unit of tessellation and* (c) *its tessellation*

Two more water wheels are created using the reflection over a line with some gaps in between in Figure 8(a). Since the reflection is done using two lines (line *l* and *m*) which form 120°, the centers of three water wheels form an equilateral triangle and a flower is drawn in the middle of this triangle (Figure 8b). This equilateral triangle serves as a unit of tessellation (Figure 8c).

4. Buricho

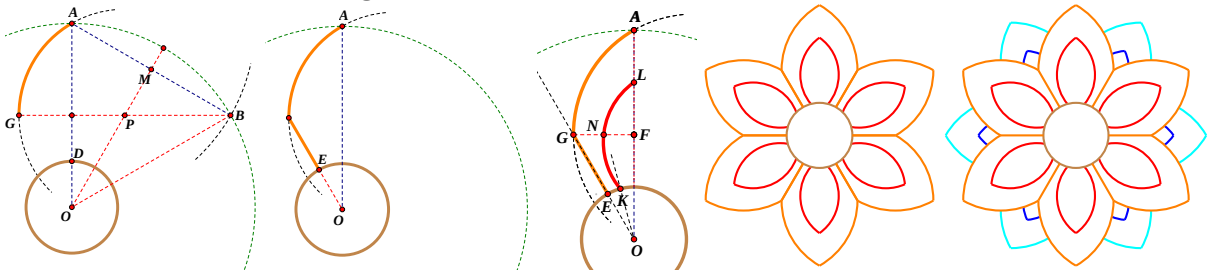
At the end of a rafter, a single pattern called *buricho* is applied. The *Gyeongbokgung* is the main royal palace of Joseon Dynasty, and *Geunjeongjeon* is the throne hall in it, where the king granted audiences to his officials, presided over large official functions and met foreign envoys [4]. Figure 9 shows the rafters with lotus shape buricho of *Geunjeongjeon*. The same pattern can be found from all five royal palaces in Seoul including *Changdeokgung*, *Changgyunggung*, *Deoksugung*, and *Gyeonghuigung*. The pattern with six petals is the most popular, though sometimes patterns with five or eight petals are found [2] [5] [6].

In Figure 10, a detailed picture of a six petal lotus *buricho* and an outcome of the geometric construction are shown side by side. Figure 11 shows construction steps of a lotus *buricho* with six petals. We tried to create a simple and precise construction method which can generate the basic pattern of *buricho*. Figure 11(a) shows the first step of the construction to draw an arc of a bigger petal. ΔOAB is a equilateral triangle, and P is the centroid of the triangle. The line $\overline{BG}$ is the extension of one of medians. The arc AG is drawn using P as its center. Figure 11(b) shows that G and O are simply connected until the boundary of the inner circle. The inner circle is one quarter of the outer circle. Figure 11(c) illustrates how to draw an arc of the inner petal. F is the midpoint of $\overline{AO}$. L and N are the midpoints of $\overline{AF}$ and $\overline{GF}$. K is the midpoint of the arc $\overline{EH}$. The inner petal arc LNK is decided by these three points. Using Geometer's Sketchpad the arc can be constructed easily [10].

Since it has six petals, the result of steps in (a)-(c) is reflected to create one petal, and repeated six times with 60° rotation to get (d). The radius of small circle is a quarter of the radius of the big circle. The petals behind can be created the same way with 30° rotation.

Figure 9: *Lotus Buricho On rafters of Geunjeongjeon Gyeongbokgung*

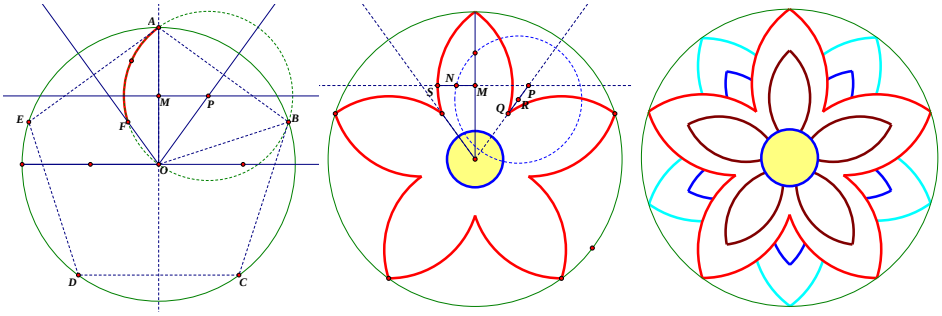
Figure 10: *Lotus Buricho and its construction*



(a) (b) (c) (d) (e)

Figure 11: *Lotus Buricho with six petals and its construction*

Figure 12 illustrates how to create a lotus *buricho* with 5 petals. The outer petal is created similarly to the case of six petals. Figure (b) shows that the center of an arc for the smaller petal, R is the midpoint of P and Q. Using the point as the center, the arc is a part of circle which passes the point N which is the midpoint of M and S. The reflection and five rotations should be done to generate the final result. In Figure 13, the final result of the geometric construction is shown with a detailed picture of a five petal *buricho*.



(a) (b) (c)

Figure 12: *Step-by-step construction of five petal lotus buricho*

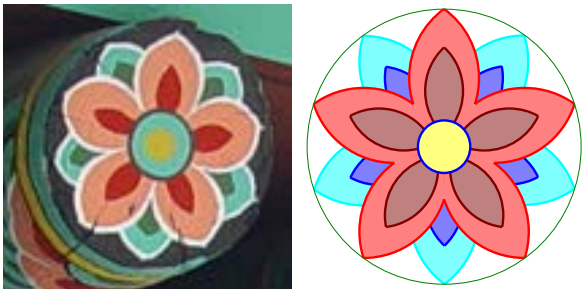


Figure 13: *Lotus Buricho with five petals and its construction*

5. Mahn (卍) Pattern

This pattern is called *Mahn* (卍**)** pattern and it is normally used for the background of main pictures such as flowers or Chinese charaters in *Danchong*. Also it is often found from the surface of pearl decorated wardrobes or the wall of royal palaces such as Gyungbokgung (Figure 14). The character *mahn* (卍) is the symbol of Buddhism in Korea but it is also known as swastika in Sanskrit which means well-being. Though swastika is a mirror image of *Mahn*, they have same geometric properties. Geometrically, they are both irregualar icosahedron and have 90 degree rotational symmetry. In Korean culture, *Mahn* character symbolizes eternity and continuity because it is endlessly connected using lines, and people wished the long life for the king by applying them on the wall of royal palace.

Figure 16 shows the step-by step construction of fundamental region of *mahn* pattern. It is a right triangle, which is one eighth of the square. Using this fundamental region, reflect over the hypotenuse followed by 90 degree counterclockwise rotation will complete the unit design. This unit will be used for making a cube during the workshop.

(a) (b) (c) (d)

Figure 14: *(a) Mahn pattern used for a background of flowers and (b)the wall of Gyotaejeon of Gyeonbokgung (c) Mahn character (d) swastika (Sanskit)*

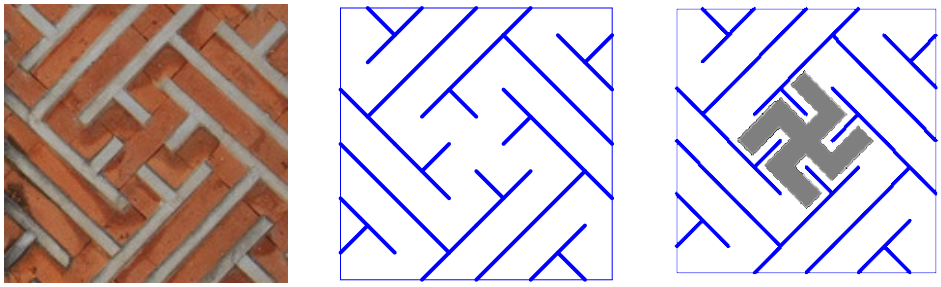


Figure 15: *Detailed picture of mahn pattern on the wall of Gyotaejeon*

We want to use a part of the wall design from Gyungbokgung to cover a cube. Figure 15 shows the square unit that will be used for this purpose. You can see the Mahn pattern in the middle of this unit.

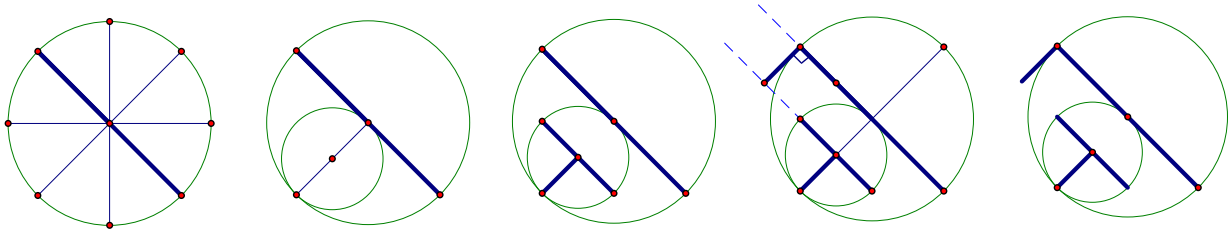
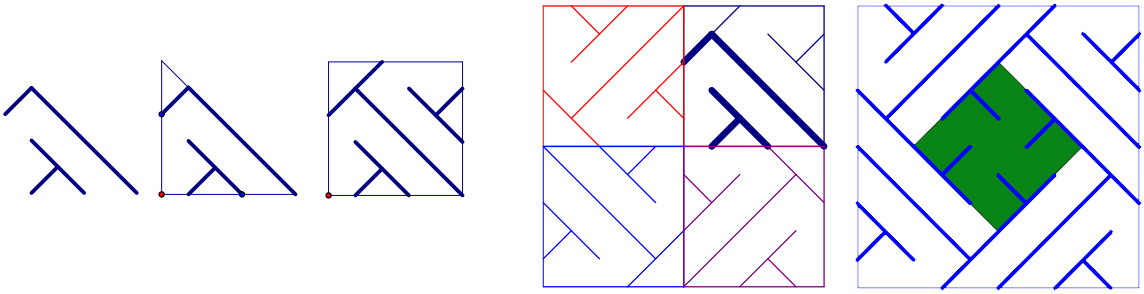


Figure 16: *Step-by-step construction of a fundamental region*

Figure 16 shows the step-by step construction of fundamental region of the *mahn* pattern. Start with a diameter of a circle as a longest segment, draw a smaller circle with half the radius, the diameter of this circle that is parallel to the original diameter becomes the second segment. Then the shortest length of line segment becomes a quarter of the diameter.



(a) (b) (c)

Figure 17: (a) *reflection over a hypotenuse* (b) counterclockwise rotation *and* (c) *a unit design of the mahn pattern to be used for covering a cube*

The fundamental region is found as a right triangle, which is one eighth of the square shown in Figure 17(b). In Figure 17(a) and (b), this fundamental region is reflected over the hypotenuse of the right triangle and 90 degree counterclockwise rotation completes the unit design (Figure 7c). This unit will be used for making a cube during the workshop.

6. Applying Patterns to Platonic Solids

In our workshop, details of *danchong* construction will be introduced. Participants will create patterns with prepared worksheets. One of good ways to see the beauty of *danchong* is to apply them onto mathematical figures. So, as in Figure 18, nets of five Platonic solids with faces filled with patterns discussed in this paper will be provided. After cutting the nets and attaching edges, final outcomes that participants will have are shown in Figure 19.

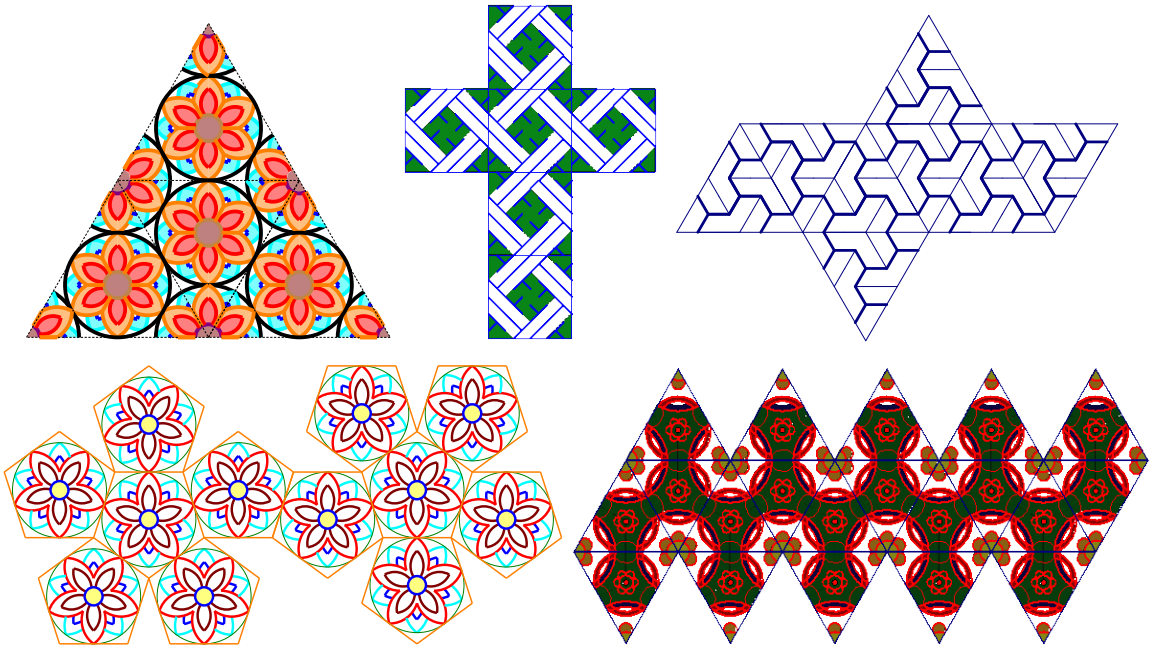


Figure 18: *Nets to build Platonic solids of danchong pattern faces*

7. Conclusion

A geometric construction of traditional patterns is a popular topic of mathematics classroom, and in past Bridges conferences there have been papers and workshops about the geometric constructions of Islamic patterns [6] [7] [8] [9]. However there was not any study about how to construct Korean traditional patterns, *danchong* before. In this paper and workshop we introduced geometric constructions of some typical *danchong* patterns, and provide hands-on practices to see the beauty of those patterns by building Platonic solids with faces of constructed patterns.

Figure 19: *Platonic solids built using nets of Figure 18*

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